The Coefficient of Volume Absorption of Second Sound and the Viscosity of the Normal Component of Helium II down to 0.83° K.

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(Submitted to JETP editor March 2, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 31-36 (July, 1956)

Measurements of the absorption of second sound[1] have been extended to 0.83° K for frequencies from 200 cycles to 4 kc by the method of the determination of the width of resonance curves in standing waves. Values of the coefficient of volume absorption α/ω^2 and of viscosity η of the normal component of He II have been obtained by separation of the volume and surface losses. It is established that the losses due to thermal conduction predominate in the volume absorption. The analogue of the thermal conductivity in He II has been computed. The results of the experiment completely verify the theory of Landau and Khalatnikov for He II kinetic coefficients.

INTRODUCTION

THE absorption of second sound increases rapidly with drop in temperature, as was shown in a previous paper[1], and in other confirmatory researches[5,6]; in this case the surface absorption is due primarily to the viscosity, and the volume absorption to the thermal conductivity of He II.

The theoretical temperature dependence of the absorption coefficient has been computed by Khalatnikov[3,4], who showed that the absorption effect increased in the direction of low temperature; here the losses in second sound due to thermal conductivity are significantly greater than the losses determined by the first and second viscosity.

For the total coefficient of volume absorption, Khalatnikov obtained the expression

$$\frac{\alpha}{\omega^2}=\frac{1}{2\rho u_2^3}\left\{\frac{\rho_s}{\rho_n}\left[\frac{4}{3}\eta+\zeta_2-\rho(\zeta_1+\zeta_4)+\rho^2\zeta_3\right]+\frac{\varkappa}{c}\right\}, \qquad (1)$$

where η is the first viscosity coefficient, ζ_1, ζ_2, ζ_3, ζ_4 are the coefficients of second viscosity, c the heat capacity and $\varkappa$ the analogue of thermal conductivity in He II. Previous measurements of (1) did not permit a complete comparison of experiment with theory, since the experimental values of α/ω^2 were obtained in a comparatively narrow temperature range (1.0 – 1.2 ° K.).

In the present work, we have carried out measurements down to 0.83° K, using methods previously developed. In an attempt to make more precise the values and the temperature dependence of the kinetic coefficients we have also carried out more accurate measurements from 1 to 1.3 ° K.

METHODS OF MEASUREMENT

As in the previous work, the measurements were carried out in cylindrical glass resonators by the method of investigating the widths of the resonance curves for standing waves at frequencies from 200 cycles to 4 kc. We used three different resonators; two of them had internal diameters of 5.8 mm and lengths 91.2 and 44.5 mm, the third had an internal diameter 5.35 mm and length 86.2 mm. The plates at the ends of the resonator were made of glass instead of the plexiglass used previously, and were attached through a thin layer of oil to the polished faces of the resonator by means of a spring. A heater of constantan wire (50 μ) and a resistance thermometer of phosphor bronze (30 μ) were prepared in the form of flat, single-layered spools with bifilar windings, and were fastened to the disks. Replacement of the plexiglass reflectors by glass improved the frequency characteristics of the resonator in the region of the first transverse types of vibration, where earlier one had always observed an additional increase in the damping. The frequencies of the transverse resonances are given by [7]

$$(\omega/u_2)^2=(l\pi/l_0)^2+(r'_{nm}/a)^2, \qquad (2)$$

where $l = 1, 2, 3, \ldots$, r'_{nm} = roots of the derivatives of the Bessel functions, l_0 is the length and a the radius of the resonator. At the minimum velocity of second sound (at $T = 1.1$ ° K) two such transverse resonances exist up to 4 kc in the resonator of diameter 5.35 mm ($\nu' = 2000$ cycles and $\nu'' = 3620$ cycles), and in the resonator of diameter 5.8 mm, three ($\nu' = 1800$, $\nu'' = 3010$ and $\nu''' = 3840$ cycles). (The frequencies of transverse resonances increase with increase in the sound velocity.) Testing of the width of the resonance curves in regions close to transverse resonances showed that no noteworthy increase in the absorption of second sound could be observed at the fundamental resonances. This permitted us to extend the range of measurement (at not too low temperatures) to 4 kc. As in the previous research, the second sound was excited by the passage of current from the generator through the heater, and was detected by a resistance thermometer. The circuit described in Ref. 1 was used as an amplifier and was capable of measuring signals of $\sim 0.1\ \mu$ V with ease.

