The Scattering of Photons by Nucleons and Nuclear Isobars

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The scattering cross section for photons against nucleons is computed, taking into account absorption and the excited states of the nucleons (isobars). The excited states of the nucleons are described by a relativistic equation for particles of spin and isotopic spin 3/2. The coupling constants are taken from Refs. 1 and 2 where they were obtained by comparison of experiments on the scattering and photoproduction of mesons on protons. Comparison of the results of this calculation with experimental data on the scattering of photons by protons would provide and additional check on the admissibility of the isobaric representation and on the correctness of the values chosen for the constants.

1. INTRODUCTION

A SUFFICIENTLY strong interaction between nucleons and mesons produces around the nucleons a charged cloud of virtual mesons. The presence of such a charged cloud lends to the nucleons an anomalous magnetic moment $\gamma e\hbar/2mc$ instead of the normal magnetic moment $e\hbar/2mc$ predicted by the Dirac theory for spin 1/2 particles. Therefore, the scattering of photons by nucleons will not be described by the Klein-Nishina formula. The anomalous magnetic moment can be described phenomenologically by adding to the nucleonic Lagrangian the expression

$$\tfrac{1}{2}(\gamma - 1)\,(e\hbar/2mc)\,\bar{u}\gamma_\mu\gamma_\nu F_{\mu\nu}u, \qquad (1.1)$$

proposed by Pauli[3]. Computations for the scattering of photons by protons carried out by Batdorf and Thomas and by Powell[4] making use of Pauli's expression, yield a scattering cross section 25 percent greater than that obtained from the Klein-Nishina formula for 100 mev γ-rays. It is clear, however, that the virtual meson cloud is deformed in proportion to the energy of the photon, with the largest deformation expected when the frequency of the photon is $\mu c^2/\hbar$. The value of the anomalous magnetic moment therefore depends on the energy of the photon. It should also be noted that an analysis of meson photoproduction, while taking account of the anomalous magnetic moment as suggested by Pauli[5], does not bring about agreement with experiment. All this makes it appear quite doubtful that an analysis of photon-proton scattering according to Eq. (1.1) would give the correct result.

Sachs and Foldy[6] have analyzed more recently the scattering of photons by nucleons by taking into account the presence of virtual mesons and their interactions with photons. Carrying out the computations in the weak interaction approximation and neglecting nucleonic recoil, they obtain the cross section for scattered photons, for the pseudoscalar meson theory, in the form of a resonance curve with maximum at $E_\gamma = \mu c^2$. However, an analogous though more exact calculation of the anomalous magnetic moment of a nucleon[7], including nucleonic recoil, is well known to yield only the correct sign and not the correct magnitudes of the proton and neutron magnetic moments. Thus Sachs' and Foldy's computations will apparently not agree with experiment.

In the present article, the scattering of light by nucleons will be considered in the spirit of a semi-phenomenological theory[1] according to which nucleons can be found not only in the ground state with spin and isotopic spin 1/2 and mass m, but also in an excited state with spin and isotopic spin 3/2 and mass M. In this theory the cloud of virtual mesons around the nucleon is phenomenologically described by an isobaric state since the nucleon can go into this state by interacting with virtual mesons as well as real mesons and electromagnetic fields. Analyses of the processes of scattering[1] and photoproduction[2] of mesons by nucleons which take into account the excited state of the nucleon have yielded satisfactory agreement with experiment. In the process of meson photoproduction the fundamental interaction is between nucleons and photons as well as mesons. The agreement between experimental and computational results on meson photoproduction permits one to hope for a similar agreement with experiment in the analysis of the scattering of photons against nucleons by including the excited state of the nucleons; here the main nucleonic interaction takes place with photons while interaction with mesons occurs only during absorption. An analysis of the scattering of photons against nucleons which takes into account the isobaric state was carried out by Minami[8]. However, in computing the absorption of the incident wave, he approximates it by the resonance term of the transition matrix element, and this substantially changes the cross section at high energies. It is therefore worthwhile to compute the scattering of photons by the same method as was used for calculating the scattering[1] and photoproduction[2] of mesons. We start from Heitler's integral equation which relates the amplitude of the scattered wave with the transition matrix elements and takes account of absorption more consistently than Ref. 8.

2. LAGRANGIAN FOR THE INTERACTION BETWEEN NUCLEONS AND PHOTONS

We shall use the same symbols to designate Lagrangians and matrices as Ref. 2. Referring to it for details, we only give the Lagrangian for the interaction between nucleons and photons:

$$\mathscr{L}_{\text{int}} = \mathscr{L}_{\text{n}\Phi} + \mathscr{L}_{\text{in}\Phi}, \quad \mathscr{L}_{\text{n}\Phi} = -e\bar{\psi}\tau_p \hat{A}\psi; \tag{2.1}$$

$$\mathscr{L}_{\text{in}\Phi} = i\frac{e}{m}\bar{B}_\mu N F_{\mu\nu}\gamma_\nu\gamma_5\psi - i\frac{e}{m}\bar{\psi} N F_{\mu\nu}\gamma_\nu\gamma_5 B_\mu$$

$$= i\frac{e}{m}\{a^+\bar{B}_\mu^{1/2} F_{\mu\nu}\gamma_\nu\gamma_5\psi_p + b^+\bar{B}_\mu^{-1/2} F_{\mu\nu}\gamma_\nu\gamma_5\psi_n$$

$$+ a\bar{\psi}_p F_{\mu\nu}\gamma_5\gamma_\nu B_\mu^{1/2} + b\bar{\psi}_n F_{\mu\nu}\gamma_5\gamma_\nu B_\mu^{-1/2}\},$$

$$F_{\mu\nu} = \frac{\partial}{\partial x_\mu} A_\nu - \frac{\partial}{\partial x_\nu} A_\mu,$$

$$A_\mu = e_\mu \sum_k \sqrt{\frac{2\pi}{k}}\left(ce^{-i(kx)} + c^+ e^{i(kx)}\right).$$

