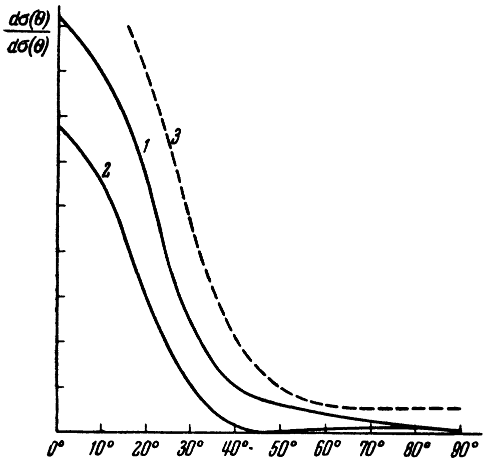

distributions at 5, 6 and 7 mev do not differ appreciably from the distribution at 8 mev, and the distribution at 5 and 6 mev for angles from 70^0 to 90^0 is in better agreement with the experimental curve than curve *1*. At 5 to 8 mev, the experimental distributions are identical within the limits of experimental error and are represented by curve 3. The reaction could also be calculated by Butler's theory[4], from the reverse reaction Be^8 $(dp)Be^9$. The corresponding distribution at E_p = 8 mev is given by curve *2*. From a comparison of curves *1* and *2* with each other on the one hand, and with the experimental curve *3* on the other hand, we can see that curve *1* is in somewhat better agreement with experiment than curve *2* (especially for angles from 35° to 70°), although it does not differ appreciably as a whole from Butler's curve *2*.

It is well known that Butler's approximation is equivalent to a Born approximation in which the interior of the nucleus is neglected. Our result shows that an approximate calculation, which includes the interior of the nucleus, does not essentially change the angular distribution of deuterons in the $Be^9(pd)Be^8$ reaction that is obtained on the basis of Butler's theory.

We mention in conclusion that we have also calculated the angular distribution for a square well (instead of a Yukawa potential) interaction between the proton and the neutron of the nucleus. The result is practically the same as the distribution obtained from Eq. (3).

I consider it an obligation to express my gratitude to Professor V. I. Mamasakhlisov for his interest and for a number of suggestions.

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Polarization of Elastically Scattered Deuterons

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It is known that the scattering of polarized particles provides important information regarding the spin dependence of their interactions. High energy polarized particles can be obtained by scattering on nuclei. The polarization is due to the spin-orbit interaction. A number of papers have presented calculations of the polarization of particles with spin 1/2. Lepore[1] has given a calculation of the scattering of polarized nucleons by nuclei with zero spin. The present note is concerned with the elastic scattering of polarized deuterons by nuclei with zero spin. This problem is similar to the problem of triplet nucleon-nucleon scattering[2].

The polarization of a deuteron beam is characterized by the three components of the spin vector and the five components of the symmetric second rank spin tensor with zero trace. Before collision the deuteron is described by the distorted plane wave

$$\Psi_0 = \exp\left[i\left(\mathbf{K}_0\mathbf{r} - \alpha \ln 2Kr\right)\right]\chi_0, \tag{1}$$

where χ_0 is the initial deuteron spin function. Polarization of the ingoing deuteron beam is characterized by the following quantities:

$$\mathbf{P}_{\text{in}} = (\chi_0 \mathbf{S} \chi_0^+), \qquad \langle T_{ik}\rangle_{\text{in}} = (\chi_0 T_{ik} \chi_0^+), \tag{2}$$

where T_{ik} is a symmetric spin tensor with zero trace:

$$T_{ik} = \tfrac{1}{2}(S_i S_k + S_k S_i) - \tfrac{2}{3}\delta_{ik}.$$

the constant $\alpha = Ze^2\mu / K\hbar$.

For the purpose of obtaining the scattered amplitude the wave function of the system must be expanded in a series of eigenfunctions of the operators

J^2, J_z, L, S, where J = L + S; L is the orbital moment of relative motion and S is the spin of the deuteron . Such an expansion of the wave function is accomplished through the use of projection operators[2]:

$$\Pi_l^+ = [l+1+(l+2)(\mathbf{SL}) + (\mathbf{SL})^2] / (l+1)(2l+1); \tag{3}$$

$$\Pi_l^0 = [\,l(l+1) - (\mathbf{SL}) - (\mathbf{SL})^2] / l(l+1);$$

$$\Pi_l^- = [-l-(l-1)(\mathbf{SL}) + (\mathbf{SL})^2] / l(2l+1).$$

These operators enable us to write the wave function of the system in the form

$$\Psi = \sum_{l=0}^{\infty} (2l+1)^{1/2} i^l \left[A_l^+ \Pi_l^+ \frac{U_l^+(Kr)}{Kr} + A_l^0 \Pi_l^0 \frac{U_l^0(Kr)}{Kr} + A_l^- \Pi_l^- \frac{U_l^-(Kr)}{Kr} \right] Y_l^0 \chi_0. \tag{4}$$

