On the Problem of Unquantized Relativistically Invariant Renormalized Equations for a Three-Dimensional Extended Particle

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A system of relativistically invariant equations for a smeared-out particle interacting with a field has been obtained by using the method previously proposed (see Ref. 1) for constructing a three-dimensional extended particle. The conservation laws for the total energy-momentum four-vector are formulated. The particle is stable without the necessity for introducing additional forces (of the Poincare pressure type). An exact mass renormalization is carried out. For comparison with earlier equations, a rigorous limiting transition to the case of a point particle is made. Interaction with the electromagnetic field and with a scalar meson field are considered.

1. THE theory of elementary particles is basically a quantum theory. However, many difficulties of the quantized theory have a classical origin, so that it appears worthwhile to consider some questions in classical theory, especially since it is simpler and more pictorial, and may suggest the way to construct a quantized theory. Among the fundamental questions which have been discussed in recent times from both points of view, the foremost are the problem of the intrinsic mass and the derivation of the equation of motion of the particle.

Historically, the first elementary particle equation was Lorentz's equation for the three-dimensionally extended spherical electron. The decisive shortcoming of this equation is its noncovariance and the need for invoking forces of nonelectromagnetic origin (Poincare pressure) to stabilize the electron. The fundamental difficulty in constructing a theory of point particles on a classical basis is associated with the appearance of infinite intrinsic masses. One can avoid this difficulty in the classical theory by taking the point of view that all the particle mass is mechanical in origin, and eliminating all sources of electromagnetic mass.

Dirac[2] obtained an equation for a point electron by calculating the flux of energy and momentum from a world tube and using an artifice for eliminating the field-mass: the solution of the equation for the proper field of the electron was taken to be half the difference between the advanced and retarded potentials. Later other attempts were made to eliminate the infinite electromagnetic mass of the point particle and to derive an equation of motion for it. The electron, and also a nucleon interacting with meson fields, was considered (cf., for example, Refs. 3 and 4), but in all cases the use of half the difference between retarded and advanced potentials is a fixed requirement in deriving the equation of motion.

Attempts have been made[1,5] to construct a theory of a three-dimensionally extended relativistically invariant particle. The equation of motion of such a particle in an external field was obtained[1], and it was shown that relativistic invariance is achieved, and that the difficulties associated with Laue's theorem (instability of the electron, nonuniqueness of the 4-momentum) are removed if a covariant form of the Poincare pressure is introduced.

We make use of the method proposed in Ref. 1 for constructing a three-dimensional invariant "smearing". However, in contrast to Ref. 1, we shall smear the interaction and not the charge, as a result of which the particle will be stable without the introduction of the Poincare pressure.

2. We consider first the case of an electron interacting with the electromagnetic field. By analogy with ordinary electrodymanics, we choose the action S in the form

$$S = S^m + S^f + S^{int}; \quad (1)$$

$$S^m = -m\int d\tau; \quad (2)$$

$$S^f = -\frac{1}{8\pi}\int d\Omega\left\{\frac{1}{2}F_{ik}^2(x) + \left(\frac{\partial A_i(x)}{\partial x_i}\right)^2\right\}; \quad (3)$$

$$S^{int} = \int d\Omega\, A_i(x)\, j_i(x), \quad (4)$$

where S^m and S^f refer to the free particle and the field, respectively, and S^{int} to the interaction.

Here, in contrast to the usual electrodynamics

$j_i(x)$ is the current of a three-dimensional extended particle and not of a point particle. As shown in Ref. 1, the conserved current for such a particle has the form

$$j_i(x) = \int d\tau \rho(r_l)\,\delta(r_k u_k)(1 + r_j \dot{w}_j)\, u_i. \quad (5)$$

The notation used here and in the sequel is the same as in Ref. 1.