3. SCATTERING MATRIX ELEMENTS

The matrix elements for the transition of the system from an initial state [nucleon and photon 4-momentum p_1 (E_1, $\mathbf{p}_1$), k_1 (k_1, $\mathbf{k}_1$)] to a final state [nucleon and photon 4-momentum p_2(E_2,$\mathbf{p}_2$), k_2 (k_2, $\mathbf{k}_2$)] are computed by the method of Feynman. Only second order matrix elements are computed. All possible matrix elements allowed by the interaction Lagrangian (2.1) are then given by four Feynman diagrams (Fig. 1). Roman numerals next to each vertex on the diagrams denote the respective interaction operators. These operators have the following forms in momentum representation:

I. Operator for transition of nucleon to isobar with absorption or emission of a photon

$$I_\mu(k) = -\frac{e}{m} N \sqrt{\frac{2\pi}{k}}\left(ce^{-i(kx)}\right.$$

$$\left.- c^+ e^{i(kx)}\right)\left[e_\mu\hat{k} - k_\mu\hat{e} - \frac{1}{4}\gamma_\mu(\hat{e}\hat{k} - \hat{k}\hat{e})\right]\gamma_5.$$

II. Operator for transition of isobar to nucleon with absorption or emission of a photon

$$II_\mu(k) = -\frac{e}{m} N \sqrt{\frac{2\pi}{k}}\left(ce^{-i(kx)} - c^+ e^{i(kx)}\right)$$

$$\times (e_\mu\hat{k} - k_\mu\hat{e})\gamma_5.$$

III. Operator for transition of nucleon to nucleon with absorption or emission of a photon

$$III(k) = e\sqrt{\frac{2\pi}{k}}\left(ce^{-i(kx)} + c^+ e^{i(kx)}\right)\tau_p\hat{e}.$$

The inner lines on the diagrams correspond to propagators. The propagators for isobars[9] and nucleons with momentum $\hat{p}$ are given in momentum space by

$$K_{\mu\nu}(p)$$

$$= (\hat{p} - M)^{-1}\left(\delta_{\mu\nu} - \frac{\gamma_\mu\hat{p}\,p_\nu}{6M^2} - \frac{p_\mu p_\nu}{3M^2} - \frac{\gamma_\mu p_\nu}{2M}\right);$$

$$(\hat{p} - m)^{-1}.$$

The matrix elements corresponding to the diagrams in Fig. 1 are then:

$$\begin{aligned} &\bar{\psi}_2 II_\upsilon(k_2) K_{\upsilon\nu}(p_3) I_\nu(k_1)\psi_1; \\ &\bar{\psi}_2 II_\mu(k_1) K_{\mu\nu}(p_4) I_\nu(k_2)\psi_1; \\ &\bar{\psi}_2 III(k_2)(\hat{p}_3 - m)^{-1} III(k_1)\psi_1; \\ &\bar{\psi}_2 III(k_1)(\hat{p}_4 - m)^{-1} III(k_2)\psi_1; \end{aligned} \tag{3.1}$$

the only terms remaining in the operators I_μ, II_μ, III are those corresponding to the absorption of a photon k_1 and the emission of a photon k_2 and $p_3 = p_1 + k_1 = p_2 + k_2$, $p_4 = p_1 - k_2 = p_2 - k_1$.

The transition matrix W for the process $\gamma + N \to \gamma' + N'$ may be written by means of (3.1) in the form

$$W = \bar{\chi}_2 NN\chi_1(A + B) + \bar{\chi}_2\tau_p\tau_p\chi_1(C + D), \tag{3.2}$$

if ψ_1 and ψ_2 are written as $\psi = \chi u$, the product of an isotopic spin function and a mechanical spin and coordinates function u. The matrix elements for the two possible scattering processes of photons against nucleons are:

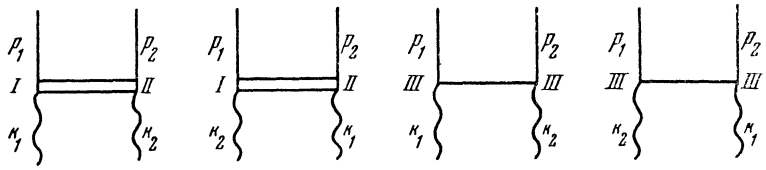

FIG. 1. Feynman diagrams of second order matrix elements for the process $\gamma + N \to \gamma' + N'$; double lines, isobars; single lines, nucleons; wavy lines, photons.

