distribution of γ-rays must have the form $A + B \cos^2\theta$, where

$$A = a + \frac{b}{2}\left[1 - \int_0^{\infty} d\varepsilon_\gamma \int_{\alpha}^{\infty} \frac{\cos^2\vartheta\, F(\varepsilon_\pi)\, d\varepsilon_\pi}{\sqrt{\varepsilon_\pi^2 - 1}}\right];$$

$$B = \frac{b}{2}\left[3 \int_0^{\infty} d\varepsilon_\gamma \int_{\varepsilon_{min}}^{\infty} \frac{\cos^2\vartheta\, F(\varepsilon_\pi)\, d\varepsilon_\pi}{\sqrt{\varepsilon_\pi^2 - 1}} - 1\right].$$

The same conclusion was reached earlier in Ref. 2. Using this result and Rosenfeld's[3] suggestion regarding the existence of an "isotropic" angle θ_N^* for charged mesons we can also conclude that the gamma flux at angle θ_N^* is independent of the ratio of a and b in the angular distribution of π^0-mesons.

The above-mentioned properties of the gamma spectrum and flux for the "isotropic" angle are retained in the more general case when the angular and energy distributions of the π^0-mesons are of the form $a(\epsilon_\pi) + b(\epsilon_\pi) \cos^2\theta$. It should be noted that in this instance when the derivative of the measured gamma spectrum is multiplied by the gamma-ray energy we obtain the function $a(\epsilon_\pi) + 1/3\, b(\epsilon_\pi)$.

I take this opportunity to acknowledge my indebtedness to B. M. Pontecorvo for a discussion of the above results.

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On the Stability of the Phase Boundaries Between Normal and Superconducting States

I. E. DZIALOSHINSKII
Institute for Physical Problems, Academy of Sciences, USSR

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NOT long ago, Beck's paper[1] appeared, in which the author, using London's equations for the magnetic field in a superconductor, found an instability in the boundary between the n- and s-phases as regards a periodic (along the surface) perturbation of the shape of the boundary. It is immediately clear from Eqs. (25) and (34) of Ref. 1 that the instability found by the author, at least as regards a perturbation with a period much greater than the penetration depth of the magnetic field in the superconductor, is explained by the well-known fact that London's equations lead to a negative surface energy on the boundary[2]. Since a negative surface tension contradicts experimental results for thin films, the analysis of the problem of stability should be based, not on London's equations, but on the theoretical calculations of Landau and Ginzburg[3], which give a positive value to the surface energy. In Landau and Ginzburg's theory, the problem of stability becomes the problem of a unique solution at infinity under corresponding boundary conditions. A strict analysis such as this can scarcely be performed by means of the non-linear equations of the theory. The only complete solution is for a perturbation with a period much greater than the penetration depth. In this case it can be made equal to zero, so that B = 0 in the superconducting phase. Also, in agreement with the theory of Landau and Ginzburg, we attribute a positive energy to the boundary between the n- and s-phases, which we write in the usual form, $(H_k^2/8\pi)\Delta$, where H_k is the critical field and Δ is a constant with the dimensions of length. The free energy change taking place with a variation in the shape of the boundary is written (for this case)

$$\delta F = \frac{H_k^2}{8\pi}\,\delta V_n + \Delta \frac{H_k^2}{8\pi}\,\delta S - \frac{1}{8\pi}\,\delta \int_{V_n} H^2 dV. \qquad (1)$$

The equilibrium of a plane boundary is studied in relation to an arbitrary (but not specifically oriented, as in Ref. 1) periodic perturbation. The stability of the boundary of arbitrary form is analyzed in the same way because any small part of the boundary can be thought of as a plane. The integral on the right side of Eq. (1) is easily transposed so that to calculate δF, correct to a second degree term over a small variation in the boundary δz, it is sufficient to know the magnetic field variation $\delta \mathbf{H}$ with an accuracy to a term of the first order of δz. By means of a simple transformation we can show that

$$-\frac{1}{8\pi}\,\delta \int_{V_n} H^2 dV = -\frac{H_0^2}{8\pi}\,\delta V_n + \frac{1}{8\pi}\int_S [\mathbf{A}_0 \delta \mathbf{H}]\, ds, \qquad (2)$$

where $\mathbf{A}_0$ is the vector potential of the unperturbed constant field $\mathbf{H}_0$; the integral on the right-hand side of Eq. (2) is taken over the surface of the perturbed boundary. In the derivation of Eq. (2) the vector potentials $\mathbf{A}$ and $\mathbf{A}_0$ are so normalized that

$A_0 = 0$ when there is no perturbation and $A = 0$ where there is a perturbation of the boundary.

