$A_0 = 0$ when there is no perturbation and $A = 0$ where there is a perturbation of the boundary.

If a small periodic perturbation is put on the boundary

$$\delta z = a(\mathbf{f})\, e^{i\mathbf{rf}},$$

then for the component δH, parallel to f, correct to a first order quantity for $a(f)$, we have

$$\delta H_{\parallel} = H_0 f a(\mathbf{f}) e^{i(\mathbf{fr}) - fz}, \qquad (3)$$

$\delta H_{\perp}$ being reduced to zero. Using (2) and (3) we determine the free energy change (at the same time $H_0 = H_k$, which appears to be a necessary condition of equilibrium).

$$\delta F = \frac{SH_k^2}{8\pi} \sum_{\mathbf{f}} |a(\mathbf{f})|^2 \left(\frac{\Delta f^2}{2} + f\cos^2\varphi\right), \qquad (4)$$

where φ is the angle between f and H_0.

In this way $\delta F \geq 0$, which shows the stability of the boundary in relation to a smoothly changing form (it should be $|\operatorname{grad} z| \ll z/\delta$, where δ is the penetration depth).

Equation (4) allows us to calculate the mean square of the fluctuation of the displacement of the boundary. By using the general theory of thermodynamic fluctuation[4] we find

$$\overline{|a(\mathbf{f})|^2} = \frac{8\pi kT}{SH_k^2(\Delta f^2 + 2f\cos^2\varphi)},$$

from which

$$\overline{(\delta z^2)} = \frac{2kT}{\pi H_k^2} \int \frac{d\mathbf{f}}{\Delta f^2 + 2f\cos^2\varphi}. \qquad (5)$$

The integral diverges logarithmically for large f, but since our analysis is correct only insofar as $f \ll 1/\delta$ (δ is the penetration depth), we should stop the integration at $f_0 = 1/\lambda$, $\lambda \sim \delta$. The calculation gives

$$\overline{(\delta z)^2} = (4kT/\Delta H_k^2)\ln(\Delta/\lambda). \qquad (6)$$

For mercury when $T \sim 1°$ K, with the exception of the region near the lambda point, $\delta z \sim 10^{-7}$ cm.

We note that a difference from the usual result for the fluctuation of the displacement of the boundary in the absence of magnetic field, where $\overline{(\delta z)^2} \sim \int f^{-2} d\mathbf{f}$, is that the integral (5) corresponds to lower limit.

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Mechanical Phase Analyzers for Treatment of Experimental Data on the Scattering of Particles without Spin on Particles with Spin 0 or 1/2

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As is known, the scattering amplitude for elastic scattering of particles without spin on particles with spin 1/2 in a state with a definite isotopic spin is of the form

$$f(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} [(l+1)(\rho_l^+ - 1) \qquad (1)$$

$$+ l(\rho_l^- - 1)] P_l(\cos\theta) + \frac{\sigma\mathbf{n}}{2k} \sum_{l=1}^{\infty} (\rho_l^- - \rho_l^+) P_l^1(\cos\theta),$$

where $P_l(\cos\theta)$ and $P_l^1(\cos\theta)$ are the Legendre polynomials and the associated Legendre functions, k and θ are wave number and scattering angle in the center-of-mass system, and n is a unit vector perpendicular to the plane of scattering. Here we introduce the notation: $\rho_l^{\pm} = \exp 2i\delta_l^{\pm}$, where $\delta_l^{\pm}$ are the scattering phases. With the plus sign we denote the magnitudes for the states in which the total momentum j is equal to $l + 1/2$, and with the minus sign, for the states in which $j = l - 1/2$. The amplitude in Eq. (1) satisfies the relationship

$$\tfrac{1}{2}\,\mathrm{Sp}\, Im f(0) = (k/4\pi)\,\sigma, \qquad (2)$$

where σ is the total scattering cross section. For the scattering on the nonpolarized particles we have for the differential cross section and polarization

$$\sigma'(\theta) = \frac{1}{2}\,\mathrm{Sp}\, f^+(\theta) f(\theta) \qquad (3)$$