Temperatures below 1 °K were obtained by removing the helium vapor through two diffusion pumps in series. The arrangement was similar to that described in Ref. 1. Temperatures of the helium inside the resonator above 1° K were measured with a resistance thermometer calibrated in terms of the helium vapor pressure. Below 1 ° K (down to 0.85 ° K), the temperature was determined from the velocity of second sound as measured by Peshkov[8] with a high degree of accuracy. The lowest temperature of 0.83 ° K obtained in the present work corresponded to a second sound velocity of 22.9 m / sec.

RESULTS

Measurements of the width of the resonance curves were carried out for minimum heat flow densities, usually not exceeding $3-5 \times 10^{-3}$ W/cm^2 since, for greater loading, a dependence of absorption on amplitude was observed. The graph of this dependence at $T = 1.27$ ° K is shown in Fig. 1. As is evident from the drawing, the damping of second sound, relative to the damping at infinitesimal amplitude, was proportional to the power for three different frequencies. We did not make any special investigation of this problem and limited the region of measurement to small amplitudes, where the nonlinear effects are small.

To separate the volume and surface loss of second sound, proportional, respectively, to the square and square root of the frequency, the observed dependence on the frequency of the resonance width for different temperatures was plotted in coordinates $\Delta\nu/\sqrt{\nu}$; $\nu\sqrt{\nu}$, where ν is the frequency and $\Delta\nu$ is the resonance width in cycles. This dependence is plotted in Fig. 2 for the resonator $d = 5.8$ mm, $l_1 = 44.5$ mm, $l_2 = 91.2$ mm. The intercepts on the ordinate determine the surface absorption of second sound, while the slope of the lines is proportional to the volume absorption. Above 1.3 ° K, the accuracy of measurement was insufficient to determine the slope of the lines — they become virtually horizontal, which testifies to the predominant role of surface absorption at these frequencies.

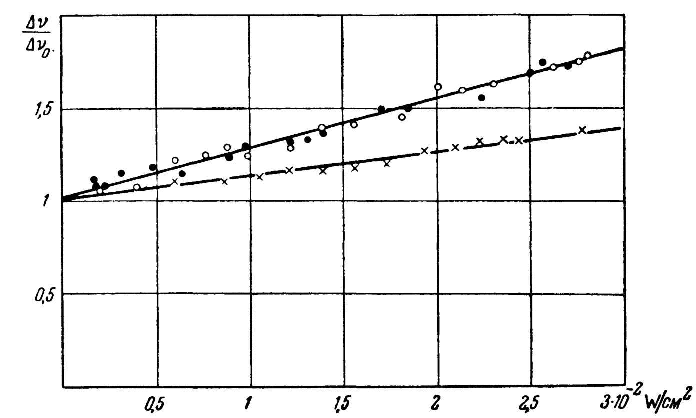

FIG. 1. Dependence of the absorption of second sound on power for $T = 1.27$ ° K in the resonator $d = 5.35$ mm and $l = 86.2$ mm; O — 984, ● — 2750, × — 3960 cycles.

Boundary losses in the resonator, which amount to several per cent at high temperatures (1.6 – 2.0 ° K) are negligible at low temperatures, in comparison with the true absorption. While the absorption was always higher (at high temperatures) in the short resonator than in the long one (because of wall losses), at low temperatures, beginning about $T \sim 1.3$ ° K, the absorption no longer depends on the length of the resonator.

We note that below 0.95 ° K, measurements are not possible in the longer resonator since the resonances for adjacent frequencies begin to overlap, due to the high damping.

The viscosity η of the normal component of He II can be computed from the magnitude of the surface absorption by the formula

$$\eta = \left(\frac{\Delta\nu}{\sqrt{\nu}}\right)_0^2 \frac{\pi r^2 \rho^2 \rho_n^2}{\rho_s^2}, \qquad (3)$$

where r is the radius of the resonator, $(\Delta\nu/\sqrt{\nu})_0$ is the ordinate intercept in Fig. 2. The data for

