The Spin of the Λ-Particle

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THERE has recently been discussion in the literature on the spin of the Λ-particle; in particular, a model has been considered in which the Λ-particle represents a system of a nucleon + π-meson, which has sufficiently small dimensions $[a \sim (M\mu)^{-1/2}$ (here M is the mass of the nucleon, μ the mass of the meson)] and sufficiently large angular momentum ($l = 5$). Such a model would enable us to explain: 1) the long lifetime of the Λ-particle, 2) the results of correlation experiments, 3) the simultaneous creation of Λ- and θ-particles. A series of attempts have been undertaken to obtain such a model from the equations of Bethe-Salpeter[1,2].

We shall show that the model of large spin does not agree with the experimental facts connected with the existence of Λ-nuclei.

As is known[3], the Λ-nuclei represent nuclear shells in which one of the neutrons is replaced by a Λ-particle. These shells last not less than 10^{-12} sec and decay in two ways: with the emission of a π-meson (with the release of energy of the order $\epsilon = 37$ mev) and without the emission of a π-meson, wherein the energy release is of the order of $\epsilon + \mu \sim \mu$ and only nucleons emerge.

The latter process is similar to internal conversion in atoms. The role of the radiation nucleus is played by the decaying Λ-particle, the role of the γ-quantum by the π-meson, and the role of the conversion electron by one of the nucleons in the nucleus, which absorbs this π-meson. We calculate the probability of such conversion, making the following assumptions:

1) The Λ-particle is an infinitely heavy particle located at the origin of the coordinates.

2) The wave function of the π-meson outside of the effective dimensions of the λ-particle ($r > a$) is a diverging wave with angular momentum l, so normalized that one particle is emitted in time:

$$\varphi_\pi = (v\tau_0)^{-1/2} R_l Y_{l_0}.$$

Here v is the velocity of the meson, Y_{l_0} is the normalized spherical harmonic, R_l is the normalized radial wave function.

$$R_l = k(\pi/2kr)^{1/2} H_{l+1/2}(kr),$$

$$R_l = \frac{(2l-1)!!}{k^l r^{l+1}} \quad (r \ll l/k), \quad R_l = \frac{1}{r} e^{i(kr - \pi l/2)} \quad (r \to \infty),$$

where k is the propagation vector of the meson.

3) The wave function of the nucleon in the initial and final states is

$$\psi_1 = \begin{cases} u_1/\sqrt{\Omega} & \text{for } r < r_n \\ 0 & \text{for } r > r_n \end{cases} \quad \psi_2 = (u_2/\sqrt{V})\, e^{i\mathbf{pr}}.$$

Here u is the spinor part of the wave function, Ω, r_n are the volume and radius of the nucleus, V is the normalized volume, $\mathbf{p}$ is the momentum of the nucleon in its final state.

The probability of conversion with the emission of a nucleon is equal to

$$1/\tau = 2\pi |U|^2 \rho_E, \tag{1}$$

$$U = g\sqrt{\frac{2\pi}{\mu}} \int \psi_1^* \left(\frac{\sigma\nabla}{2M}\varphi_\pi\right) \psi_2 dr.$$

Carrying out the computation by the usual method[4], we obtain

$$\frac{1}{\tau} = 2\pi g^2 \frac{2\pi}{\mu} \frac{1}{v\tau_0} \frac{1}{\Omega} \frac{k^2}{4M^2} \frac{Mp}{(2\pi)^3} \times \left\{\frac{l+1}{2l+1} I_{l+1}^2 + \frac{l}{2l+1} I_{l+1}^2\right\}, \tag{2}$$

where

$$I_l = \int R_l(kr)\, g_l(pr)\, r^2 dr, \tag{3}$$

$$g_l = (2\pi)^{3/2} (pr)^{-1/2} J_{l+1/2}(pr),$$

$$g_l = 4\pi \frac{p^l r^l}{(2l+1)!!} \quad (\text{for } r \ll l/p),$$

$$g_l = \frac{4\pi}{r} \sin\left(pr - \frac{\pi l}{2}\right) \quad (\text{for } r > l/p).$$