Process	$\bar{\chi}_2 N N \chi_1$	$\bar{\chi}_2 \tau_p \tau_p \chi_1$
$\gamma + p \to \gamma + p$	$\vert a \vert^2$	1
$\gamma + n \to \gamma + n$	$\vert b \vert^2$	0

The matrix elements

$$A = (e/m)^2\, 2\pi\, (k_1k_2)^{-1/2}\, \bar{u}_2\gamma_5\, (e_{2\mu}\hat{k}_2 - k_{2\mu}\hat{e}_2)\, K_{\mu\nu}(p_3) \left(e_{1\nu}\hat{k}_1 - k_{1\nu}\hat{e}_1 - \frac{1}{2}\gamma_\nu \hat{e}_1\hat{k}_1\right)\gamma_5 u_1;$$

$$B = (e/m)^2\, 2\pi\, (k_1k_2)^{-1/2}\, \bar{u}_2\gamma_5\, (e_{1\mu}\hat{k}_1 - k_{1\mu}\hat{e}_1)\, K_{\mu\nu}(p_4)\, (e_{2\nu}\hat{k}_2 - k_{2\nu}\hat{e}_2 - \frac{1}{2}\gamma_\nu \hat{e}_2\hat{k}_2)\,\gamma_5 u_1;$$

$$C = e^2 2\pi\, (k_1k_2)^{-1/2}\, \bar{u}_2\hat{e}_2\, (\hat{p}_2 - m)^{-1}\, \hat{e}_1 u_1; \qquad (3.3)$$

$$D = e^2 2\pi\, (k_1k_2)^{-1/2}\, \bar{u}_2\hat{e}_1\, (\hat{p}_4 - m)^{-1}\, \hat{e}_2 u_1$$

in the center-of-mass system where $\mathbf{p}_1 + \mathbf{k}_1 = \mathbf{p}_2 + \mathbf{k}_2 = 0$ (so that $k_1 = k_2 = k$ and the total energy of the system is $E_1 + k_1 = E_2 + k_2 = W$), we have the form

$$A = \left(\frac{e}{m}\right)^2 Q \sum_{i,\,k=0}^{5} a_{i,\,k} L_{i,\,k}^{\Phi'\Phi}; \quad C = e^2 Q \sum_{i,k=0}^{1} c_{i,\,k} L_{i,\,k}^{\Phi'\Phi};$$

$$B = -\left(\frac{e}{m}\right)^2 \frac{Q}{M^2 - m^2 + 2\,(k_2p_1)} \sum_{i,\,k=0}^{5} b_{i,\,k} L_{i,\,k}^{\Phi'\Phi} =$$

$$= -\left(\frac{e}{m}\right)^2 \frac{Q}{M^2 - m^2 + 2k\,(W-k)} \sum_{i,\,k=0}^{i,\,k=\infty} \beta_{i,\,k} L_{i,\,k}^{\Phi'\Phi}; \qquad (3.4)$$

$$D = -e^2 \frac{Q}{2\,(k_2p_1)} \sum_{i,\,k=0}^{5} d_{i,\,k} L_{i,\,k}^{\Phi'\Phi} = -e^2 \frac{1}{2}\, \frac{Q}{k\,(W-k)} \sum_{i,\,k=0}^{\infty} \delta_{i,\,k} L_{i,\,k}^{\Phi'\Phi},$$

$$a_{0,0} = \frac{k^2m^2(m+W)^2(W+2M)}{6M^2W^2}; \quad a_{1,1} = \frac{k^2m^2(m+W)^2(W-2M)}{6M^2W^2};$$

$$a_{2,2} = \frac{k^2(m+W)^2(m+3W)^2}{48W^2(W-M)};$$

$$a_{2,3} = a_{3,2} = \frac{k^3(m+W)(m+3W)}{8\sqrt{3}W(W-M)}; \quad a_{3,3} = \frac{k^4}{4(W-M)}; \quad a_{4,4} = \frac{k^2(m+W)^2(m+3W)^2}{48W^2(W+M)};$$

$$a_{4,5} = a_{5,4} = \frac{k^3(m+W)(m+3W)}{8\sqrt{3}W(W+M)}; \quad a_{5,5} = \frac{k^4}{4(W+M)};$$

$$b_{0,0} = [k^2(m+W)^2/18M^2W^2](9M^2W^3 - 2m^2MW^2 - m^4W - 6m^2M^3);$$

$$b_{1,1} = [k^2(m+W)^2/18M^2W^2](9M^2W^3 + 2m^2MW^2 - m^4W + 6m^2M^3);$$

$$b_{2,2} = [k^2(m+W)^2/144M^2W^2][9m^2W^3 + W^2(6m^3 - 2m^2M + 27M^3) + W(m^4 - 12m^3M + 18mM^3) - 18m^4M + 3m^2M^3];$$

$$b_{2,3} = b_{3,2} = [k^3(m+W)/24\sqrt{3}M^2W][3m^2W^2 + W(m^3 + 2m^2M + 9M^3) + 6m^3M + 3mM^3];$$

$$b_{3,3} = (k^4/12M^2)(m^2W + 3M^3 - 2m^2M);$$

$$b_{4,4} = \frac{k^2(m+W)^2}{144M^2W^2}[9m^2W^3 + W^2(6m^3 + 2m^2M - 27M^3) + W(m^4 + 12m^3M - 18mM^3) + 18m^4M - 3m^2M^3];$$

$$b_{4,5} = b_{5,4} = \frac{k^3(m+W)}{24\sqrt{3}M^2W}[3m^2W^2 + W(m^3 - 2m^2M - 9M^3) - 6m^3M - 3mM^3];$$

$$b_{5,5} = (k^4/12M^2)(m^2W - 3M^3 + 2m^2M);$$

$$c_{0,0} = (m+W)^3/4W^2; \quad c_{1,1} = k(m+W)/2W;$$

$$d_{0,0} = \frac{k(m+W)^3(2m-W)}{6W^2}; \quad d_{1,1} = -\frac{k^2(m+W)(2m+W)}{3W}; \quad d_{2,2} = -\frac{k^2(m+W)(m+2W)}{6W};$$

$$d_{2,3} = d_{3,2} = -k^2(m+W)/2\sqrt{3}; \quad d_{3,3} = k^2m(m+W)/2W;$$

$$d_{4,4} = -k(m+W)^4/6W^2;$$

$$d_{4,5} = d_{5,4} = -k^2(m+W)^2/2\sqrt{3}W; \quad d_{5,5} = 0; \quad Q = \pi/k(W-k)(m+W-k). \tag{3.5}$$