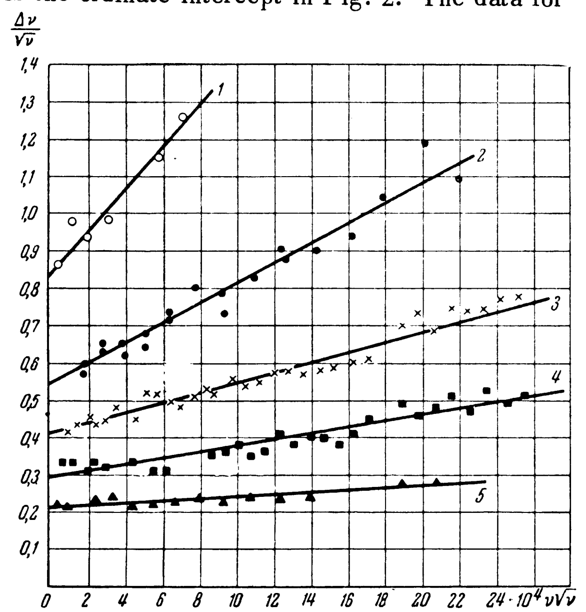

FIG. 2. Dependence of the absorption of second sound on frequency, in the coordinates $\Delta\nu/\sqrt{\nu}$, $\nu/\sqrt{\nu}$, O ● — resonator, $l = 44.5$ mm, $d = 5.8$ mm; × ■ ▲ — resonator, $l = 91.2$ mm, $d = 5.8$ mm; T is equal to: 1 – 0.86, 2 – 0.9, 3 – 0.95, 4 – 1.01, 5 – 1.1 °K.

ρ_n/ρ, computed by Peshkov on the basis of second sound velocity[8] and heat capacity[9] measurements, were employed in this formula. The values of η computed from Eq. (3) down to the temperature $T = 0.83$ °K, along with earlier measurements in the interval 1.4 – 2.1 °K,[1] are shown by the circles in Fig. 3. We note that, in comparison with the earlier data, the values of η in the region 1.0 – 1.3 °K are 20 to 30 per cent higher, which barely goes beyond the limits of error in the determination of η ($\sim$ 20 to 25 per cent). For comparison, the results of the measurement of viscosity by means of a rotating disk (Andronikashvili and others [10-12]) and the data of Heikkila and Hollis-Hallet[13], obtained by the rotating cylinder method, are plotted in the Figure. The solid line indicates the theoretical curve of Khalatnikov.[4] As is evident from the Figure, the experimental values of the viscosity agree quite satisfactorily, down to 0.83 °K, with the theoretical values of Khalatnikov. At the lower temperature, the divergence can be explained by the extraordinarily steep dependence of η on T. It is interesting to note that the viscosity according to the measurements of Heikkila and Hollis-Hallet by the method of the rotating cylinder, which does not require knowledge of ρ_n/ρ, agrees well with our measurements, while the method of the rotating disk[10-12] gives essentially different results in the low temperature region. One cannot supply the reason for the divergence, which follows from our experiments, by errors in the determination of ρ_n/ρ at low temperatures, as has been assumed earlier by several authors. Evidently, in the region of low temperatures, where the concentration of the normal component is small, the dissipation of energy by the method of the rotating disk can increase appreciably, becuuse of the nonideality of the suspension system, which leads to an entrainment of superfluid along with the normal.

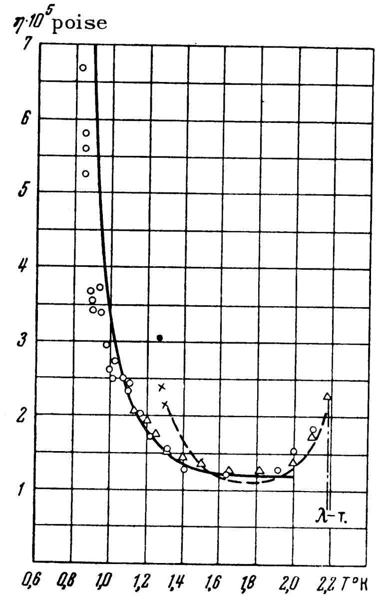

FIG. 3. Viscosity of the normal component of He II from the data: O = present measurements; ● = Ref. 12; × = Ref. 11; Δ = Ref. 13; broken line = Ref. 10; solid line = Ref. 4.