Upon integration over r for $r_n \gg l/p$, the upper limit can be set equal to ∞, since the chief contribution is biven by the inverval $r < l/p$; for $r_n < l/p$, we can use r_n as the upper limit. So far as the lower limit of integration is concerned, it can be set equal to zero for large l and small a, since the contribution of the interval $0 < r < a$ to the integral is small. In fact, using the expressions for R_l and g_l for small r, we get

$$I_l^a = \int_0^a R_l(kr)\, g_l(pr)\, r^2 dr = \frac{2\pi}{2l+1} \left(\frac{p}{k}\right)^l a^2,$$

while

$$I_l = \int_0^\infty R_l(kr)\, g_l(pr)\, r^2 dr = 4\pi \left(\frac{p}{k}\right)^l \frac{1}{p^2 - k^2}.$$

For $\alpha \leq 1/p$, which is the case in this model, $I_l^q / I_l \leq 1/2\,(2l+1)$ and is small for large l. It is easy to see that if r_n is used as the upper limit, then $I_l^a / I_l = a^2 / r_n^2 \ll 1$.

For large l, $I_{l-1} \ll I_{l+1}$; Substituting I_{l+1} in Eq. (2) and multiplying by the number of nucleons in the nucleus, we obtain

$$\frac{1}{\tau} = \frac{3(l+1)}{2(2l+1)} g^2 \frac{\mu^3 k}{p^3 M} \left(\frac{p}{k}\right)^{2l+2} \frac{1}{\tau_0}. \qquad (4)$$

for $g^2 = 10$, $l = 5$, $\epsilon = 37$ mev, $k = \sqrt{2\mu\epsilon}$, $p = \sqrt{2M\mu}$, we get for the conversion coefficient $\tau_0 / \tau \sim 5 \times 10^6$. For $p = \sqrt{M\mu}$, we get $\tau_0 / \tau \sim 2 \times 10^5$.

If $r_n < l/p$, then we put I_{l+1} in place of I_{l+1}^a, with $a = r_n$. The value of the conversion coefficient in this case is also $\sim 10^6 - 10^5$. The large values of the conversion coefficient are explained by the large l, on the one hand, and on the other, by the fact that the momentum of the nucleon which absorbed the meson is larger than the momentum of the meson ($p/k \gg 1$), so that the centrifugal barrier is more transparent.

Thus the lifetime of the bound Λ-particle is approximately 10^6 times smaller than for the free Λ-particles, and the decay ought to be almost exclusively conversion, which contradicts experiment. Our conclusions do not depend on the character of the internal region, since only the small dimension of the Λ-particle is essential for it. It appears improbable that the contribution from the internal region could compensate the contribution of the external region, since such a compensation would have to be one of extraordinarily great exactness.

The problem of the spin of the Λ-particle is of interest also, aside from any dependence on the model considered by us. Suppose that the origins of the metastability of the Λ-particle are not due to a large angular momentum, but to some sort of forbidden principle[5,6] connected, for example, with the isotopic spin. In this case, the proof carried out above that there can be no large spin associated with the Λ-particle, is not valid. This is connected with the fact that now the effective dimensions of the Λ-particle can be of the order of $l \geq 4$, and in this case we cannot draw any conclusions on the magnitude of conversion in this interval. However, even in this case, it can be shown that the presence of a very large spin in the Λ-particle leads to an anomalously large "conversion coefficient". In fact, it follows from Eqs. (3) and (4) that the region of integration from $1/\mu$ to ∞, in the case of $l \geq 4$, gives a contribution which is of the order of magnitude of the integral from 0 to ∞ and, consequently, all our estimates maintain their force.

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Binding Energy of Hyper-Nuclei

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ONE of the fundamental discoveries of recent times has been the discovery of Danisz and Pinewski[1] of new types of nuclei--the so-called hyper- or Λ-nuclei, which contain a Λ^0-particle in addition to neutrons and protons. It has been shown that in many cases the binding energy of the Λ^0-particle to the nuclei is positive and that the comparatively short life of the Λ-nuclei ($\tau \sim 10^{-10}$ sec) is connected with the instability of the Λ^0-particle itself[1-12]. The character of the interaction of the Λ^0-particles with nucleons (N) has not yet been established[3,7,13]. Observations have permitted us to establish, although with low accuracy, the binding energy of a series of light nuclei[1-12] We have plotted the most trustworthy values for the binding energy (see Figure), which illustrates the dependence of the binding energy