The normalized angular polynomial-matrices $L_{i,k}^{\Phi'\Phi}$ for the reaction $\gamma + N \to \gamma' + N'$, in terms of which the matrix elements A, B, C, D are expanded, are obtained and described in detail in Ref. 2. Each of them describes an angular distribution of the reaction $\gamma + N \to \gamma' + N'$ for given angular momenta of the photons γ and γ' and of the total system $\gamma + N$. We shall use here the first 10 polynomials where

$$L^{\Phi'\Phi}_{0,0}(1_E\,{}^1/_2\,1_E) = (\sigma e')(\sigma e);\quad L^{\Phi'\Phi}_{1,1}(1_M\,{}^1/_2\,1_M) = (\sigma s')(\sigma s);$$
$$L^{\Phi'\Phi}_{2,2}(1_M\,{}^3/_2\,1_M) = 3(s's) - (\sigma s')(\sigma s);$$
$$L^{\Phi'\Phi}_{2,3}(1_M\,{}^3/_2\,2_E) = -i\sqrt{3}[(\sigma e)(s'k) + (\sigma k)(s'e)];$$
$$L^{\Phi'\Phi}_{3,2}(2_E\,{}^3/_2\,1_M) = i\sqrt{3}[(\sigma e')(k's) + (\sigma k')(e's)];$$
$$L^{\Phi'\Phi}_{3,3}(2_E\,{}^3/_2\,2_E) = (\sigma e')(\sigma e)(k'k) + (\sigma k')(\sigma e)(e'k) + (\sigma e')(\sigma k)(k'e) + (\sigma k')(\sigma k)(e'e);$$
$$L^{\Phi'\Phi}_{4,4}(1_E\,{}^3/_2\,1_E) = 3(e'e) - (\sigma e')(\sigma e);$$
$$L^{\Phi'\Phi}_{4,5}(1_E\,{}^3/_2\,2_M) = i\sqrt{3}[(\sigma s)(e'k) + (\sigma k)(e's)];$$
$$L^{\Phi'\Phi}_{5,4}(2_M\,{}^3/_2\,1_E) = -i\sqrt{3}[(\sigma s')(k'e) + (\sigma k')(s'e)];$$
$$L^{\Phi'\Phi}_{5,5}(2_M\,{}^3/_2\,2_M) = (\sigma s')(\sigma s)(k'k) + (\sigma k')(\sigma s)(s'k) + (\sigma s')(\sigma k)(k's) + (\sigma k')(\sigma k)(s's). \qquad (3.6)$$

Here $s = [ke]$, $s' = [k'e']$, and the symbols in parenthesis following the $L^{\Phi'\Phi}_{i,k}$'s denote, reading right to left, the angular momentum and multipolar character of the incident quantum, the angular momentum of the system and the angular momentum and multipolar character of the scattered quantum. The coefficients $\beta_{i,k}$ and $\delta_{i,k}$ in (3.4) are related to the coefficients $b_{i,k}$ and $d_{i,k}$ by means of formulas obtained by expanding the functions

$$F = \sum_{i,k=0}^{5} f_{i,k} L^{\Phi'\Phi}_{i,k} / (1 + \tau\cos\theta) = \sum_{i,k=0}^{\infty} \varphi_{i,k} L^{\Phi'\Phi}_{i,k}, \quad \cos\theta = (k'k)/k'k$$

in series of polynomials $L^{\Phi'\Phi}_{i,k}$:

$$\varphi_{0,0} = {}^1/_4\{2J_0 f_{0,0} + J_1(2f_{1,1} + f_{2,2} - \sqrt{3}f_{2,3} - \sqrt{3}f_{3,2} + 3f_{3,3}) + ({}^3/_2 J_2 - {}^1/_2 J_0)(f_{4,4} - \sqrt{3}f_{4,5} - \sqrt{3}f_{5,4} + 3f_{5,5})\};$$

$$\varphi_{1,1} = {}^1/_4\{2J_0 f_{1,1} + J_1(2f_{0,0} + f_{4,4} - \sqrt{3}f_{4,5} - \sqrt{3}f_{5,4} + 3f_{5,5}) + ({}^3/_2 J_2 - {}^1/_2 J_0)(f_{2,2} - \sqrt{3}f_{2,3} - \sqrt{3}f_{3,2} + 3f_{3,3})\};$$

$$\varphi_{2,2} = {}^1/_8\{4J_0 f_{2,2} + J_1(f_{0,0} + 5f_{4,4} + \sqrt{3}f_{4,5} + \sqrt{3}f_{5,4} - 3f_{5,5}) + ({}^3/_2 J_2 - {}^1/_2 J_0)(f_{1,1} + f_{2,2} + \sqrt{3}f_{2,3} + \sqrt{3}f_{3,2} + 3f_{3,3}) + 6J_3 f_{5,5}\};$$