$$= \frac{1}{4k^2}\left\{\left|\sum_{l=0}^{\infty} [(l+1)(\rho_l^+ - 1) + l(\rho_l^- - 1)) P_l(\cos\theta)\right|^2\right.$$

$$+\left|\sum_{l=1}^{\infty}(\rho_l^- - \rho_l^+)\, P_l^1(\cos\theta)\right|^2\Bigg\} = \frac{1}{k^2}\sum_{l=0}^{\infty} A_l P_l(\cos\theta),$$

$$\sigma'(\theta)\,P(\theta) = \frac{1}{2}\,\mathrm{Sp}\, f^+(\theta)\,\sigma \mathbf{n} f(\theta) \qquad (4)$$

$$= \frac{1}{4k^2}\,2\left\{\sum_{l=0}^{\infty}[(l+1)(\rho_l^+ - 1) + l(\rho_l^- - 1)]\,P_l(\cos\theta)\right\} \times \left\{i\sum_{l=1}^{\infty}(\rho_l^- - \rho_l^+)\,P_l^1(\cos\theta)\right\},$$

where the product of the complex numbers is taken in the sense of the scalar product $2Z_1Z_2 \equiv Z_1^*Z_2 + Z_2^*Z_1$. The coefficients A_l are determined experimentally. From Eqs. (3) and (4) it is obvious that $\sigma'(\theta)[1 \pm P(\theta)]$ and particularly $\sigma'(0)$, $\sigma'(\pi)$ and $\sigma'(\pi/2)[1 \pm P(\pi/2)]$ are expressed as the squares of the moduli of some linear combinations of the quantities $\rho_l^{\pm}$. If we add an arbitrary real number λ under the square sign of the modulus in the expression for $\sigma'(0)$, then, because of Eq. (2), we obtain

$$4k^2\left\{\sigma'(0) - \lambda\frac{\sigma}{4\pi}\right\} + \lambda^2 \qquad (5)$$

$$= \left|\sum_{l=0}^{\infty}[(l+1)(\rho_l^+ - 1) + l(\rho_l^- - 1)] + \lambda\right|^2.$$

Furthermore, if the number of phases different from zero is finite, and there are present both phases of the highest orbital momentum p taking part in the interaction, then for the last coefficient of the expansion of the differential cross section we get the following, as can be easily verified:

$$|(p+1)\rho_p^+ + p\rho_p^- - p - 1|^2 \qquad (6)$$

$$= p^2 + \frac{2(4p-1)!\,[(p-1)!]^2\,p!\,(p+1)!}{[(2p-1)!]^3\,(2p+1)!}\,A_{2p}.$$

If the last phases are both phases of the maximum total momentum $j = p - 1/2$ then

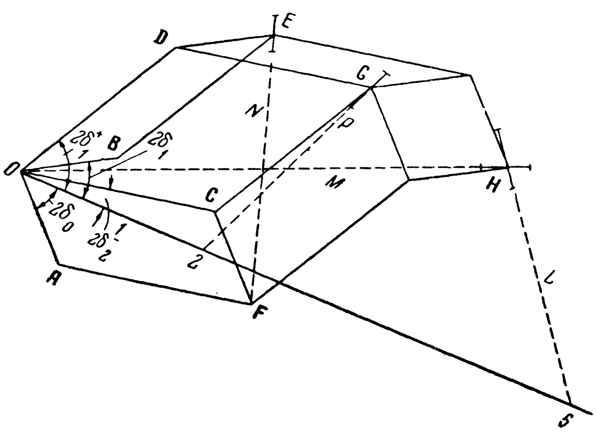
$$|\rho_{p-1}^+ + \rho_p^- - 1|^2 \qquad (7)$$

$$= 1 + \frac{4(4p-3)!\,[(p-1)!]^4}{(2p-2)!\,[(2p-1)!]^3}\,A_{2p-1},$$

and the coefficient for $P_{2p}(\cos\theta)$ in σ' is identically zero. In cases, when the polarization is not experimentally known, one can use the relationships which follow directly from Eq. (3); for instance, letting $\theta = \pi/2$.

If in Eqs. (1)-(3), (5) and (6) we let $\rho_l^+ = \rho_l^-$ then we get the relationship for the scattering of particles without spin.

