antinucleons with nuclei. However, in the collisions of antinucleons with nucleons bound to the nucleus, there is the possibliity of other processes ("extraordinary" annihilation) in which the number of π-mesons emitted is less than or equal to one.

Annihilation with the emission of a single π-meson can take place in the collisions of an antinucleon with a nucleus of atomic mass $A \geq 2$. Annihilation which is not accompanied by emission of even a single meson is possible only in the collisions of an antinucleon with a nucleus of atomic mass $A \geq 3$, It is not difficult to see that the processes of one-meson and zero-meson annihilation of antinucleons occur in processes inverse to those in which antinucleons are created in the collisions of π-mesons and nucleons with nucleons.

Keeping in mind the possibility of setting up experiments, we have considered below several processes of "extraordinary" annihilation of antinucleons which are characterized by the fact that the number of particles in the final state is equal to 2.

In the case of collisions with deuterons, the following reactions are possible:

$$\text{a)}\ \tilde{p}+d \rightleftarrows \pi^0+n;\quad \text{a}')\ \tilde{n}+d \rightleftarrows \pi^0+p;$$
$$\text{b)}\ \tilde{p}+d \rightleftarrows \pi^-+p;\quad \text{b}')\ \tilde{n}+d \rightleftarrows \pi^++n.$$

According to the principle of charge symmetry, the cross sections of reactions of type a) are equal; the cross sections of reactions of type b) are also equal.

It is not difficult to show that charge independence requires that the cross sections of type b) be twice those of reactions of type a). From the experimental point of view, the reactions b) are especially interesting. Here four charged particles take part. The ratio of the cross sections of the direct and inverse reactions of b), for the conditions of identical energy in the center-of-mass system, is equal to

$$\frac{\sigma(\tilde{p}+d\rightarrow\pi^-+p}{\sigma(\pi^-+p\rightarrow\tilde{p}+d)}=\frac{(2S_\pi+1)(2S_p+1)}{(2S_{\tilde{p}}+1)(2S_d+1)}\frac{k^2}{q^2}=\frac{k^2}{3q^2},$$

where $S_{\tilde{p}}$, S_d are the magnitudes of the spins of the antiproton, the deuteron, etc.; k and q are the momenta of the π-mesons and the antiprotons in the center-of-mass system. Investigation of the direct and reverse reactions of b) give the possibility of verifying the correctness of the assumption that the spin of a negative particle with proton mass is equal to one-half. For example, the cross section of the reaction b) of the annihilation into a deuteron of an antiproton with kinetic energy 500 mev ought to be 1.6 times greater than the cross section of the reaction of the creation of a deuteron and an antiproton in the collision with a proton of a π-meson with energy 4.6 bev.

Let us consider processes of single meson annihilation of an antinucleon in a nucleus with $A = 3$:

$$\text{c)}\ \tilde{p}+\text{He}^3 \rightleftarrows d+\pi^0;\quad \text{c}')\ \tilde{n}+\text{H}^3 \rightleftarrows d+\pi^0;$$
$$\text{d)}\ \tilde{p}+\text{H}^3 \rightleftarrows d+\pi^-;\quad \text{d}')\ \tilde{n}+\text{He}^3 \rightleftarrows d+\pi^+.$$

The ratio of the cross section of reactions of types c) and d), according to charge independence, is equal to 2. From the experimental viewpoint, the reverse reactions c′) and d′) present interest.

Zero-meson annihilation of an antinucleon is illustrated by the following reactions:

$$\text{e)}\ \tilde{p}+\text{H}^3 \rightleftarrows n+n;\quad \text{e}')\ \tilde{n}+\text{He}^3 \rightleftarrows p+p;$$
$$\text{f)}\ \tilde{p}+\text{He}^3 \rightleftarrows n+p;\quad \text{f}')\ \tilde{n}+\text{H}^3 \rightleftarrows p+n.$$

Here the reverse reaction to e′)--the formation of He^3 in the collision of two protons--is of experimental interest.

Experimental investigation of the above-mentioned processes is of fundamental significance. It is reasonable to expect that the processes of single-meson and zero-meson annihilation are significantly less probable than the process of multiple meson annihilation. This follows, for example, from Fermi's statistical theory of multiple production of mesons.

It should be emphasized that the probability of processes of "extraordinary" annihilation of antinucleons could be increased if one could have especially strong nucleon-antinucleon interactions of the type assumed in the Fermi-Yang model.

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Solution of the Schwinger Equation in the Bloch-Nordsieck Model

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IN the consideration of the scattering of an electron in an external field, Bloch and Nordsieck[1] assumed a method of approximate solution of the

Dirac equation, which is valid in the region of low frequencies. The zeroth approximation of this method is equivalent to substitution in the initial equations of the c-numbers for the Dirac matrices. In the determination of the Green's function, there is definite methodological interest in making the same replacement of the matrices γ^α by the c-numbers u^α ($\alpha = 0, 1, 2, 3$) in the corresponding equations. After such a substitution the resultant equations are solved exactly.