Varying S with respect to the coordinates of particle and field we obtain, respectively, the equations of motion and the field equations:

$$m w_i = \int d\Omega \rho(r)\,\delta(r_k u_k)(1 + r_l w_l)\, F_{ij}(x)\, u_j \quad (6)$$

$$- \frac{d}{d\tau}\int d\Omega \rho(r)\,\delta(r_k u_k)\{A_j(x)\, u_j u_i r_l w_l + A_i(x)\, r_l w_l - A_j(x)\, r_i w_j\};$$

$$\Box A_i(x) = -4\pi j_i(x). \quad (7)$$

By direct substitution of $A_k + \partial\varphi/\partial x_k$ for A_k, one can verify that Eq. (6) is gauge invariant.

The equation of motion (6) is not identical with Eq. (28) of Ref. 1, differing from it by the term

$$\frac{d}{d\tau}\int d\Omega \rho(r)\,\delta(r_k u_k)\{A_j(x)\, u_j u_i r_l w_l + A_i(x) r_l w_l - A_j(x)\, r_i w_j\}. \quad (8)$$

This term is connected with the Thomas precession: a particle displaced relativistically along a closed contour will, when it returns to its initial position, turn out to have rotated through some angle about its axis. This is unimportant for a point particle, and in fact Eq. (8) goes to zero for $r_k \to 0$. For rectilinear motion, even if accelerated, this term again gives zero. It describes forces which appear only during curvilinear motion and are associated with rotation of the particle about its axis. Thus, for a rigid relativistic particle in a field there appears, so to speak, an additional degree of freedom, so that initially we must assign not only the coordinates and velocities, but also the angular velocity of the Thomas precession, which is uniquely determined by the normal component of the acceleration. This is the reason why the term (8) contains $\dot{w}_k$, the third derivative with respect to τ. If the particle is initially free, then (8) vanishes and the assignment of the auxiliary initial condition is unnecessary.

3. To obtain the total conserved 4-momentum P_i, we use a method developed by Pauli[6]. In this paper there was obtained for the first time an explicit expression for the energy-momentum integral in a quantized nonlocal theory. It satisfies exact conservation laws, and not asymptotic ones as previously supposed. Thus one can at any stage of the computation make the transition to a local theory which coincides with the usual theory.

Although we are not considering the interaction of fields, but rather of a field and particle, we shall also use the method proposed in Ref. 6. As a result of computations similar to those of Ref. 6, the total 4-momentum is obtained in the form

$$P_i(t) = P_i^m(t) + P_i^f(t) + P_i^{int}(t); \quad (9)$$

$$P_i^{int}(t) = \int d\Omega' j_l(x')\,\frac{\partial A_l(x')}{\partial x_i'}\,\frac{1}{2}\{\varepsilon(t-T) - \varepsilon(t-t')\}$$

$$- \int d\Omega' j_l(x')\,\frac{\partial A_i(x')}{\partial x_l'}\,\frac{1}{2}\{1 + \varepsilon(t-t')\}$$

$$+ \int d\Omega' \int dT \delta(T - t)\{A_j(x')\, u_j u_i r_l' w_l + A_i(x')\, r_l' w_l - A_j(x')\, r_i' w_j\}; \quad (10)$$

$$P_i^m(t) = m\int dT u_i \dot{\delta}(T - t); \quad (11)$$

$$P_i^f(t) = \frac{1}{4\pi i}\int d\Omega' \dot{\delta}(t - t') \times \left\{F_{il}(x')\, F_{4l}(x') - \frac{\delta_{i4}}{4} F_{jl}^2(x')\right\}, \quad (12)$$

where

$\varepsilon(\tau) = +1$ for $\tau > 0$ and -1 for $\tau < 0$; $T(\tau) = -iX_4(\tau)$.

One can verify by direct test that the 4-momentum defined by Eqs. (9)-(12) is conserved, i.e.,

$$dP_i(t)/dt = 0. \quad (13)$$

4. Using the expression for $P_i(t)$, we get the completely renormalized particle mass M. For this purpose we solve the system of Eqs. (6) and (7) for an electron at rest in the absence of external fields and substitute the solution into the expression $-iP_4 = M$; we get

$$\varphi = -iA_4 = \int dV\rho(\mathbf{r}) \frac{1}{|\mathbf{r}-\mathbf{r}'|}; \quad (14)$$

$$u_i = (0, 0, 0, i); \quad \mathbf{A} = 0;$$

$$M = m + m^f;$$

where

$$m^f = \frac{1}{2}\int dV\rho(\mathbf{r})\int dV\rho(\mathbf{r}')\frac{1}{|\mathbf{r}-\mathbf{r}'|} \quad (15)$$

is the finite electromagnetic mass of the electron. In the special case where the charge is distributed over the surface of a sphere of radius r_0, $m^f = e^2/2r_0$, while for a volume distribution, $m^f = 3e^2/5r_0$. m is the primitive mechanical mass, which enters explicitly into the equation of motion (6), M is the renormalized or experimental mass. In contrast to the quantum mechanical case, the renormalization is exact.