For the construction of mechanical apparatus which would allow the determination of phases $\delta_l^{\pm}$ from the experimental data for angular distribution and polarization of the particles, we will consider the magnitudes $\rho_l^{\pm}$ as vectors in a plane. Then it is obvious that the mentioned relationships determine the absolute magnitudes of some linear combinations of these vectors.



Scheme of the apparatus for 4 phases for spins (0, 1/2):

$$A = \rho_0,\ B = \rho_1^-,\ C = 2\rho_2^-,\ D = 2\rho_1^+,\ E = 2\rho_1^+ + \rho_1^-,$$

$$F = 2\rho_2^- + \rho_0,\ G = 2\rho_1^+ + 2\rho_2^-,\ H = \rho_0 + \rho_1^- + 2\rho_1^+ + 2\rho_2^-,$$

$$L = 2k\sqrt{\sigma'(0)},\ M = \sqrt{4k^2\{\sigma'(0) - 6\sigma/4\pi\} + 36},$$

$$N = 2k\sqrt{\sigma'(\pi)},\ P = 2\sqrt{1 + {}^{10}/_{9}A_2}.$$

Let us consider in more detail the case of finding four phases for the scattering of particles with spins (0, 1/2). From vectors ρ_0, ρ_1^-, $2\rho_1^+$ and $2\rho_2^-$, let us construct a lever system, shown on the picture, and superimpose on it four connections which are determined by relationships (5) (for $\lambda = 6$), (7), and by relationships following from the expressions for $\sigma'(0)$ and $\sigma'(\pi)$. On the picture the connections are shown by the dotted lines. Moreover we fix all 4 degrees of freedom of the system and can read off the angles 2δ which the vectors ρ in point 0 make with the abscissa. Because of the arbitrariness of λ in Eq. (5), the point H can be connected with two arbitrary points of the real axis, choosing them in such a way that the angle between these two connections is closest to a right angle--which increases the accuracy of the apparatus. For this purpose, because of Eq. (2), it is convenient to choose λ_1 and λ_2 in such a way that $\lambda_1 < 2k^2\sigma/4\pi < \lambda_2$, wherein, in the joining of the connection to the point x of the abscissa, $\lambda = (1+p)^2 - x$ or $\lambda = p^2 + p - x$ [depending on which of the Eqs. (6) or (7), is applicable]. Because of Eq. (2), the left side of (5) is positive, and the experimental data, which do not satisfy this condition, are not compatible, since for them the relationship $|f(0)|^2 \geq [\operatorname{Im} f(0)]^2$ is not satisfiel. As is known (Ref. 1) the cross section (3) does not change if one substitutes for all phases the phase of the states of the same total momentum but of the opposite parity. This property of the cross section can be utilized to double the region of the angles which can be found by this apparatus without moving any of its parts through the point 0; for this purpose it is sufficient to rename the vectors according to the scheme $\rho_0 \leftarrow \rho_1^-$, $2\rho_1^+ \leftarrow 2\rho_2^-$. Note that only 4 relations [(5), (6) or (7) and relations for $\sigma'(0)$ and $\sigma'(\pi)$] have a simple appearance and at the same time do not include polarization data. Therefore, for finding a larger number of phases it is necessary either to consider polarization data, or to realize more complicated relations of the type (3), or, finally, to extrapolate some phases based on the measurements for other energies.

Mechanical analyzers, with an adequate amount of experimental data, permit one quickly to find phases and thus to solve the system of trigonometric equations which arise during a comparison of the expansion coefficients for the experimental angular distributions and polarization with the expressions of these coefficients by means of phases. In this type of apparatus, the connection between the phases and magnitudes, introduced from the experiment, is two-sided. Therefore, if the limits are known in which each of n independent experimental magnitudes can be found with the probability q, the apparatus allows us to find the limits in which the phases lie with the probability Q, where $q^n \leq Q \leq q$ (the right equality is realized for the complete correlation of errors of the experimental data, the left for the complete absence of the correlation). For this purpose the lengths of the connections must allow a free change within the errors of the corresponding magnitudes (in the picture these limits are shown by the segments of the couplings). Varying the position of the apparatus within the limits allowed by the connections, one can find the limits for the phases and the sensitivity of the solution to a change of experimental data.