The equation for the electronic Green's function[2] has the form

$$\left\{ iu^\alpha \frac{\partial}{\partial x^\alpha} - m - \sqrt{4\pi}\, eu^\alpha A_\alpha(x) - \sqrt{4\pi}\, ieu^\alpha \int D_{\alpha\beta}(\xi, x|A) \frac{\delta}{\delta A_\beta(\xi)} d\xi \right\} G(x, y \mid A) = -\delta(x-y), \tag{1}$$

where $D_{\alpha\beta}(\xi, x \mid A)$ is the photon Green's function. In the given model, as can be easily shown, vacuum polarization is absent; consequently, we have, in momentum representation,

$$D_{\alpha\beta}(p) = \frac{g^{\alpha\beta}}{\lambda^2 - p^2 - i\varepsilon} - \frac{g^{\alpha\beta}}{M^2 - p^2 - i\varepsilon},$$

$$M \to \infty, \; g^{00} = 1, \; g^{ii} = -1.$$

Equation (1) takes the form

$$\left\{ (uk) - m - \int (up) A_\alpha(p) \frac{\delta}{\delta A_\alpha(p)} dp - \sqrt{4\pi} \frac{e}{(2\pi)^2} \int u^\alpha A_\alpha(p)\, dp - i\sqrt{4\pi} \frac{e}{(2\pi)^2} \times \int u^\alpha D_{\alpha\beta}(p) \frac{\delta}{\delta A_\beta(p)} dp \right\} G(k \mid A) = -1, \tag{2}$$

in momentum representation, where $(uk) = u^\alpha k_\alpha$; $m \to m - i\epsilon$. Here we have used the invariance of the Green's function under the transformation

$$G(x, y \mid A) = G(y - x \mid T_x A);$$

$$D(\xi, x \mid A) = D(\xi - x \mid T_x A),$$

where T_x is the displacement operator $T_x A(\xi) = A(\xi + x)$.

For the solution of Eq. (2) we make use of the method of proper time of Fock[3]:

$$G(k \mid A) = i \int_0^\infty \Phi(\nu, k \mid A)\, d\nu.$$

We then have for the function $\Phi(\nu, k \mid A)$ the equation

$$-i \frac{\partial}{\partial \nu} \Phi(\nu, k \mid A) = H\Phi(\nu, k \mid A); \quad \Phi(0, k \mid A) = 1,$$

where H is an operator which appears on the left side of Eq. (2). Here the relation

$$\frac{\delta}{\delta A_\alpha(p)} \Phi(\nu, k \mid A) = -\sqrt{4\pi} \frac{eu^2}{(2\pi)^2} \frac{e^{i(up)\nu} - 1}{(up)} \Phi(\nu, k \mid A)$$

holds. As a result, we can write

$$G(k \mid A) = i \int_0^\infty d\nu e^{-i[m-(uk)]\nu + f(\nu)} \times \exp\left\{ \sqrt{4\pi} \frac{e}{(2\pi)^2} \int \frac{e^{-i(up)\nu} - 1}{(up)} u^\alpha A_\alpha(p)\, dp \right\}; \tag{3}$$

$$f(\nu) = \frac{e^2}{4\pi^3} u^\alpha u^\beta \int_0^\nu d\nu \int \frac{e^{-i(up)\nu} - 1}{(up)} D_{\alpha\beta}(p)\, dp.$$

Integrating over p-space, we get

$$f(\nu) = -\frac{i}{2} e^2 M\nu + \frac{e^2}{\pi} \ln \frac{M}{\lambda} - \frac{e^2}{\pi} \int_0^\infty \frac{\exp\{-i\sqrt{\lambda^2 + p^2}\,\nu\}}{(\lambda^2 + p^2)^{3/2}} p^2\, dp. \tag{4}$$

For $\lambda = 0$ we have

$$f(\nu) = -\frac{i}{2} e^2 M\nu + \frac{e^2}{\pi} \ln \frac{M}{m'} + \frac{e^2}{\pi} \ln m'\nu. \tag{5}$$

The first two terms in Eqs. (4) and (5) are removed by renormalization of the electron mass and by the Z-factor of the electronic Green's function.