From consideration of the stationary case and using condition (13) it follows that the electron is stable without introducing any additional forces of the Poincare pressure type.

We can introduce the completely renormalized mass M into the equation of motion (6) in place of the primitive mass m. The equation of motion with the renormalized mass takes the form:

$$Mw_i = m^f w_i + \int d\Omega\rho(r)\,\delta(r_k u_k)(1 + r_l w_l)F_{ij}(x)u_j - \frac{d}{d\tau}\int d\Omega\rho(r)\,\delta(r_k u_k) \times \{A_j(x)u_j u_i r_l w_l + A_i(x)r_l w_l - A_j(x)r_i w_j\}, \quad (16)$$

where m^f is to be taken from (15).

But this does not complete the renormalization process. We must similarly renormalize the field, i.e., eliminate from the right side of the equation of motion that part of the proper field of the particle which is already included in m^f.

In (16), $A_i(x)$ is the sum $A_i + A_i^s$, where A_i is the electromagnetic field external to the particle; A_i^s is the field produced by the particle. The latter consists of two parts: the field $A_{i\,r}^s$ radiated by the particle, and the field $A_{i\,a}^s$ permanently attached to it. $A_{i\,a}^s$ is just that part of the field which must be taken out to get complete renormalization. Obviously we must require that in the stationary case $A_{i\,a}^s$ coincide with the field of the free particle. However, the choice of A_i^s is not unique.

We shall take for A_i^s a solution of Eq. (7) in the form of a retarded potential, which corresponds to the natural physical description:

$$A_i^s(x) = \int d\Omega' 4\pi G^{ret}(x - x')j_i(x'); \quad (17)$$

$$G^{ret}(x - x') = (1/4\pi)\,\delta\{|\mathbf{x}-\mathbf{x}'| - (t - t')\}/|\mathbf{x}-\mathbf{x}'|, \quad (18)$$

where G^{ret} is the familiar Green's function for the retarded solution of the d'Alembert equation.

Substituting $A_i^s(x)$ into the right side of (6), we get the self-force

$$F_i^s(\tau) = \int d\Omega\rho(r)\,\delta(r_k u_k) \times (1 + r_l w_l)\int d\Omega' 4\pi G^{ret}(x - x') \times \int d\tau'\delta(r_n' u_n')\Big\{u_j(w_i' u_j' - w_j' u_i')\rho(r') + u_j\Big(u_j'\frac{\partial\rho(r')}{\partial x_i'} - u_i'\frac{\partial\rho(r')}{\partial x_j'}\Big)(1 + r_m' w_m')\Big\} - \frac{d}{d\tau}\int d\Omega\rho(r)\,\delta(r_k u_k)\{r_l w_l(u_i u_j + \delta_{ij}) - r_i w_j\} \times \int d\Omega' 4\pi G^{ret}(x - x')\int d\tau'\delta(r_n' u_n') \times (1 + r_m' w_m')\rho(r')u_j', \quad (19)$$

where $r' = x' - X'$; $X' = X(\tau')$.

The equation then takes the form

$$Mw_i = m^f w_i + F_i + F_i^s, \quad (20)$$

where m^f is taken from (15), F_i^s from (19), and F_i from the right side of Eq. (6), where it is understood that now $A_i(x)$ includes only external fields satisfying the free-field equation

$$\Box A_i(x) = 0. \quad (21)$$

The system of equations (20 and (21) is a complete system of exactly renormalized equations describing an extended electron in interaction with the electromagnetic field. The total 4-momentum of the system is conserved despite the presence of odd time derivatives in the equation of motion.