The usual method of calculating the scattered amplitude[1,3] gives

$$F(\theta) = A(\theta) + B(\theta)(\mathbf{Sn}) + C(\theta)(\mathbf{Sn})^2 + {}^1/_2 D(\theta)\{(\mathbf{Sk}_0)(\mathbf{Sk}) + (\mathbf{Sk})(\mathbf{Sk}_0)\}, \tag{5}$$

where $\mathbf{k}_0$ is a unit vector parallel to the momentum of the ingoing deuteron; $\mathbf{k}$ is a unit vector parallel to the momentum of the scattered deuteron; $\mathbf{n} = [\mathbf{k}_0, \mathbf{k}] / \sin\theta$;

$$A(\theta) = \frac{1}{K} \sum_{l=0}^{\infty} \frac{1}{(2l+1)^{1/2}} \Big\{ \exp(i\delta_l^+) \sin\delta_l^+ \left(Y_l^0 + \frac{2\cos\theta}{l+1} \frac{\partial V_l^0}{\partial(\cos\theta)} \right) + (2l+1)\exp(i\delta_l^0)\sin\delta_l^0 \left(Y_l^0 - \frac{2\cos\theta}{l(l+1)} \frac{\partial Y_l^0}{\partial(\cos\theta)} \right) - \exp(i\delta_l^-)\sin\delta_l^- \left(Y_l^0 - \frac{2\cos\theta}{l} \frac{\partial Y_l^0}{\partial(\cos\theta)} \right) \Big\};$$

$$B(\theta) = \frac{i\sin\theta}{K} \sum_{l=0}^{\infty} \frac{1}{(2l+1)^{1/2}} \left\{ \frac{2l+3}{2(l+1)} \exp(i\delta_l^+)\sin\delta_l^+ - \frac{2l+1}{2l(l+1)} \exp(i\delta_l^0)\sin\delta_l^0 - \frac{2l-1}{2l} \exp(i\delta_l^-)\sin\delta_l^- \right\} \frac{\partial Y_l^0}{\partial(\cos\theta)};$$

$$C(\theta) = \frac{1}{K} \sum_{l=0}^{\infty} \frac{1}{(2l+1)^{1/2}} \left\{ \frac{1}{l+1} \exp(i\delta_l^+)\sin\delta_l^+ - \frac{2l+1}{l(l+1)} \exp(i\delta_l^0)\sin\delta_l^0 + \frac{1}{l}\exp(i\delta_l^-)\sin\delta_l^- \right\} \left\{ l(l+1) Y_l^0 - 2\cos\theta \frac{\partial Y_l^0}{\partial(\cos\theta)} \right\};$$

$$D(\theta) = -\frac{1}{K} \sum_{l=0}^{\infty} \frac{1}{(2l+1)^{1/2}} \left\{ \frac{1}{l+1} \exp(i\delta_l^+)\sin\delta_l^+ - \frac{2l+1}{l(l+1)} \exp(i\delta_l^0)\sin\delta_l^0 + \frac{1}{l}\exp(i\delta_l^-)\sin\delta_l^- \right\} \frac{\partial Y_l^0}{\partial(\cos\theta)},$$

where δ_l^+, δ_l^0, δ_l^- are scattering phases corresponding to the total quantum number $j = l+1,\ l,\ l-1$.

We can now calculate the differential cross section and the polarization. It is convenient to use a density matrix[4,5] for the calculation of these quantities. For a deuteron beam this matrix is[5]

$$\rho = {}^1/_3 \{1 + {}^3/_2 (\mathbf{P}_{\text{in}}\mathbf{S}) + {}^3/_2 \langle T_{ik}\rangle_{\text{in}} (\mathbf{S}_i\mathbf{S}_k + \mathbf{S}_k\mathbf{S}_i)\}, \tag{6}$$

so that

$$\mathbf{P}_{\text{in}} = \mathrm{Sp}(\mathbf{S}\rho),$$

$$\langle T_{ik}\rangle_{\text{in}} = \mathrm{Sp}\{[{}^1/_2(S_iS_k + S_kS_i) - {}^2/_3\delta_{ik}]\rho\}.$$

Two special cases are of interest: a) the polarization of an unpolarized incident deuteron beam, and b) the differential cross section for a polarized beam. In the first case the polarization vector and the polarization tensor will be expressed as follows:

$$\mathbf{p} = \frac{4}{3} \frac{\mathrm{Re}\left[\left(A + C + D\frac{\cos\theta}{2}\right)B^*\right]}{(d\sigma/d\Omega)_0} \mathbf{n} = P(\theta)\,\mathbf{n}, \tag{7}$$