$$\sqrt{3}\varphi_{2,3} = {}^1/_8\{4J_0\sqrt{3}f_{2,3} + 3J_1(-f_{0,0} + f_{4,4} + \sqrt{3}f_{4,5} - \sqrt{3}f_{5,4} + 3f_{5,5}) + ({}^3/_2 J_2 - {}^1/_2 J_0)(-3f_{1,1} + 3f_{2,2} - \sqrt{3}f_{2,3} + 3\sqrt{3}f_{3,2} - 3f_{3,3}) - 6J_3(2f_{5,5} - \sqrt{3}f_{5,4})\};$$

$$\varphi_{3,3} = {}^1/_8\{4J_0 f_{3,3} + 3J_1(f_{0,0} - f_{4,4} + \sqrt{3}f_{4,5} + \sqrt{3}f_{5,4} - f_{5,5}) + ({}^3/_2 J_2 - {}^1/_2 J_0)(3f_{1,1} + 3f_{2,2} - \sqrt{3}f_{2,3} - \sqrt{3}f_{3,2} + f_{3,3}) + 2J_3(3f_{4,4} - 2\sqrt{3}f_{4,5} - 2\sqrt{3}f_{5,4} + 4f_{5,5})\};$$

$$\varphi_{4,4} = {}^1/_8\{4J_0 f_{4,4} + J_1(f_{1,1} + 5f_{2,2} + \sqrt{3}f_{2,3} + \sqrt{3}f_{3,2} - 3f_{3,3}) + ({}^3/_2 J_2 - {}^1/_2 J_0)(f_{0,0} + f_{4,4} + \sqrt{3}f_{4,5} + \sqrt{3}f_{5,4} + 3f_{5,5}) + 6J_3 f_{3,3}\};$$

$$\sqrt{3}\varphi_{4,5} = {}^1/_8\{4J_0\sqrt{3}f_{4,5} - 3J_1(f_{1,1} - f_{2,2} - \sqrt{3}f_{2,3} + \sqrt{3}f_{3,2} - 3f_{3,3}) - ({}^3/_2 J_2 - {}^1/_2 J_0)(3f_{0,0} - 3f_{4,4} + \sqrt{3}f_{4,5} - 3\sqrt{3}f_{5,4} + 3f_{5,5}) - 6J_3(2f_{3,3} - \sqrt{3}f_{3,2})\};$$

$$\varphi_{5,5} = {}^1/_8\{4J_0 f_{5,5} + 3J_1(f_{1,1} - f_{2,2} + \sqrt{3}f_{2,3} + \sqrt{3}f_{3,2} - f_{3,3}) + ({}^3/_2 J_2 - {}^1/_2 J_0)(3f_{0,0} + 3f_{4,4} - \sqrt{3}f_{4,5} - \sqrt{3}f_{5,4} + f_{5,5}) + 2J_3(3f_{2,2} - 2\sqrt{3}f_{2,3} - 2\sqrt{3}f_{3,2} + 4f_{3,3})\}.$$

Formulas for $\varphi_{3,2}$ and $\varphi_{5,4}$ are obtained from the formulas for $\varphi_{2,3}$ and $\varphi_{4,5}$ by permuting corresponding indices.

$$J_n(\tau) = \int_{-1}^{1} \frac{x^n\,dx}{1+\tau x},$$

where

$$\text{for } \beta_{i,k} \quad \tau = \frac{2k^2}{M^2 - m^2 + 2k(W-k)},$$

$$\text{for } \delta_{i,k} \quad \tau = \frac{k}{W-k}.$$

Thus the transition matrix (3.2) for scattering of photons agains nucleons can be written with the help of tables and formulas (3.4), (3.5) in the form

$$W(\gamma'p',\gamma p) = e^2 Q \sum_{i,\,k=0}^{\infty} \left\{\frac{a^2}{m^2}\left(a_{i,k} - \frac{\beta_{i,k}}{M^2 - m^2 + 2k(W-k)}\right) + c_{i,k} - \frac{\delta_{i,k}}{2k(W-k)}\right\} L_{i,k}^{\Phi'\Phi}. \quad (3.7)$$

4. EFFECTIVE SCATTERING CROSS SECTION

In order to obtain the effective scattering cross section, it is necessary to know the amplitude $F(\gamma', \gamma)$ of the scattered wave. As shown in Ref. 2, $F(\gamma', \gamma)$ can be obtained by solving the following system of Heitler integral equations (formula 4.5 of Ref. 2; the notations are identical):

$$\begin{aligned}
F^{3/2}(\pi,\gamma) &= V^{3/2}(\pi,\gamma) - i\eta_\pi \int U^{3/2}(\pi,\pi') F^{3/2}(\pi',\gamma)\,d\Omega_{\pi'} - i\eta_\gamma \int V^{3/2}(\pi,\gamma') F(\gamma',\gamma)\,d\Omega_\gamma;\\
F^{1/2}(\pi,\gamma) &= V^{1/2}(\pi,\gamma) - i\eta_\pi \int U^{1/2}(\pi,\pi') F^{1/2}(\pi',\gamma)\,d\Omega_{\pi'} - i\eta_\gamma \int V^{1/2}(\pi,\gamma') F(\gamma',\gamma)\,d\Omega_{\gamma'};\\
F(\gamma',\gamma) &= W(\gamma',\gamma) - i\eta_\pi \int \left\{\frac{1}{3}[V^{3/2}(\pi,\gamma')]^+ F^{3/2}(\pi,\gamma) + \frac{2}{3}[V^{1/2}(\pi,\gamma')]^+ F^{1/2}(\pi,\gamma)\right\} d\Omega_\pi - i\eta_\gamma \int W(\gamma',\gamma'') F(\gamma'',\gamma)\,d\Omega_{\gamma''};\\
\eta_\pi &= (1/8\pi^2)\, q^2 dq/dW, \quad \eta_\gamma = (1/8\pi^2)\, k^2 dk/dW.
\end{aligned} \quad (4.1)$$

