in the transition points takes place for compounds of Ta and Nb, but not of V, although all these elements belong to the subgroup Va. There is a high scattering capacity in vanadium, yielding only to Ti and Zr, which also brings about a lowering of the transition point.

In compounds of Nb and Ta, there is evidently a very favorable relation of the value of 1 /Nn of the metal and φ of the metalloid; therefore in the series Me (V) B → Me (V) C → Me (V) N there is a more clearly expressed rise in T_k ; moreover, the transition points are high in absolute value.

For a transition to compounds of W and Mo, the values of T_k are much higher than for compounds of Ti, Zr, Hf and V, but lower than for compounds of Nb and Ta; this is possible as a consequence of the increase of screening of the d-band of these metals by the natural excitations of the electrons,[6] which makes difficult the excitation of the valence electrons of the metalloid. In each case, it is characteristic that the sharp decrease in the number 1/Nn from 0.167 – 0.100 for Ti, Zr, V, Hf to 0.5 – 0.67 for Ta, Nb, W and Mo_+, is accompanied by such a sharp increase in the value of T_k.

It should also be noted that in a number of cases the value T_k increases with increase in the metallic content, for example, for Nb_2N, $T_k = 9.5°$ K, but for NbN, $T_k = 15$ ° K, Mo_2C, $T_k = 2.9$ ° K, and for MoC — 8° K, for Mo_2N, $T_k = 5$ ° K, for MoN — 12° K, for W_2C, $T_k = 2.74°$ K, for WC, 2.5 — 4.21 ° K.

The relatively lower transition values for all borides in comparison with carbides and nitrides are probably explained by the presence of strong covalent bonds between the boron atoms , which leads to the formation of the characteristic structure of the elements — little chains, lattices, shells of boron atoms in boride crystals.[7] In this connection, the fraction of electrons capable of completing the electron deficiency of the atoms of the transition metals is not large and T_k is correspondingly decreased.

Actually, in the case of all metallic compounds, especially Nb, Ta, W and Mo, the increase of T_k in the transition from Me — B to Me — C, is, as a rule, much sharper than for the transition from Me — C to Me — N. Therefore the borides are essentially different from the carbides and nitrides. The latter are close to each other in values of T_k. but the smaller ionization potential of carbon in comparison with nitrogen enhances the effect of the increase of T_k in series MeB — MeC — MeN, especially for compounds of the transition metals with low electron deficiencies. Compounds of silicon, which have still lower ionization potentials than boron ought, from this point of view, to possess still lower transition temperatures, i.e., there ought to be the series: MeSi → MeB → MeC → MeN, which actually takes place in most cases, with the exception of some silicides (V_3Si, TaSi, W_sSi_2) whose superconductivity is related chiefly to purely structured factors, for example, in V_3Si, which has the structure β — W).

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*N is the principal quantum number, n is the number of electrons of the incomplete d-level.

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Production of Nuclear Stars by γ-Quanta

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IN this article is investigated that particular mechanism of the production of nuclear stars by γ- quanta in which the γ-quantum creates a virtual π-meson pair at a large distance from a nucleus. This pair is thereupon absorbed by this nucleus and a star is produced. All considerations are conducted in the region of high energies $\omega >> \mu$* (where ω is the frequency of the quantum, μ is the meson rest mass).

In Ref. 1 there was considered a process in which only one member of the pion pair created by the γ-quantum is absorbed by that nucleus, creating a star, and the remaining pion carried away an energy of the order of the total energy of the star. The method used in the calculation of this process, we also use in the present case, i.e., the cross section can be found with the help of the matrix element of the radiative transition for which a form of the ψ-function describing the absorbed meson was determined in Ref. 1.

We will consider the nucleus to be perfectly black to the pion, and subsequently generalize the result to semi-transparent nuclei.

The matrix element is

$$M=\frac{2e}{i}\sqrt{\frac{2\pi}{\omega}}\int[\psi_1^*(\mathbf{j}\nabla)\,\psi_2^*]\,e^{i\omega\mathbf{r}}\,d\mathbf{r} \qquad (1)$$

(we set $\hbar = c = 1$), where $\mathbf{j}$ is the polarization of the quantum, ψ_1 (ψ_2) is a spherical wave collapsing to an arbitrary point on the cross section of the nucleus $\mathbf{s}_1$ ($\mathbf{s}_2$).