5. The self-force F_i^s can be integrated approximately under the condition on the smallness of the acceleration

$$wr_0 \ll 1, \tag{22}$$

where r_0 is the size of the particle.

Multiplying (22) by M we get a simple interpretation: the change in energy (or momentum) along the path r_0 must not exceed the self-energy of the particle. For r_0 approaching zero and the point particle, the condition (22) drops out.

We carry out the computation in the rest system. In the final result we transform to an arbitrary system. We integrate in turn with respect to t, t', τ', using the three δ-functions

$$\delta(r_k u_k), \quad \delta(r_n' u_n'), \quad \delta\{|\mathbf{x}-\mathbf{x}'|-(t-t')\}$$

and make use of the normalization condition $\int dV\rho(\mathbf{r}) = 1$. The τ' integration is approximate. Introducing the new integration variable $s = \tau' - \tau$, we expand the integrand in series subject to the conditions

$$ws \ll 1 \tag{23}$$

and (22). But because of the δ-function in the integrand, s is of the same order of smallness as r_0. Therefore, conditions (22) and (23) are equivalent. As a result of the integration we get a series in powers of r_0. The first term of this series (of degree minus one in r_0),

$$\frac{1}{2}e^2 w_i \int dV\rho(\mathbf{r}) \int dV'\rho(\mathbf{r}') \frac{1}{|\mathbf{r}-\mathbf{r}'|} \tag{24}$$

corresponds to $A^s_{i\,a}$ [the Green's function is taken in the form $\frac{1}{2}(G^{ret} + G^{adv})$]. In the equation, it combines with $m^f w_i$ from (15).

The second term of the series (of zero degree in r_0) is

$$\tfrac{2}{3}e^2(\dot{w}_i + \dot{w}_k u_k u_i) \tag{25}$$

and is the well-known term for the self-force or reaction of the radiation from a point electron interacting with the electromagnetic field. It corresponds to $A^s_{i\,r}$ (here the Green's function is $\frac{1}{2}(G^{ret} - G^{adv})$,

$$A^s_{i\,ir} + A^s_{i\,ia} = A^s_i \tag{26}$$

and correspondingly,

$$\tfrac{1}{2}(G^{ret} - G^{adv}) + \tfrac{1}{2}(G^{ret} + G^{adv}) = G^{ret}.$$

The remaining terms of the series (first and higher degree in r_0) are small under the condition $wr_0 \ll 1$, and can simply be dropped. For large accelerations they cease to be small, and the series expansion and integration of the self-force lose their meaning.

The final form of the renormalized equation of motion is

$$Mw_i = \int d\Omega\rho(r)\,\delta(r_k u_k)(1 + r_l w_l) F_{ij}(x) u_j \tag{27}$$

$$+ \frac{2}{3}e^2(\dot{w}_i + \dot{w}_k u_k u_i) + \ldots,$$

where terms of first and higher degree in r_0 have been dropped (they are expressed in the form of integrals). Equation (27) reminds one of Lorentz's equation but differs from it in being relativistically invariant. In addition, the electron is stable without the introduction of auxiliary forces (Poincare pressure). Together with Eq. (21) for the free external field, Eq. (27) gives a complete system of exactly renormalized equations for a three-dimensional extended particle interacting with the electromagnetic field. We have already given the conservation law (13) for the 4-momentum P_i defined by Eqs. (9)-(12).

For comparison with Dirac's equation, we go to the limit of a point particle. Then the terms of first and higher degree in r_0 vanish. The condition $wr_0 \ll 1$ no longer restricts the acceleration, and the equation of motion takes the form

$$Mw_i = e^2 F_{ik}(X)\, u_k + \tfrac{2}{3}e^2(w_i + w_k u_k u_i). \tag{28}$$

In this form, the equation of motion coincides with Dirac's equation. However, unlike Ref. 2, the physical interpretation of Eq. (28) is clear. Throughout the whole computation, we used finite quantities, considered the physically understandable case of retarded potentials, achieved exact mass renormalization, and also obtained rigorous conservation laws (9)-(13).