The coefficient of volume absorption α/ω^2 was computed from the magnitude of the volume loss of second sound in the temperature range 0.83 – 1.3 °K. Computations were made according to the formula

$$\frac{\alpha}{\omega^2} = \frac{\Delta\nu/\nu^2}{4\pi u_2}, \qquad (4)$$

where $\Delta \nu / \nu^2$ is the slope in Fig. 2, u_2 is the velocity of second sound. The temperature dependence of α / ω^2 is represented in a semi-logarithmic plot in Fig. 4. Here we have plotted the mean results of Hanson and Pellam [5] and the data of Atkins and Hart,[6] along with the theoretical curve of Khalatnikov[4] (solid line). It is easy to see that our experimental values of α / ω^2, determined to within 20 per cent, agree both with the theoretical values of Khalatnikov and with the values obtained by other authors at higher temperatures.

An estimate of the viscous loss in volume absorption of second sound according to values of the coefficient of first viscosity η, taken from our experiments (see Fig. 3), gives the broken curve in Fig. 4. As is evident from the Figure,

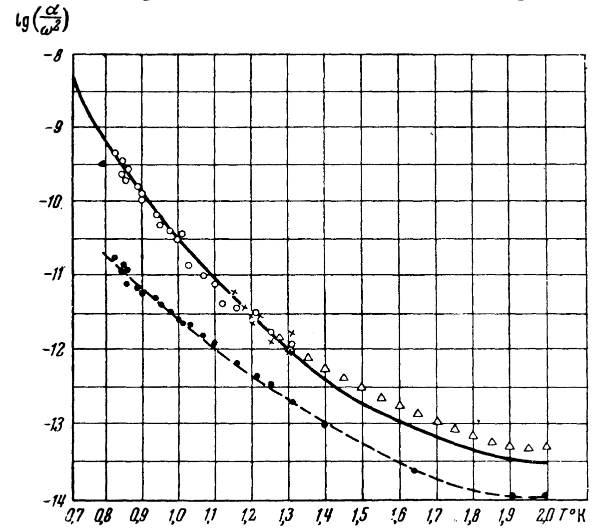

FIG. 4. Temperature dependence of the coefficient of volume absorption of second sound according to: O — present work, Δ — Ref. 5; × — Ref. 6; solid line — Ref. 4; ● — computed value of volume absorption of second sound, obtained from first viscosity.

viscosity losses are smaller by an order of magnitude than the total losses, and the relative contribution of the viscous loss decreases with decreasing temperature. Neglecting losses from second viscosity (in order of magnitude, they are comparable with losses from first viscosity), and attributing the remaining amount of absorption to the thermal conduction losses, we can estimate the coefficient of heat conductivity $\varkappa$ of He II to within 25 to 30 per cent. In this estimate, the values of the heat capacity were taken from the recent measurements of Kramers, Wasscher and Görter[14] which agree well with those of Ref. 9.

The temperature dependence of $\varkappa$ is plotted in Fig. 5 on a semi-logarithmic scale. The solid line denotes the theoretical curve of Khalatnikov. The agreement of the experimental data with the theoretical is excellent. The high values of $\varkappa$ in the previous measurements[1] must evidently be

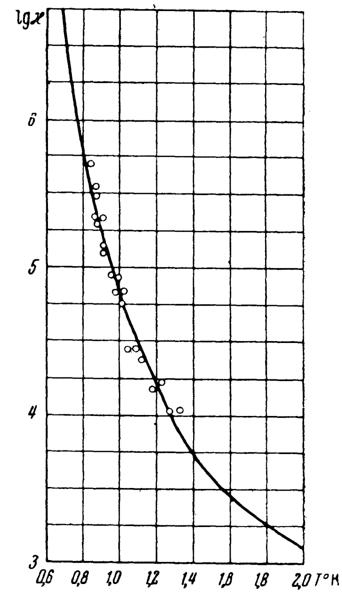

FIG. 5. Temperature dependence of the coefficient of thermal conductivity $\varkappa$ in He II: O = data of present work; solid curve = Ref. 4.

explained by the additional increase in damping at high frequencies because of the effect of transverse vibrations.

The experimental values of α / ω^2, η and $\varkappa$, taken from several experiments on different resonators, together with the data on u_2, ρ_n / ρ and c used by us are given in the Table. (Values of η above 1.4 °K are taken from the previous measurements.) From the values of $\varkappa$ it is not difficult to estimate the mean free path length of the phonons, making use of the classical formula in the quantum gas: $\varkappa = 1/3\, l_{ph}\, c_{ph}\, v_{ph}$. For the lowest temperature of our measurements, 0.83 °K, the mean free path length of the phonons was ~ 0.1 mm.