$$\psi_1=\frac{\sqrt{p_1}}{4\pi^{3/2}}\frac{e^{-ip_1|\mathbf{r}-\mathbf{s}_1|}}{|\mathbf{r}-\mathbf{s}_1|} \qquad (2)$$

$$=\frac{\sqrt{p_1}}{8\pi^{7/2}}\int\frac{e^{-i\mathbf{q},\mathbf{r}-\mathbf{s}_1}}{q^2-p_1^2+i\varepsilon}\,d\mathbf{q};$$

$$\psi_2=\frac{\sqrt{p_2}}{4\pi^{3/2}}\frac{e^{-ip_2|\mathbf{r}-\mathbf{s}_2|}}{|\mathbf{r}-\mathbf{s}_2|}$$

$$=\frac{\sqrt{p_2}}{8\pi^{7/2}}\int\frac{e^{-i\mathbf{q}',\mathbf{r}-\mathbf{s}_2}}{q'^2-p_2^2+i\varepsilon}\,d\mathbf{q}',\quad \varepsilon\to 0.$$

Setting $\mathbf{q}=\mathbf{q}_\omega+\mathbf{g}$, $\omega\cdot\mathbf{g}=0$, and carrying out the integration, we obtain

$$M=\frac{e}{i}\sqrt{\frac{p_1p_2}{32\,\pi^5\omega^3}} \qquad (3)$$

$$\times\int\frac{d\mathbf{g}(\mathbf{j}\times\mathbf{g})e^{i\mathbf{g},\,\mathbf{s}_2-\mathbf{s}_1}}{\mu^2+g^2}\left(\frac{E_1}{\sqrt{p_1^2-g^2}}+\frac{E_2}{\sqrt{p_2^2-g^2}}\right),$$

where the integration over $\mathbf{g}$ is taken over the entire plane perpendicular to ω. The effective cross section of the process will be:

$$\sigma=2\pi\int|M|^2\,d\mathbf{s}_1\,d\mathbf{s}_2\,dE_1\,dE_2\delta \qquad (4)$$

$$\times(\omega-E_1-E_2)\,|\overline{F}(\omega)|^2.$$

Here the integration over $\mathbf{s}_1$, $\mathbf{s}_2$ are taken over the cross section of a nucleus of radius R, where πR^2 is equal to the cross section of all the inelastic processes in the collision of a pion with a nucleus. The integration over E is taken from O to ω.

We used the conservation law $\omega = E_1 + E_2$ in view of the fact that in the present process, as also in creation processes of free pion pairs,[2] the mesons are created in the main at large distances from the nucleus $r_{eff} >> R > 1/\mu$ (this is related to the small transfer of longitudinal momentum to the nucleus in the pair creation process). But then there is an indeterminacy in the energy (Because the pions are virtual) $\Delta E \sim 1/\Delta t \sim 1/r_{eff} << \mu$, and one can disregard this quantity. In the expression for the cross section there also entered a factor $\overline{F}$, a pion form factor averaged over angle and energy,[2] dependent on the possible finiteness of the pions and on their mutual interaction.

Carrying out the integration, we obtain, to logarithmic order,

$$\sigma=(e^2R^2/12)\ln(\omega^3R/\mu^2)\,|\overline{F}|^2. \qquad (5)$$

If we stipulate that $31n(\omega/\mu) >> \ln\mu R$, then

$$\sigma=(e^2R^2/4)\ln(\omega/\mu)\,|\overline{F}|^2. \qquad (6)$$

If we set $\overline{F} = 1$, then

$$\sigma=(e^2R^2)\ln(\omega/\mu). \qquad (7)$$

In this case the cross section increases logarithmically with the energy of the γ-quantum. One should keep in mind, however, that the logarithm is determined in the region of large angles between the momenta of the mesons and the γ-quantum. In this region neglecting the form factor may appear to be invalid, since the form factor can appreciably lower the contribution of this region to the total cross section. If, as a consequence of this, we limit ourselves to the consideration of the region of small angles, then in Eq. (3) for the matrix element we must cut off the integration over g at some $g_{max} \sim \mu$.

For the effective cross section of the process, then, we obtain, considering $R >> 1/\mu$.

$$\sigma=\frac{e^2R^2}{12}\left[\ln\frac{\mu^2+g_{max}^2}{\mu^2}-\frac{g_{max}^2}{\mu^2+g_{max}^2}\right], \qquad (8)$$

i.e., the cross section in this case does not depend on the energy of the γ-quantum. The obtained cross section is consistent with the total cross section for the creation of free pion pairs.[2]

An answer to the question as to how significant the form factor of the pion is, can be obtained from a comparison of the theoretical results (for example Refs. 2, 1) with the experimental data.