This system of equations can be solved by means of the angular polynomial matrices $L_k^{M'M}$, $L_{i,k}^{M\Phi}$, $L_{i,k}^{\Phi'M}$ otbained in Ref. 2. Expanding all the transition matrices and amplitudes of scattered waves in series of corresponding angular polynomials, and denoting the expansion coefficients by corresponding small script letters, we obtain the system of algebraic equations

$$f_{i,k}^{3/2} = v_{i,k}^{3/2} - i4\pi\eta_\pi u_i^{3/2} f_{i,k}^{3/2} - i4\pi\eta_\gamma \sum_l v_{i,l}^{3/2} f_{l,k};$$

$$f_{i,k}^{1/2} = v_{i,k}^{1/2} - i4\pi\eta_\pi u_i^{1/2} f_{i,k}^{1/2} - i4\pi\eta_\gamma \sum_l v_{i,l}^{1/2} f_{l,k};$$

$$f_{l,k} = w_{l,k} - i4\pi\eta_\pi \left(\frac{1}{3} v_{i,l}^{3/2} f_{i,k}^{3/2} + \frac{2}{3} v_{i,l}^{1/2} f_{i,k}^{1/2}\right) - i4\pi\eta_\gamma \sum_n w_{l,n} f_{n,k}; \quad (4.2)$$

$i = 0$ if $k = 0$ and i equals the integer given by either $(k+1)/2$ or $(k+2)/2$ for the remaining values of k. The system (4.2) can be solved exactly. An exact solution is cumbersome, however. Neglecting quantities of the order of g^2 and higher with respect to 1, we obtain in the first approximation the following solution to system (4.2):

$$f_{l,k} = \frac{w_{l,k}}{1 + i4\pi\eta_\pi(u_i^{3/2} + u_i^{1/2})};$$

$$f_{i,k}^{3/2,\,1/2} = \frac{v_{i,k}^{3/2,\,1/2}}{1 + i4\pi\eta_\pi u_i^{3/2,\,1/2}}, \quad (4.3)$$

where i depends on k as in (4.2). Therefore,

$$F(\gamma',\gamma) = \sum_{l,\,k=0}^{\infty} f_{l,k} L_{l,k}^{\Phi'\Phi} = \sum_{l,k} \frac{w_{l,k}}{1 + i4\pi\eta_\pi(u_i^{3/2} + u_i^{1/2})} L_{l,k}^{\Phi'\Phi}, \quad (4.4)$$

where the $w_{l,k}$ are obtained from Eqs. (3.8), (3.5) and (3.7), and $u_i^{3/2}$, $u_i^{1/2}$ are related to $\tan\delta_i^{3/2}$, $\tan\delta_i^{1/2}$ of Ref. 1 by the formulas

$$\operatorname{tg}\delta_{i+1}^{3/2} = -4\pi\eta_\pi u_i^{3/2}, \quad \operatorname{tg}\delta_{i+1}^{1/2} = -4\pi\eta_\pi u_i^{1/2}.$$

We have limited ourselves to the first ten terms of the series in computing the amplitude $F(\gamma', \gamma)$. This implies that we consider all possible scatterings of dipolar and quadrupolar quanta against nucleons, assuming that the angular momentum of the system is no higher than 3/2. Mesonic absorption is included in the first six terms of the series (processes $1_E \to {}^1\!/_2 \to 1_E$, $1_M \to {}^1\!/_2 \to 1_M$, $1_M \to {}^3\!/_2 \to 1_M$, $1_M \rightleftarrows {}^3\!/_2 \rightleftarrows 2_E$, $2_E \to {}^3\!/_2 \to 2_E$). This means that absorption is considered in conjunction with s and p meson waves.

In order to determine the effective cross section, it is necessary to obtain $|F|^2$, average it over the spins of the initial states of the nucleon, sum it over the spins of the final states of the nucleon (these operations are equivalent to $1/2\,\mathrm{Sp}|F|^2$), average it over the polarizations of the incident photons and sum over the polarizations of the scattered photons. The last two operations are equivalent to an averaging over polarizations of both incident and scattered photons and multiplication by two. It is therefore necessary to obtain $\mathrm{Sp}\overline{|F|^2}$ where the bar denotes averaging over the polarizations of both photons.

