Letters to the Editor

The Role of Radiational Losses in Cyclic Accelerators

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WE showed in Refs. (1,2) that during the movement of relativistic electrons in cyclic accelerators, there appears a strong damping of betatron oscillations. In the present work we give the generalization of the results of Refs. 1 and 2.

We start with the classical relativistic equation of Dirac-Lorentz for the motion of an electron (Ref. 3).

$$mc\frac{du_i}{ds} = \frac{e}{c}F_{ik}u_k + \frac{2e^2}{3c}\left[\frac{d^2u_i}{ds^2} + u_iu_k\frac{d^2u_k}{ds^2}\right], \quad (1)$$

where u_i are components of 4-velocity of an electron, F_{ik} — components of a tensor of the electromagnetic field. The sum in the parenthesis is the self-action force due to the presence of radiation.

Let us consider the movement of an electron in an axially symmetric magnetic field $H(r)$ in the neighborhood of an orbit of a constant radius R with an accelerating electric field $\mathcal{E}_r = \mathcal{E}_z = 0$, $\mathcal{E}_\theta = \mathcal{E}(\theta, t)$. In a linear approximation we obtain equations from Eq. (1) which are necessary for the description of the radial and azimuthal motion:

$$\ddot{\rho} + \left(\frac{\dot{E}}{E} + \frac{W}{E}\right)\dot{\rho} + (1-n)\frac{c^2}{R^2}\rho \quad (2)$$

$$= \frac{c^2}{R^2}\frac{E - eH_0R}{eH_0R} + \frac{R^2}{c^2}\left(\frac{mc^2}{E}\right)^2\frac{W}{E}\dddot{\rho},$$

$$\dot{E} = e\mathcal{E}c - W + eH_0R\dot{\rho}, \quad (3)$$

$$\dot{\theta} = (c/R)(1-\rho) = c/R - \dot{\varphi}, \quad (4)$$

where the following dependence was used: $H = H_0(1 - n\rho)$, $\rho = r/R - 1$, E = energy of an electron, $W = \left(\frac{2e^2c}{3r^2}\right)\left(\frac{E}{mc^2}\right)^4$ = magnitude of the radiational losses.

The term containing $\dddot{\rho}$ in Eq. (2) describes the "proper" attenuation of the betatron oscillations. Practically, it does not influence the motion in comparison with the main dissipation $\sim W\rho/E$, if $|1-n| \ll (E/mc^2)^2$, which is always fulfilled. In the main orbit of an accelerator, the radiational losses are compensated, and the energy of the electrons grows in time in correspondence with the growth of the magnetic field. The solution of equations (2) – (4) appears as follows: $\rho = \rho_M + \rho_{bet}$, with

$$\rho_M = (E - eH_0R)/eH_0R(1-n), \quad (5)$$

where ρ_{bet} are betatron oscillations, with the frequency $(c/R)\sqrt{1-n}$, which take place near the instantaneous orbit with the radius $R(1+\rho_M)$. From Eqs. (2) – (5) we obtain an equation of the synchrotron and betatron oscillations (Ref. 4), whose amplitude changes according to the following law*

$$a_{bet} \sim E^{-1/2}\exp\left[-\frac{1}{2}\frac{n}{(1-n)}\int_{t_0}^{t}\frac{W}{E}dt\right]; \quad (6)$$

$$a_M \sim E^{-3/4}[e\dot{H}_0R + \dot{W}]^{1/4}\exp\left[-\frac{1}{2}\frac{3-4n}{1-n}\int_{t_0}^{t}\frac{W}{E}dt\right].$$

These expressions are the generalization of the laws of variation of $a_{bet} \sim E^{-1/2}$, $a_M \sim E^{-3/4}$ for the case when the radiation losses, which are compensated by the external field (accelerator), are present.

If the acceleration takes place in the curl field (betatron), then, for the presence of radiation, the condition $\bar{H} = 2H$ is changed in the corresponding manner so that the condition R = const. holds. In this case, for ρ_{bet} we obtain the same law (Eq. 6) and

$$\rho_M \sim E^{-1}\exp\left[-\frac{3-4n}{(1-n)}\int_{t_0}^{t}\frac{W}{E}dt\right]. \quad (6a)$$

We note that damping of the type (6) is in fact not connected with the actual form of radiation losses, which are compensated by the external field. An analogous result could be obtained, for instance, in the case of losses for the bremsstrahlung on the nuclei of the residual gas. The quantity W, which appears in Eq. (6), can have the interpretation of power spent by the accelerating system for the compensation of all similar losses.