The obtained results may be generalized using a semi-transparent model of the nucleus.[3] Introducing into the expression for the ψ-function a factor characterizing the absorption of the meson,

$$\psi_i=\frac{\sqrt{p_i}}{4\pi^{3/2}}\left[1-\exp\left(-2\varkappa_i\sqrt{R^2-s_i^2}\right)\right]^{1/2} \qquad (9)$$

$$\times\frac{e^{-ip_i|\mathbf{r}-\mathbf{s}_i|}}{|\mathbf{r}-\mathbf{s}_i|},$$

($i = 1, 2$), where $\varkappa_i$ is the absorption coefficient of the i th meson, we obtain the following expression for the effective cross section:

$$\sigma = \frac{e^2}{2\pi} |\bar{F}(\omega)|^2 \int_0^\omega \frac{E_1(\omega - E_1)}{\omega^3} \tag{10}$$

$$\times \ln \frac{E_1^2(\omega - E_1)^2 R}{\mu^2 |2E_1 - \omega|} (\sigma_{\varkappa_1} + \sigma_{\varkappa_2} - \sigma_{\varkappa_1+\varkappa_2})\, dE_1,$$

where $\sigma_\varkappa$ is the cross section for the capture, by a nucleus of radius R, of a pion with an absorption coefficient $\varkappa$.

The integration cannot be carried out in general, since the dependence of $\sigma_\varkappa$ on energy is unknown. If one takes $\varkappa$ to be independent of energy, $\varkappa_1 = \varkappa_2 = \varkappa$, then we obtain

$$\sigma = (e^2/4\pi) \ln(\omega/\mu) |\bar{F}|^2 (2\sigma_\varkappa - \sigma_{2\varkappa}). \tag{11}$$

If we introduce a cut-off in angle, then under these conditions the cross section becomes

$$\sigma = \frac{e^2}{12\pi} \left[\ln \frac{\mu^2 + g_{\max}^2}{\mu^2} - \frac{g_{\max}^2}{\mu^2 + g_{\max}^2} \right] (2\sigma_\varkappa - \sigma_{2\varkappa}). \tag{12}$$

In addition to the one considered, there are possible a series of other processes for the formation of nuclear stars by γ-quanta. However, in view of the fact that in this process an effective role is played by a region large in comparison to nuclear dimensions, one can expect that the considered mechanism is the fundamental one at high energies $\omega \gg \mu$.

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The Influence of the Proximity of an External Resonance on the Magnitude of the Transition Energy in a Strong Focussing Accelerator

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THE magnitude of the transition energy in a strong focussing accelerator is determined by the formula

$$E_{\rm cr} = mc^2 \alpha^{-1/2}; \quad \alpha = d \ln L / d \ln p, \tag{1}$$

where L is the length of the orbit, p is the momentum of the particle. Usually, only the direct dependence of the orbit length on the momentum is taken into account, resulting from the equation

$$\frac{d^2 r}{d\theta} - \left(\frac{l}{2\pi}\right)^2 \frac{\partial H/\partial r}{H_0 \rho} r = -\left(\frac{l}{2\pi}\right)^2 \frac{\Delta p}{p} \frac{1}{\rho}, \tag{2}$$

where $\rho = \rho(\theta)$ is the radius of curvature of the unperturbed trajectory (for $(\Delta p/p)_{\rm synch} = 0$)*, l is the length of a periodic sector, $\theta = 2\pi s/l$, s is the coordinate along the unperturbed trajectory, H is the magnetic field, r is the horizontal deviation from the equilibrium orbit. However, in the vicinity of resonances, L obviously depends on the distance from the resonances, ϵ_r, ϵ_z, and these distances depend sharply on $(\Delta p/p)_{\rm synch}$. Therefore

$$\alpha = \frac{\partial \ln L}{\partial \ln p} + \frac{\partial \ln L}{\partial \varepsilon_{0r}} \frac{d\varepsilon_{0z}}{d \ln p} + \frac{\partial \ln L}{\partial \varepsilon_{0z}} \frac{d\varepsilon_{0z}}{d \ln p} = \alpha_p + \alpha_\varepsilon, \tag{3}$$

$$\varepsilon_r = \nu_r - \nu_{\rm res} \quad \varepsilon_z = \nu_z - \nu_{\rm res} \tag{4}$$

where $\nu_{r,z}$ are the betatron quasi-frequencies of the transverse oscillations. The derivatives in Eq. (3) are taken at $(\Delta p/p)_{\rm synch} = 0$, i.e. ϵ_{0r}, ϵ_{0z} correspond to the mid-position of the synchrotron oscillations. The quantity α corresponds to a high energy $mc^2 \alpha^{-1/2}$. At such energies, the betatron oscillations about the equilibrium orbit are already sufficiently small; therefore parametric resonance, generally speaking, plays a weak role in the effect.**

Thus we have to understand L to be the length of the perturbed equilibrium periodic orbit. Clearly,