In the center-of-mass system, we then have

$$d\sigma / d\Omega = (1/4\pi^2)\, k^2 (W-k)^2 W^{-2}\, \mathrm{Sp}\,\overline{|F|^2};$$

$$\mathrm{Sp}\,\overline{|F|^2} = M_0 + M_1\cos\theta + M_2\left({}^3\!/_2\cos^2\theta - {}^1\!/_2\right) + M_3\cos^3\theta;$$

$$\begin{aligned} M_0 = {} & 2[(0,0;\ 0,0) + (1,1;\ 1,1)] + 4[(2,2;\ 2,2) + 2(2,3;\ 2,3) + (3,3;\ 3,3) \\ & + (4,4;\ 4,4) + 2(4,5;\ 4,5) + (5,5;\ 5,5)]; \end{aligned} \tag{4.5}$$

$$\begin{aligned} M_1 = {} & 2[2(0,0;\ 1,1) + (0,0;\ 2,2) - 2\sqrt{3}(0,0;\ 2,3) + 3(0,0;\ 3,3) + (1,1;\ 4,4) - \\ & - 2\sqrt{3}(1,1;\ 4,5) + 3(1,1;\ 5,5) + 5(2,2;\ 4,4) + \\ & + 2\sqrt{3}(2,2;\ 4,5) - 3(2,2;\ 5,5) + 2\sqrt{3}(2,3;\ 4,4) + 6\sqrt{3}(2,3;\ 5,5) - \\ & - 3(3,3;\ 4,4) + 6\sqrt{3}(3,3;\ 4,5) - 3(3,3;\ 5,5)]; \end{aligned} \tag{4.6}$$

$$\begin{aligned} M_2 = {} & (2,2;\ 2,2) + 4(2,3;\ 2,3) + (3,3;\ 3,3) + (4,4;\ 4,4) + 4(4,5;\ 4,5) + \\ & + (5,5;\ 5,5) + 2[(0,0;\ 4,4) - 2\sqrt{3}(0,0;\ 4,5) + 3(0,0;\ 5,5) + (1,1;\ 2,2) - \\ & - 2\sqrt{3}(1,1;\ 2,3) + 3(1,1;\ 3,3) + 2\sqrt{3}(2,2;\ 2,3) + 3(2,2;\ 3,3) - 2\sqrt{3}(2,3;\ 3,3) + \\ & + 2\sqrt{3}(4,4;\ 4,5) + 3(4,4;\ 5,5) - 2\sqrt{3}(4,5;\ 5,5)]; \end{aligned}$$

$$\begin{aligned} M_3 = {} & 4[3(2,2;\ 5,5) + 6(2,3;\ 4,5) + 3(3,3;\ 4,4) - 4\sqrt{3}(2,3;\ 5,5) \\ & - 4\sqrt{3}(3,3;\ 4,5) + 4(3,3;\ 5,5)], \end{aligned}$$

$$2(i,\ k;\ l,\ m) = f^{+}_{i,k} f_{l,m} + f^{+}_{l,m} f_{i,k}; \qquad f_{i,k} = f_{k,i}. \tag{4.6}$$

The total cross section is

$$\sigma = \left(k^2 (W-k)^2 / \pi W^2\right) M_0. \tag{4.7}$$

5. RESULTS

We present in Figs. 2 and 3 the total effective cross section σ as a function of the energy of the incident photon k_0 in the laboratory system, and the differential cross section $d\sigma/d\Omega$ in the center-of-mass system for various energies k_0. The cross sections were computed assuming*

$$a = 1.61; \quad G^2 = 0.12; \quad g^2 = 0.10;$$

$$f^2 = 0.529; \quad M = 1.316m. \tag{5.1}$$

* The meson mass is assumed to be exactly $\mu = 276\, m_e$.

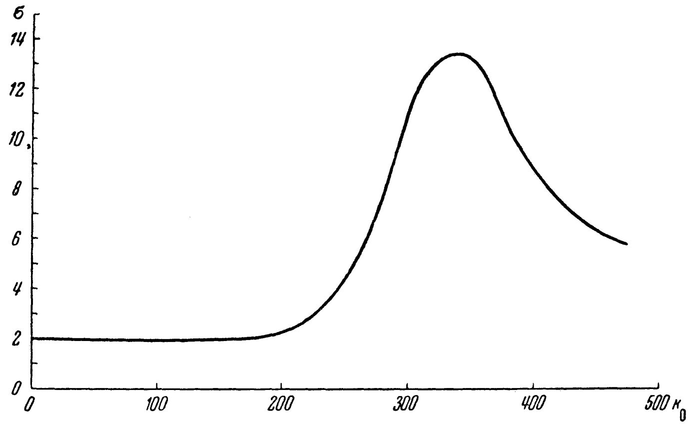

FIG. 2. Total effective cross section $\sigma(10^{-31}\,\mathrm{cm}^2)$ for the scattering of photons agains protons as a function of the photon energy k_0 (mev) in the laboratory system.

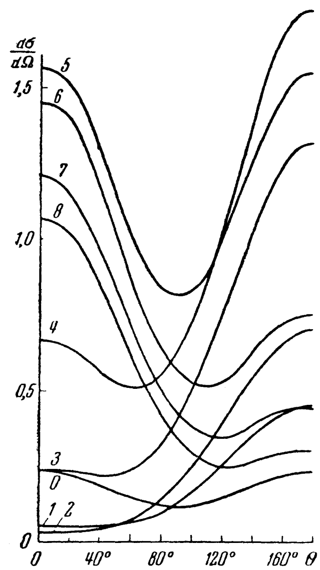

FIG. 2. Differential cross section $d\sigma/d\Omega\,(10^{-31}/\mathrm{cm}^2/$ steradian) for the scattering of photons against protons as a function of the scattering angle θ in the center-of-mass system. Energy of the photons: *0* -- 0, *1* -- 192, *2* -- 230, *3* -- 272, *4* -- 296, *5* -- 344, *6* -- 392, *7* -- 430, *8* -- 470 mev.