CONCLUSIONS

1. Experiments on the absorption of second sound down to 0.83 °K entirely confirm the validity of the theory of kinetic coefficients of He II of Landau and Khalatnikov.[2-4]

2. The volume absorption for second sound below 1 °K increases rapidly with decreasing temperatures, in full agreement with the theory. The losses due to the thermal conductivity of He II exceed the viscous losses by an order of magnitude.

3. The viscosity of the normal component of He II below 1 °K increases extremely rapidly with decrease in temperature. In absolute

T °K	α/ω^2, sec^2/cm	$\eta \times 10^5$ poise	$\varkappa$ $\frac{cal}{deg\text{-}cm\text{-}sec}$	u_2, m/sec	ρ_n/ρ	c $\frac{cal}{gm - deg}$
0,83	$4.5 \cdot 10^{-10}$	6.7	$1,21 \cdot 10^{-2}$	22.9	$1.5 \cdot 10^{-3}$	$8.0 \cdot 10^{-2}$
0.85	$3,58 \cdot 10^{-10}$	5.8	$8.7 \cdot 10^{-3}$	22.0	$1,95 \cdot 10^{-3}$	$8.2 \cdot 10^{-3}$
0.85	$2.17 \cdot 10^{-10}$	5.26	$5,2 \cdot 10^{-3}$	22.0	$1,95 \cdot 10^{-3}$	$8.2 \cdot 10^{-3}$
0.855	$1,95 \cdot 10^{-10}$	5.6	$5.05 \cdot 10^{-3}$	21,95	$2,08 \cdot 10^{-3}$	$9,0 \cdot 10^{-3}$
0.86	$2.62 \cdot 10^{-10}$	3.7	$7,4 \cdot 10^{-3}$	21.6	$2.2 \cdot 10^{-3}$	$1.0 \cdot 10^{-3}$
0.89	$1.64 \cdot 10^{-10}$	3.66	$5,5 \cdot 10^{-3}$	20.6	$3.0 \cdot 10^{-3}$	$1,2 \cdot 10^{-3}$
0.9	$1.23 \cdot 10^{-10}$	3.42	$3.46 \cdot 10^{-3}$	20.3	$3.3 \cdot 10^{-3}$	$1.22 \cdot 10^{-2}$
0.9	$1.08 \cdot 10^{-10}$	3.55	$3.0 \cdot 10^{-3}$	20.3	$3.3 \cdot 10^{-3}$	$1,22 \cdot 10^{-2}$
0.94	$6.5 \cdot 10^{-11}$	3.72	$2.2 \cdot 10^{-3}$	19.5	$4,8 \cdot 10^{-3}$	$1,7 \cdot 10^{-2}$
0.95	$4.8 \cdot 10^{-11}$	3.38	$1,63 \cdot 10^{-3}$	19,35	$5.2 \cdot 10^{-3}$	$1,78 \cdot 10^{-2}$
0.98	$4.2 \cdot 10^{-11}$	2.96	$1,59 \cdot 10^{-3}$	18.9	$6.5 \cdot 10^{-3}$	$2.1 \cdot 10^{-2}$
1.0	$3,2 \cdot 10^{-11}$	2.61	$1,39 \cdot 10^{-3}$	18,7	$7.5 \cdot 10^{-3}$	$2,5 \cdot 10^{-2}$
1.01	$3.75 \cdot 10^{-11}$	2.5	$1.72 \cdot 10^{-3}$	18.7	$7,7 \cdot 10^{-3}$	$2,6 \cdot 10^{-2}$
1.03	$1.43 \cdot 10^{-11}$	2.74	$6.6 \cdot 10^{-4}$	18.5	$9.2 \cdot 10^{-3}$	$3.0 \cdot 10^{-2}$
1.07	$1.09 \cdot 10^{-11}$	2.49	$6.6 \cdot 10^{-4}$	18.35	$1.19 \cdot 10^{-2}$	$4,0 \cdot 10^{-2}$
1.1	$8,35 \cdot 10^{-12}$	2.45	$5.7 \cdot 10^{-4}$	18.30	$1.45 \cdot 10^{-2}$	$4.56,10^{-2}$
1.16	$3.76 \cdot 10^{-12}$	2.02	$3.6 \cdot 10^{-4}$	18.4	$2.3 \cdot 10^{-2}$	$6.4 \cdot 10^{-2}$
1,21	$3,2 \cdot 10^{-12}$	1.72	$4.1 \cdot 10^{-4}$	18,55	$2.9 \cdot 10^{-2}$	$8.0 \cdot 10^{-2}$
1.25	$1.76 \cdot 10^{-12}$	1.8	$2,6 \cdot 10^{-4}$	18.85	$4.0 \cdot 10^{-2}$	$1.0 \cdot 10^{-1}$
1.31	$1,24 \cdot 10^{-12}$	1.56	$2,7 \cdot 10^{-4}$	19.2	$5,3 \cdot 10^{-2}$	$1.28 \cdot 10^{-1}$
1.4		1.3				
1,64		1.25				
1.91		1.3				
2.0		1.55				
2,1		1.8				