Up to the present time the solution of the corresponding system of equations (e.g., for the scattering of π-mesons by nucleons, or neutrons by α-particles) was either carried out numerically (Refs. 2-5), or with the help of electrical analogues (Ref. 6), or graphically (Refs. 7-9). The first two methods have the disadvantage that the connection between the experimental data and phases is one-sided: only analytical expressions for the coefficients in the expansion in phases of the cross sections and polarization are known, and not vice versa. Therefore, the solution is obtained by a large number of successive approximations and does not indicate the shortcomings of the phases. True, the numerical calculation allows one to obtain the "best" phases using the method of least square; however, the speed of numerical processing will doubtless be considerably greater if the phases are defined more accurately within the limits determined by the apparatus. Graphical solution is possible only for the case when the number of obtained phases does not exceed three; it also does not allow one to find the errors in the phases.

The lever apparatus has already been described in the literature (Refs. 10 and 11) for determining three phases for the scattering of π-mesons by nucleons, but in this apparatus only the relation (2) is utilized and the relation for $\sigma'(0)$; also, instead of utilizing the relations for $\sigma'(\pi)$ or (6), there is used a very inconvenient condition for the equality of two angles whose apexes are at different points for all situations of the apparatus.

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Electrical Properties of Germanium at Very Low Temperatures

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HUNG[1] found that there is a change in the activation energy of the carrier current in germanium in the region of helium temperatures and later this was verified by experiments[2,3]. But until the present time there has not appeared a satisfactory theoretical clarification. It seems to be proper at this time then to study the properties of germanium at very much lower temperatures.

We obtained very low temperatures by the adiabatic demagnetization of iron-ammonium alum. The apparatus allowed us to cool the germaium sample to 0.15° K. The temperature of the sample was controlled by a calibrated carbon thermometer, with an accuracy of 5×10^{-3}° K. The electrical conductor for measuring the resistance was passed into the dewar to the sample through an evacuated steel tube, and covered with polystyrene washers. The conductor is cooled to helium temperatures by means of a quartz rod and to very low temperatures by means of a block of alum. This made possible the attainment of very low temperatures in a matter of a few hours, so that we could carry on the experiments at a temperature that remained relatively stable.

The thermal and electrical contacts to the sample of germanium were made using springs tightly enveloping the sample, the ends of which were electrolytically covered with copper. The electrical resistance of the sample was measured with an electrometer with a reversable connection and with a current sensitivity equal to 1×10^{-14} A.

For measurements in the region of hydrogen and helium temperatures, the apparatus was filled with atmospheric helium. For measurements at very low temperatures, several centimeters of gaseous helium were put in at room temperature by which the isothermal magnetization was hastened. The adiabatic demagnetization depends on the adsorption of the gaseous helium over the surface of the cooled salt.

Several samples of germanium were studied having a specific resistance of the orderof 1 ohm-cm at room temperature. The samples were prepared in the Institute of the Metallurgical Academy of Sciences, USSR, and in the semiconductor section of Moscow State University.

The temperature dependence of the specific resistance of the sample is shown in Fig. 1. The resistance of the sample was measured at a gradient of 50-100 mv/cm. At this voltage the resistance for all practical purposes still does not depend on the field. The effect of geometry on the resistance was studied. To clarify the role of the contacts, the space between them was variedfor one of the samples from 15.8 to 8 mm at a constant cross section area of about 0.25 cm^2. Within the limits of the accuracy of the experiment (around 20%) the calculated specific resistances were equal. There is the same degree of accuracy in the results for samples subjected to these different surface treatments: 1. polishing, 2. dipping in a boiling mixture of hydrochloric and nitric acids. The results did not change when the ends of the sample were coated with copper by means of electrolysis.

We have shown two sets of curves on Fig. 1. One of these curves corresponds to the resistance of the samples in the region of hydrogen and helium temperatures, the other, in the region of helium and very low temperatures. The results of the measurements allow us to make some conclusions on the existence in the region of temperature from 0.15 to 1°K of an energy of activation smaller than in the region of temperature between 1.6 and 4.2° K. The small amount of activation energy of the current carrier in the region of helium and very low