The fundamental constants in this computation are a, G^2 and M, and these are taken as in Ref. 2; the constants g^2 and f^2 differ from those assumed in Ref. 2, but do not play an essential role here as they only enter in secondary terms in the absorption**; therefore, the cross section does not change very much if one adopts the constants given in Eq. (5.1) of Ref. 2 instead of the constants (5.1) of the present article.

For small energies, the total cross section equals the Thompson cross section $(8\pi/3) \times (e^2/mc^2)^2 = 1.973 \times 10^{-31}\,\mathrm{cm}^2$, for as $k \to 0$, isobaric terms and terms due to absorption go to zero. For $k_0 \sim 180$ mev the cross section begins to grow, and for $k_0 = 344$ mev*** it reaches a maximum of $1.34 \times 10^{-30}\,\mathrm{cm}^2$, exceeding the Thompson cross section by a factor of almost 7. Upon further increase in k_0, the cross section falls. For small energies the angular distribution of scattered photons coincides with $1 + \cos^2\theta$; then as the energy increases, a preferential backward scattering appears. At resonance ($k_0 = 344$ mev) the angular distribution is again symmetrical with respect to $\theta = \pi/2$, and for energies beyond resonance a preferential forward scattering appears, the asymmetry increasing with energy. The change in the position of the maximum from backward to forward angles takes place extremely rapidly in the resonance region.

Comparison of these results with those of Sachs and Foldy[6] and Minami[8] shows that the width of the curve obtained for the total cross section is several times larger while its maximum is several times smaller than for curves of Refs. 6

** In distinction from Ref. 2 where g^2 and f^2 enter in the transition matrix as well as in the absorption.

*** When $k_0 = 344$ mev, the energy of the system W equals the mass M of the isobar in the center-of-mass system.

and 8* obtained with the same parameter values. Furthermore, the maximum is not found at $k_0 = \mu c^2$ as in Ref. 6, but at a considerably higher energy. The angular distribution resembles those of Refs. 6 and 8. only for very small and very large k_0, and strongly differs at other values of k_0; this is especially true of the backward scattering in the pre-resonance region and of the minimum for $\theta = \pi/2$ at resonance.

In conclusion, I wish to thank O. P. Ryibalkin, V. I. Petukhov and A. G. Trunov for carrying out the numerical computations.

Note added in proof: After completing this analysis, the author discovered that an analogous calculation was carried out independently by R. Gurzhi.

1 Tamm, Gol'fand and Feinberg, J. Exptl. Theoret. Phys. (U.S.S.R.) 26, 649 (1954).

2 V. I. Ritus, J. Exptl. Theoret. Phys. (U.S.S.R.) 27, 660 (1954).

3 W. Pauli, Rev. Mod. Phys. 13, 203 (1941).

4 S. B. Batdorf and R. Thomas, Phys. Rev. 59, 621 (1941); J. L. Powell, Phys. Rev. 75, 32 (1949).

5 A. M. Baldin and V. V. Mikhailikov, J. Exptl. Theoret. Phys. (U.S.S.R.) 21, 562 (1951); Usp. Fiz. Nauk 44, 200 (1951).

6 R. G. Sachs and L. L. Foldy, Phys. Rev. 80, 824 (1950).

7 K. M. Case, Phys. Rev. 76, 1 (1949).

8 V. Ia. Fainberg, J. Exptl. theoret. Phys. (U.S.S.R.) 25, 644 (1953).

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On the S-Matrix for Particles with Arbitrary Spin

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This is a discussion of the perturbation theory for particles with arbitrary spin. The properties of the singular functions for such particles are discussed. It is shown that the elements of the *S*-matrix may be found by the Feynman rules.

THE present theory of wave fields permits us to consider fields corresponding to particles with any integer or half integer spin or with any number of integer or half integer spin states.[1-4] In spite of the fact that particles with spin larger than unity are not observed experimentally, the theory of such particles can present some interest in the interpretation of the properties of the newly discovered mesons and in the study of the isobaric states of nucleons. For instance, the theory of particles with spin 3/2 has been applied to the construction of the partly phenomenological theory of interaction of π-mesons with nucleons[4] - and the results obtained compare satisfactorily with the experimental data.

The basic problem in the study of the interaction of particles with arbitrary spin with other fields amounts to the construction of the scattering matrix *S*. It has already been assumed previously that the elements of the *S*-matrix for particles with arbitrary spin may be obtained by the Feynman rules.[5-6, 7] in analogy to the quantum electrodynamics case. However, until now no basis was given to this assumption.

A difficulty arises in the construction of the *S*-matrix for particles with arbitrary spin. The difficulty is related to the fact that for higher spins not all the components of the wave function $\psi(x)$ are dynamically independent.[1] This fact is the main impediment in the attempt to generalize, for the case of arbitrary spin, the *S*-matrix theory; the latter was derived by Dyson[7] for quantum electrodynamics (spin 1/2 in the interaction representation. (For more details on these difficulties see Refs. 5, 6).

In the present paper, it will be shown how one can obtain the elements of the *S*-matrix for particles with arbitrary spin in the Heisenberg representation. We follow the Yang-Feldman method.[8] For the sake of simplicity, we will consider the interaction with an electromagnetic field (the interaction with other fields can be treated analo-

* The constants used in this article are almost identical to those of Ref. 8.