6. Let us now consider the case of a nucleon interacting with a scalar meson field. We proceed in complete analogy to the calculation for the electromagnetic field, so that everything previously stated also applies here. We shall only note the differences and peculiarities in the present case, and give the final results.

We choose the action function S in the form

$$S^m = -m\int d\tau; \tag{29}$$

$$S^f = -\frac{1}{8\pi}\int d\Omega\,\{\varphi_k^2(x) + \chi^2\varphi^2(x)\}; \tag{30}$$

(31)

$$S^{int} = -g\int d\tau\int d\Omega\rho(r)\,\delta(r_k u_k)(1 + r_l w_l)\,\varphi(x),$$

where $\varphi(x)$ is the potential of the meson field, $\varphi_k(x) = \partial\varphi(x)/\partial x_k$, χ is a constant numerically equal to $\mu/\hbar$, μ is the meson mass, g is the scalar coupling constant (the nuclear "charge").

We then get the equation of motion

$$m w_i = -g\int d\Omega\varphi_i(x)\,\rho(r)\,\delta(r_k u_k)(1 + r_l w_l) \tag{32}$$
$$-g\int d\Omega\,\frac{d}{d\tau}\{\rho(r)\,\delta(r_k u_k)\,\varphi(x)\,u_i\}$$

and the field equation

$$(\Box - \chi^2)\,\varphi(x) \tag{33}$$
$$= 4\pi g\int d\tau\rho(r)\,\delta(r_k u_k)(1 + r_l w_l).$$

It is interesting to note that the equation of motion (32) contains no derivatives with respect to τ higher than the second.

The action for vector coupling is

$$S_f^{int} = -f\int d\Omega\,\frac{\partial\varphi}{\partial x_k}\,j_k(x) = -f\int d\Omega\,\frac{\partial}{\partial x_k}(\varphi j_k)$$

because the divergence of the current vanishes identically, and is thus the integral of a divergence and therefore gives no contribution to the equation.

The total conserved 4-momentum has the form

$$P_i^{int}(t) = g\int d\Omega'\varphi_i(x')\int d\tau\rho(r')\,\delta(r'_k u_k)(1 + r'_l w_l)$$
$$\times\,\frac{1}{2}\{\varepsilon(t - T) - \varepsilon(t - t')\}$$
$$+ g\int d\Omega'\varphi(x')\int dT\rho(r')\,\delta(r'_k u_k)\,u_i\delta(T - t);$$
$$P_i^m(t) = m\int dT u_i\delta(T - t);$$
$$P_i^f(t) = \frac{1}{4\pi i}\int d\Omega'\delta(t - t')$$
$$\times\left\{\varphi_i(x')\,\varphi_4(x') - \frac{1}{2}\,\delta_{i4}(\varphi_l^2(x') + \chi^2\varphi^2(x'))\right\}. \tag{34}$$

The renormalized equation of motion is

$$M w_i = m^f w_i + F_i(\varphi) + F_i^s, \tag{35}$$

where m^f is the finite field-mass of the nucleon, equal to

$$m^f = -\frac{1}{2}\,g^2\int dV\rho(\mathbf{r})\int dV'\rho(\mathbf{r}')\,\frac{e^{-\chi|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r} - \mathbf{r}'|}, \tag{36}$$

$F_i(\varphi)$ is the right side of Eq. (32), where $\varphi(x)$ now includes only external fields, satisfying the free-field equation

$$(\Box - \chi^2)\,\varphi(x) = 0. \tag{37}$$

F_i^s, the self-force, is equal to

$$F_i^s(\tau) = g^2\int d\Omega\delta(r_k u_k)\,\rho(r)(1 + r_l w_l) \tag{38}$$
$$\times\int d\Omega'\,.\,4\pi G^{ret}(x - x')$$
$$\times\int d\tau'\delta(r'_n u'_n)(1 + r'_m w'_m)$$
$$\times\left\{\frac{\partial\rho(r')}{\partial x'_k}(\delta_{ik} + u_i u_k) + \rho(r')\,w_i(1 + r_l w_l)^{-1}\right\}.$$