magnitude, the values of the viscosity agree with the results of the measurements of Heikkila and Hollis-Hallet[13], obtained by the rotating cylinder method, while a considerable divergence is observed from the results of Andronikashvili and others [10-12], observed by means of the rotating disk method below 1.5 °K.

4. The analogue to the coefficient of thermal conductivity in He II, $\varkappa$, increases rapidly with drop in temperature, ranging in the interval 0.83 – 1.31° K from 1.2×10^{-2} to 2.7×10^{-4} cal/deg-cm-sec.

I take this opportunity to express my deep gratitude to V. Peshkov for valued advice and constant interest in the work, and also to N. I. Iakovlev, who helped in setting up the apparatus and carrying out the experiments.

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Translated by R. T. Beyer
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Other Errata

Page	Column	Line	Reads	Should Read
Volume 4				
38	1	Eq. (3)	$\ldots \frac{\pi r^2 \rho^2 \rho^2{}_n}{\rho_s^2},$	$\ldots \frac{\pi r^2 \rho^2 \rho_n}{\rho_s^2},$
196		Date of submittal	May 7, 1956	May 7, 1955
377	1	Caption for Fig. 1	$\delta_{35} = \eta - 21 \cdot \eta^5$	$\delta_{35} = -21^\circ \eta^5.$
377	2	Caption for Fig. 2	$\alpha_3 = 6.3^\circ\ \eta$	$\alpha_3 = -6.3^\circ \eta$
516	1	Eq. (29)	$s^2/c^2 \ldots$	s/c
516	2	Eqs. (31) and (32)	Replace $A_1 s^2/c^2$ by A_1	
497		Date of submittal	July 26, 1956	July 26, 1955
900	1	Eq. (7)	$\ldots \frac{i}{4\pi} \sum_{c,\alpha} \frac{\partial w_a(t, P)}{\partial P^\alpha} \ldots$	$\ldots 2\pi^2 i \sum_{c,\alpha} \ldots$
			(This causes a corresponding change in the numerical coefficients in the expressions that result from the calculation of the effects of the plasma particles on each other).	
804	2	Eq. (1)	$\ldots \exp\{-(\overline{T} - V')\}$	$\ldots \exp\{-(\overline{T} - V')\tau^{-1}\}$
Volume 5				
59	1	Eq. (6)	$v_l(1\partial F_0/\partial x) + \ldots$ where E_l is the projection of the electric field E on the direction 1	$\overline{(v\partial F_0/\partial x)} + \ldots$ where the bar indicates averaging over the angle θ and E_l is the projection of the electric field E along the direction 1
91	2	Eq. (26)	$\Lambda = 0.84\,(1+22/A)$	$\Lambda = 0.84/(1+22/A)$
253		First line of summary	$Tl^{204,\ 206}$	$Tl^{203,\ 205}$
318	1	Figure caption	$\ldots e^2 mc^2 = 2.8 \cdot 10^{-23}$ cm,	$\ldots e^2/mc^2 = 2.8 \cdot 10^{-13}$ cm,
398		Figure caption	... to a cubic relation. A series of points etc.	... to a cubic relation, and in the region 10 – 20°K to a quadratic relation. A series of points ●, coinciding with points O, have been omitted in the region above 10°K.