Until now all our results followed completely from the classical equation of Dirac-Lorentz (Eq. 1). We want to consider the corrections due to the consideration of quantum effects. For the description of the motion of the electron up to the energies $E \ll E_{1/2} = mc^2 (Rmc/\hbar)^{1/2} (\simeq 10^{15}$ ev for usual conditions) one can, with good reason, use classical concepts and consider only the statistical (quantum) character of radiation. For this purpose the following term should be added to Eq. (3):

$$W(R) - \sum_i \varepsilon_i \delta(t - t_i), \quad (7)$$

where ϵ_i is the energy of a separate photon, emitted at the moment t_i, δ is the delta function. The physical meaning of the changes introduced in the motion because of the introduction of (7) into (3), was considered by us in Refs. 1 and 2. The solution of Eqs. (2), (7), (4) gives for the average square values of $\overline{\rho}^2_{bet}$ and $\overline{\rho}_M^2$ very similar expressions which also hold for the synchrotron (Refs. 5,6) and for the betatron (Refs. 1, 2).

$$\overline{\rho^2_{bet}} \leqslant \frac{55\sqrt{3}}{96} \frac{\Lambda}{Rn(1-n)} \left(\frac{E}{mc^2}\right)^2; \quad (8)$$

$$\rho_M \leqslant \frac{55\sqrt{3}}{96} \frac{\Lambda}{R(1-n)(3-4n)} \left(\frac{E}{mc^2}\right)^2,$$

where $\Lambda = \hbar / \nu c$ is the Compton wavelength.

The expression for $\overline{\rho}^2_{bet}$ was found also by a different, more complicated, method in the works of Sokolov and Ternov.[7] However, since they did not consider the damping (6), their results are erroneous.

It is interesting to follow the mechanism of damping, z_{bet}, in more detail for the case when the statistical character of losses is taken into consideration. At the time of emission of the i th quantum, a vertical recoil force acts on the electron

$$(d/dt)(m\dot{z}) = (\varepsilon_i/c^2)\dot{z}\delta(t - t_i)$$

(we omit quasi-elastic forces). On the other hand, $(d/dt)(m\dot{z}) = \dot{z}\dot{m} + m\ddot{z}$. The change of mass can be broken up into two parts: loss of energy during the radiation, and its receipt from the accelerating field

$$\dot{m} = -(\varepsilon_i/c^2)\delta(t - t_i) + \dot{m}_{accel}$$

$$m_{accel} = \frac{1}{c^2}(W + e\dot{H}_0 R).$$

It is not difficult to see that the increase of the oscillation amplitude at the expense of the mass at the time of radiation is fully compensated by the radiation friction. Consequently only the increase of the mass m_{accel} influences the amplitude of the betatron oscillations. This increase is due to the compensating field. From this, there immediately follows an equation of the type (6).

*Here we generalize our result from Refs. 1,2.

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244

Superconductivity of Barides, Carbides, Nitrides and Silicides of Transition Metals

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DORFMAN and Kikoin[1] concerned themselves with the interesting fact that the transition point to the superconducting state increases in the series TiB —TiC—TiN; the same increase was noted in the series VC—VN and ZrB—Zr—C—ZrN.

The data available in the literature on the transition temperatures of similar compounds are given in Refs. 2–4. In spite of the wide spread of the transition points obtained by different investigators, one can make the following preliminary observations. The value of T_k is connected with the electron density distribution, i.e., it depends on the acceptor capability of the atom of the transition metal 1 / Nn[5] and of the ionization potential of the non metal (φ). *

In titanium compounds, the transition points are very low, and although some increase of T_k probably takes place in the series TiB_2 — TiC—TiN, i.e., with increase in the ionization potential of the metalloid, the probabilities of realization of high acceptor probability of the 3d-level of Ti are lessened. This appears more clearly for the compounds of Zr. A significant increase