In contrast to the electron case, F_i^s consists of two parts--a singular part $\tilde{F}_i^s$ and a nonsingular part F_i^χ

$$F_i^s = \tilde{F}_i^s + F_i^\chi, \tag{39}$$

corresponding to two parts of the Green's function

$$G^{ret} = \tilde{G}^{ret} + G^{\chi\,ret}, \tag{40}$$

where the singular part $\tilde{G}^{ret}$ is given by (18) while the nonsingular part $G^{\chi\,ret}$ is

$$G^{\chi\,ret}(x - x') = \begin{cases} -\dfrac{1}{4\pi}\,\dfrac{\chi J_1(\chi\sqrt{(t - t')^2 - (x - x')^2})}{\sqrt{(t - t')^2 - (x - x')^2}} \\ 0 \end{cases} \tag{41}$$
$$\times\quad \begin{matrix} t - t' > |x - x'| \\ t - t' < |x - x'| \end{matrix}$$

The singular part of the self-force $\tilde{F}_i^s$ is integrated, subject to the condition (22) just as in the case of the electron. The nonsingular part F can be expanded in series with respect to r_0 and approximately integrated only if we impose on the acceleration the new condition

$$w \ll \chi. \qquad (42)$$

The integrand is expanded in series subject to conditions $wr_0 \ll 1$ and $ws \ll 1$. However, in contrast to the case of F_i^s and that of the electron, here s is a variable taking on values from R to ∞ (the limits of the s integration). It proves to be sufficient to integrate, not to ∞, but to some s_0 such that $\chi s_0 \gg 1$, while $ws_0 \ll 1$, so that the expansion in series can be accomplished. These two conditions together give $w \ll \chi$. This new condition which did not occur for F_i^s gives a stronger bound on the acceleration than the earlier $wr_0 \ll 1$ (for the chosen values of the constants $1/\chi = \hbar/\mu \sim 1.4 \times 10^{-11}$ cm) and is not eliminated when we go to the limit of a point particle.

The renormalized equation of motion (35) under the conditions $w \ll \chi$, $w \ll 1/r_0$ has the form

$$Mw_i = F_i(\varphi) - {}^1/_3\, g^2 (\dot{w}_i + \dot{w}_k u_k u_i) + \ldots, \qquad (43)$$

where terms of first degree and higher in r_0 have been dropped and $F_i(\varphi)$ is the same as in (35).

For accelerations limited only by the one condition $w \ll 1/r_0$, the self-force F_i^χ cannot be expanded in series and integrated. In this case the equation of motion is expressible in the form

$$Mw_i = F_i(\varphi) + {}^1/_3\, \dot{g}^2 (w_i + w_k u_k u_i) + F_i^\chi - {}^1/_2\, g^2 \chi w_i + \ldots \qquad (44)$$

Together with the Eq. (37) for the free external field, the equation of motion (35) [or, in the special cases, Eqs. (43) and (44)], gives the complete system of exactly renormalized equations for a three-dimensionally smeared nucleon interacting with a scalar meson field.

From Eq. (43), we obtain by rigorous transition to the point charge:

$$Mw_i = -g\,\{\varphi_i(X) + (d/d\tau)(\varphi(X)\,u_i)\} - {}^1/_3\, g^2 (\dot{w}_i + \dot{w}_k u_k u_i). \qquad (45)$$

This is the equation for a point nucleon interacting with scalar meson fields, subject to the condition $w \ll \chi$. The constant χ does not appear explicitly in this equation (although the equation would be different, if $\chi = 0$.

For arbitrary accelerations, by making the limiting transition we obtain from Eq. (44)

$$Mw_i = -g\left\{\varphi_i(X) + \frac{d}{d\tau}(\varphi(X)\,u_i)\right\} + \frac{1}{3} g^2 (\dot{w}_i + \dot{w}_k u_k u_i) - \frac{1}{2} g^2 \chi w_i + \frac{1}{2} g^2 \chi^2 u_i - g^2 \chi^2 \int_{-\infty}^{\tau} \frac{Y_i}{Y^2} J_2(\chi Y)\, d\tau' - g^2 \chi \frac{d}{d\tau}\left\{u_i \int_{-\infty}^{\tau} \frac{1}{Y} J_1(\chi Y)\, d\tau'\right\},$$

where $Y_k - X_k - X_k'$, $Y = \sqrt{-(X_k - X_k')^2}$.