On the basis of Eq. (5), we have

$$m' = m + {}^1\!/_2 e^2 M;$$

$$G'(k \mid A) = Z^{-1} G(k \mid A), \quad Z = (M/m')^{e^2/\pi},$$

where m' is the experimental mass of the electron, $G'(k \mid A)$ is the renormalized Green's function. Finally, renormalization of the electronic Green's function for $\lambda = 0$ takes the form

$$G'(k \mid A) = i \int_0^\infty d\nu e^{-i(m' - (uk))\nu} (m'\nu)^{e^2/\pi} \times \exp\left\{ \sqrt{4\pi} \frac{e}{(2\pi)^2} \int \frac{e^{-i(up)\nu} - 1}{(up)} u^\alpha A_\alpha(p)\, dp \right\}. \tag{6}$$

In particular,

$$G'(k \mid 0) = \frac{1}{m' - |k|} \left| 1 - \frac{|k|}{m'} \right|^{-e^2/\pi}; \tag{7}$$

here, we have set $u^\alpha = k^\alpha / |k|$. A formula similar to Eq. (7) has been obtained by Abrikosov[4].

Applying the Green's function (6), it is not difficult to obtain the following formula for the effective cross section of the scattering of an electron in an external field with radiation of n photons having energy in the interval from E_1 to E_2, independent of the number of longitudinal photons radiated in this case:

$$\sigma_n = \sigma_0 \frac{1}{n!}\left\{\frac{2e^2}{\pi}\ln\frac{E_2}{E_1}\right\}^n \quad \text{при} \quad \varepsilon_i \ll E_{\text{эл}},\ \mathbf{p}_i \ll \Delta\mathbf{P}_{\text{эл}}.$$

In conclusion, I express my deep gratitude to Acad. N. N. Bogoliubov for his direction of the work.

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On the Theory of Hyperons

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THE proposed idea consists in attributing to the nucleon an internal structure very suggestive of the structure of the hydrogen atom (see Ref. 1). We shall consider the nucleon as a system of two hypothetical particles ("m-particles"), which interact by means of a certain strong field χ. The essential difference of this model from the hydrogen atom is that the strong field is not Coulombic and is different from potentials considered earlier (the Yukawa potential and others).

In Ref. 2, the author has shown that, within the framework of the basic principles of the existing theory of elementary particles, a consideration of the relativistic field is possible, which gives (for a point source) a potential which falls off very rapidly with distance. The potential can be represented by the approximate formula

$$V(r) = -(\pi a / \sqrt{2} r)\, e^{-r^2/2\lambda^2}, \tag{1}$$

where r is the distance from the source, a is the χ-charge of the source, λ is some positive constant-the "elementary length" (we limit ourselves to the case $\varkappa = 0$ [2], which presents the greatest interest). The field which leads in the final count to the potential (1) is defined by the equation

$$(\Box - \lambda^{-4}(x^\mu - x_0^\mu)(x^\mu - x_0^\mu))\,\chi = -4\pi a \delta(x - x_0). \tag{2}$$

It is important, from the physical point of view, to emphasize that 1) for the χ-field, the case of a free field makes no sense, i.e., a field without sources does not exist; further, assuming formally that $a = 0$, we get on the left side of Eq. (2) an isolated point x_0 which corresponds to the position of a "virtual" source; 2) the quantization of Eq. (2) cannot give a state with definite 4-momentum, only states with definite 4-angular momentum. In particular, there does not exist for Eq. (2) a state with definite energy, i.e., there is no stationary state. Therefore, the particles of the χ-field, if they exist in nature, cannot be observed by experiments with counters, Wilson chambers, etc. One can show that, from the experimental viewpoint, these would not be particles, and in this sense, we have come across a possible limit of the applicability of the corpuscular-wave dynamics.

The potential (1) can be understood in dual fashion (limiting ourselves to fields of the Bose type with spin not greater than 1): either as the fourth component of a 4-vector, analogous to the Coulomb potential, or as a scalar or pseudoscalar. The first possibility presents the greater interest, since in this case all the constants of the problem can be determined up to the determination of the mass spectrum (but not by the mass spectrum).

Neglecting the spin of the particles which make up the nucleon by our hypothesis in first approximation, we obtain the wave equation (after separating out the motion of the center-of-mass):

$$\left\{-\left(v - i\hbar\frac{\partial}{\partial t} + aV\right)^2 + c^2\left(-i\hbar\nabla - \frac{a}{c}\mathbf{V}\right)^2 + c^2\left(mc + \frac{\gamma}{c}\Phi\right)^2\right\}\psi = 0, \tag{3}$$

where $(V, \mathbf{V})$ are the vector, Φ the scalar, potential, a, γ are corresponding binding constants, m is the reduced mass for the v of internal motion. Let us assume that in our system of coordinates $\mathbf{V} = 0$, V is given by Eq. (1) and that Φ is the potential of the scalar meson field. For simplicity we assume that both particles of our system have the same mass m and possess unit χ-charges and meson charges γ, in which $\gamma = g/2$ (g is the meson charge of the nucleon as a whole)*. In view of this,

$$\gamma^2/\hbar c = g^2/4\hbar c \approx (^{25}/_4)\,e^2/\hbar c \approx {}^6/_{137} \ll 1, \tag{4}$$