$$\langle T_{ik}\rangle = P_1(\theta)(n_in_k - {}^1/_3\delta_{ik})$$

$$+ P_2(\theta)(k_{0i}k_k + k_ik_{0k} - {}^2/_3\delta_{ik}\cos\theta)$$
$$+ P_3(\theta)(k_{0i}k_{0k} - {}^1/_3\,\delta_{ik}) + P_4(\theta)(k_ik_k - {}^1/_3\,\delta_{ik}),$$
$$P_1(\theta) = \frac{|B|^2 + |C|^2 + 2\mathrm{Re}\,[(A + D\cos\theta)\,C^*]}{3\,(d\sigma/d\Omega)_0};$$
$$P_2(\theta) = \frac{{}^3/_4\,|D|^2\cos\theta + \mathrm{Re}\,[(A+C)\,D^*]}{3\,(d\sigma/d\Omega)_0};$$
$$P_3(\theta) = \frac{-{}^1/_4\,|D|^2 - \frac{1}{\sin\theta}\,\mathrm{Im}\,(B^*D)}{3\,(d\sigma/d\Omega)_0};$$
$$P_4(\theta) = \frac{-{}^1/_4\,|D|^2 + \frac{1}{\sin\theta}\,\mathrm{Im}\,(B^*D)}{3(d\sigma/d\Omega)_0}.$$

In the second case,

$$d\sigma/d\Omega = (d\sigma/d\Omega)_0 + (d\sigma/d\Omega)_1 + (d\sigma/d\Omega)_2; \qquad (8)$$
$$\left(\frac{d\sigma}{d\Omega}\right)_0 = |A|^2 + \frac{2}{3}|B|^2 + \frac{2}{3}|C|^2 + \left(\frac{\cos^2\theta}{2} + \frac{1}{6}\right)|D|^2 + \frac{4}{3}\mathrm{Re}\,[A^*(C + D\cos\theta)] + \frac{2}{3}\mathrm{Re}\,(C^*D)\cos\theta; \qquad (9)$$
$$(d\sigma/d\Omega)_1 = 2\,\mathrm{Re}\,[(A + C + {}^1/_2\,D\cos\theta)\,B^*]\,(\mathbf{P}_{\mathrm{in}}\mathbf{n}); \qquad (10)$$
$$(d\sigma/d\Omega)_2 = \{|B|^2 + |C|^2 + 2\,\mathrm{Re}\,[(A + D\cos\theta)\,C^*]\}\,n_in_k\langle T_{ik}\rangle_{\mathrm{in}} + \{{}^3/_4\,|D|^2\cos\theta + \mathrm{Re}\,[(A+C)\,D^*]\}(k_{0i}k_k + k_ik_{0k})\langle T_{ik}\rangle_{\mathrm{in}} + \{-{}^1/_4\,|D|^2 + (\sin\theta)^{-1}\,\mathrm{Im}\,(B^*D)\}\,k_{0i}k_{0k}\langle T_{ik}\rangle_{\mathrm{in}} - \{{}^1/_4\,|D|^2 + (\sin\theta)^{-1}\,\mathrm{Im}\,(B^*D)\}\,k_ik_k\langle T_{ik}\rangle_{\mathrm{in}}. \qquad (11)$$

The cross section for an unpolarized deuteron beam is given by (9), whereas (10) and (11) result from initial polarization of the deuteron beam, with (10) corresponding to the polarization vector and (11) corresponding to the polarization tensor.

I take this opportunity to express my thanks to G. R. Khutsishvili for his interest and for valuable discussions.

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Charge Distribution of Mesons in Nucleon-Antinucleon Annihilation

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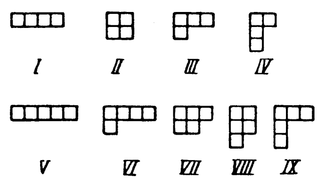
BELEN'KII and Rozental'[1] have studied the production of stars in antinucleon annihilation. On the basis of a statistical theory of multiple particle production they calculated the probabilities for processes of different multiplicites. We



present here the charge distribution which is calculated on the basis of isotopic spin conservation (see Refs. 2 and 3). As usual, p and n denote a proton and a neutron, while $\overline{p}$ and $\overline{n}$ denote an antiproton and an antineutron; annihilation products (π-mesons) are denoted by the signs of their charges. The charge distribution for $p\overline{n}$ is obtained from the distribution for $\overline{p}n$ by reversing the signs of meson charges. Table I shows the subdivision of processes of given multiplicity according to the charge states. If, for example, the annihilation cross section for $p\overline{p}$ into two mesons is σ_2, it can be seen from Table I that 0.167 of this cross section is due to the process $p + \overline{p} \to \pi^0 + \pi^0$ and 0.833 is due to $p + \overline{p} \to \pi^+ + \pi^-$.

If statistical theory is not used but only conservation of total isotopic spin, the charge distribution for a given multiplicity can be obtained only for processes that are characterized by a definite isotopic spin T, its projection T_3 and a definite Young scheme[4]. Such distributions are given in Refs. 5-7 for two and three mesons. We have done the same for four and five mesons. The results are given in Tables II and III. The Roman numerals at the top of the Tables indicate the Young schemes which correspond to the numerals in the Figure.