Comparison of Eq. (46) with earlier equations for a point nucleon interacting with a scalar meson field shows that for the case of $w \ll \chi$ Eq. (46) leads to the well-known form of the equation (cf., for example, Eq. (50c) in Ref. 7], which is obtained using G^χ in the form $\frac{1}{2}(G^{\chi\, ret} - G^{\chi\, adv})$. But in the general case, (46) differs from Eq. (50c) of Ref. 7. In addition, precisely for the case of $w \ll \chi$, Eq. (46) was obtained by us in the entirely new form (45) where its interesting feature (opposite sign of the self-force) is explicitly clear.

Equation (46) differs from the equation obtained by using for G^χ the retarded solution $G^{\chi\, ret}$ [cf., for example, Eq. (21c) in Ref. 7] by having the additional term $-\frac{1}{2} g^2 \chi w_i$ on the right side of the equation. The effective mass correction $-\frac{1}{2} g^2 \chi$ obtained in Refs. 8 and 9 combined with our additional term, so that the results of these papers concerning changes of effective mass are incorrect.

7. The fundamental problem facing the present relativistic quantum theory is to obtain in closed form a system of renormalized equations not containing infinities. This problem is solved in the quantum theory of fields in the weak coupling approximation. The solutions (or the renormalized equations) are obtained in the form of series in the fine structure constant. It is of interest to try to solve the problem in another way: by first renormalizing the classical equations and then quantizing them. In this paper the first step along this path is carried out: systems of completely renormalized classical equations are obtained for particles interacting nonlocally with electromagnetic fields (20,21), (27,21) and meson fields (35,37), (43,37) and (44,37). It is important to note that the exact renormalization was carried out for a smeared interaction and is finite. The transition to the point particle is accomplished after the renormalization so that infinite expressions are absent not only from the final equations (28,21) and (45,37), (46,37), but also from every stage of the calculation. The attempt to quantize the renormalized equation is a fascinating, but obviously not simple, problem.

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Temperature Dependence of the Viscosity of Liquid Nitrogen At Constant Density

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The results of measurements on the temperature dependence of the viscosity of liquid nitrogen at constant density are presented. These measurements were conducted over the density interval from $\rho = 0.38$ to $\rho = 0.86$ gm/cm^3 for temperatures ranging from the condensation temperature at each density to 300° K. The gas-like character of the variation of viscosity with temperature in liquid nitrogen at low densities is revealed for the first time.

IN 1950 Verkin and Rudenko[1] first investigated the temperature dependence of the viscosity of liquefied nitrogen and argon for constant values of the density. These experiments, in which it proved possible to distinguish quite fully the effects of the two factors of temperature and variation in molar volume upon the coefficient of viscosity, were conducted over a narrow density range.

The purpose of the present experiment has been to extend such an investigation to cover a wide range of densities--from a value of the density near that at the triple point to the value at the critical point. Liquid nitrogen was selected as the subject of the experiment, inasmuch as it belongs to the class of simple liquids; i.e., nonpolar, nonassociative liquids consisting of particles having spherical or nearly spherical symmetry. The full and systematic study of the properties of such substances is of great interest in connection with the problem of constructing a theory of the liquid state.

APPARATUS AND METHOD OF MEASUREMENT

In investigating the viscosity of this liquid use was made of the viscometer constructed by Verkin and Rudenko[1] and loaned for the present experiment by the Low-Temperature Laboratory of the Physico-Technical Institute, Academy of Sciences, Ukrainian SSR. The viscosity was determined from the time required for a cylindrical weight to fall along a tube of slightly greater diameter filled with the liquid to be investigated. Since pressures as high as 3000-4000 kg/cm^2 are developed as the temperature is increased with the density held constant, the tube and its contents were enclosed in a thick-walled bomb. Tube, weight and bomb were all made from the same material--beryllium bronze--and were placed within a metal Dewar flask. A core of iron-nickel magnetic alloy was pressed into the weight. Two pairs of induction coils were