If a small periodic perturbation is put on the boundary

$$\delta z = a(\mathbf{f})\, e^{i\mathbf{rf}},$$

then for the component δH, parallel to f, correct to a first order quantity for $a(f)$, we have

$$\delta H_{\parallel} = H_0 f a(\mathbf{f}) e^{i(\mathbf{fr}) - fz}, \quad (3)$$

$\delta H_{\perp}$ being reduced to zero. Using (2) and (3) we determine the free energy change (at the same time $H_0 = H_k$, which appears to be a necessary condition of equilibrium).

$$\delta F = \frac{SH_k^2}{8\pi} \sum_{\mathbf{f}} |a(\mathbf{f})|^2 \left(\frac{\Delta f^2}{2} + f\cos^2\varphi\right), \quad (4)$$

where φ is the angle between f and H_0.

In this way $\delta F \geq 0$, which shows the stability of the boundary in relation to a smoothly changing form (it should be $|\text{grad } z| \ll z/\delta$, where δ is the penetration depth).

Equation (4) allows us to calculate the mean square of the fluctuation of the displacement of the boundary. By using the general theory of thermodynamic fluctuation[4] we find

$$\overline{|a(\mathbf{f})|^2} = \frac{8\pi kT}{SH_k^2(\Delta f^2 + 2f\cos^2\varphi)},$$

from which

$$\overline{(\delta z^2)} = \frac{2kT}{\pi H_k^2} \int \frac{d\mathbf{f}}{\Delta f^2 + 2f\cos^2\varphi}. \quad (5)$$

The integral diverges logarithmically for large f, but since our analysis is correct only insofar as $f \ll 1/\delta$ (δ is the penetration depth), we should stop the integration at $f_0 = 1/\lambda$, $\lambda \sim \delta$. The calculation gives

$$\overline{(\delta z)^2} = (4kT/\Delta H_k^2)\ln(\Delta/\lambda). \quad (6)$$

For mercury when $T \sim 1°$ K, with the exception of the region near the lambda point, $\delta z \sim 10^{-7}$ cm.

We note that a difference from the usual result for the fluctuation of the displacement of the boundary in the absence of magnetic field, where $\overline{(\delta z)^2} \sim \int f^{-2} d\mathbf{f}$, is that the integral (5) corresponds to lower limit.

I gratefully acknowledge the help of E. M. Lifshitz in preparing this paper and in obtaining the results.

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Mechanical Phase Analyzers for Treatment of Experimental Data on the Scattering of Particles without Spin on Particles with Spin 0 or 1/2

N. P. KLEPIKOV
Institute for Nuclear Problems, Academy of Sciences, USSR

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As is known, the scattering amplitude for elastic scattering of particles without spin on particles with spin 1/2 in a state with a definite isotopic spin is of the form

$$f(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} [(l+1)(\rho_l^+ - 1) \quad (1)$$

$$+ l(\rho_l^- - 1)] P_l(\cos\theta) + \frac{\sigma \mathbf{n}}{2k} \sum_{l=1}^{\infty} (\rho_l^- - \rho_l^+) P_l^1(\cos\theta),$$

where $P_l(\cos\theta)$ and $P_l^1(\cos\theta)$ are the Legendre polynomials and the associated Legendre functions, k and θ are wave number and scattering angle in the center-of-mass system, and n is a unit vector perpendicular to the plane of scattering. Here we introduce the notation: $\rho_l^{\pm} = \exp 2i\delta_l^{\pm}$, where $\delta_l^{\pm}$ are the scattering phases. With the plus sign we denote the magnitudes for the states in which the total momentum j is equal to $l + 1/2$, and with the minus sign, for the states in which $j = l - 1/2$. The amplitude in Eq. (1) satisfies the relationship

$$\tfrac{1}{2}\,\mathrm{Sp}\, Im f(0) = (k/4\pi)\,\sigma, \quad (2)$$

where σ is the total scattering cross section. For the scattering on the nonpolarized particles we have for the differential cross section and polarization

$$\sigma'(\theta) = \frac{1}{2}\mathrm{Sp}\, f^+(\theta) f(\theta) \quad (3)$$

$$= \frac{1}{4k^2}\left\{\left| \sum_{l=0}^{\infty} [(l+1)(\rho_l^+ - 1) + l(\rho_l^- - 1) P_l(\cos\theta)\right|^2\right.$$