TABLE 2

The deviation (decrease) in % of the c. i .c. from the calculations of Rose (Z=92)

K	α_1	α_2	α_3	α_4	α_5	β_1	β_2	β_3	β_4	β_5
0,5	1	5.3	9,2	9.9	8.8	41.8	21.3	17,2	14,5	11
1	1,7	9,8	12,3	13	13,2	44.4	22,2	18	16	14,8
4	3,42	14	14.5	14.2	14	54.2	26,2	20,7	18,1	16,6
5	3,1	13	14,3	14.3	14.1	56.5	27.7	21.1	18.6	17.2

[1]K. Siegbahn, *Beta and Gamma-ray Spectroscopy*, (1955).

[2]M. Metropolis and J. R. Reitz,J. Chem. Phys.19, 555 (1951).

[3]L' Brillouin, *L'Atom de Thomas, Fermi et la metode du champ "Self-Consistent "*, (1934). J. R. Reitz, Phys. Rev., 77, 10 (1950).

[4]L. A. Sliv, J. Exptl. Theoret. Phys. (U.S.S.R.) 21, 770 (1951).

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Selection Rules for Antiproton Annihilation Into π – Mesons

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IN a previously published article Amati and Vitale[1] have given selection rules for the annihilation of antiprotons into two and three π– mesons. However, following from general considerations which take into account the strong interaction of π– mesons with nucleons, and also on the basis of the already available experimental data[2], the greatest practical interest lies in the effects of the multiple meson annihilation of antiprotons. The present article is devoted to a possible generalization of the selection rules to an arbitrary number of π– mesons being produced, and also contains some remarks on the dependence which follows from these rules of the effective cross sections for annihilation on the velocity of the antiproton in the region of nonrelativistic energies. We shall restrict ourselves to a consideration of strict selection rules which are based on the requirement that the laws of conservation of the total angular momentum and of the space and charge parities be satisfied simultaneously.

We recall that the requirement indicated above leads to the fact that the annihilation of antiprotons in singlet states of the nucleon-antiproton system into two π– mesons is strictly forbidden. On the other hand, in triplet states of such a system, the effects

$$1)\ \tilde{p}+p\rightarrow\pi^-+\pi^+,\quad 2)\ \tilde{p}+n\rightarrow\pi^-+\pi^0$$

can occur only under the condition

$$L_i=0\rightarrow L_f=1,\quad L_i=2n\rightarrow L_f=2n\pm1, \qquad (a)$$

or

$$L_i=1\rightarrow L_f=0,\quad L_i=2n\pm1\rightarrow L_f=2n \qquad (b)$$

$$(n=1,\,2,\ldots),$$

where L_i is the orbital angular momentum of the nucleon-antiproton system; L_f is the orbital angular momentum of the system of two π– mesons. Thus in the triplet nucleon-antiproton system annihilation into two π– mesons [effects (1) and (2)] may occur only under the condition

$$\Delta L=L_f-L_i=\pm1.$$

A characteristic property of the selection rules for the annihilation into two neutral π – mesons

$$3)\ \tilde{p}+p\rightarrow\pi^0+\pi^0$$

is the additional restriction on the states of the π– mesons being formed which comes from the requirement that the wave function of the final system should be symmetric with respect to interchanges of identical particles. The last requirement excludes the possibility of states of odd parity for a system of two π^0– mesons and leads to the additional requirement [in comparison with (1) and (2)] that the transitions (a) be forbidden. Thus the only allowed transitions with formation of two π^0 – mesons for the system of proton-antiproton mutually annihilating each other are the transitions in the spin-triplet state for which

$$L_i = 1 \rightarrow L_f = 0, \quad L_i = 2n \pm 1 \rightarrow L_f = 2n.$$

Thus in the system of a proton and an antiproton being mutually annihilated into two π^0 – mesons the states with the lowest value of the orbital angular momentum are the states $^3P_0$ and $^3P_2$. This last circumstance makes the transition matrix element for the effect (3) in the region of small energies proportional to the momentum of the antiproton (q). In connection with this, the probability of effect (3) turns out to be proportional to the square of the momentum of the antiproton:

$$\omega_3 = A_3 q^2 / M^2,$$

while the cross section for the effect (3) in a nonrelativistic approximation becomes proportional to the velocity of the antiproton (in the observer's system):

$$\sigma_3 = \omega_3 / v_{\tilde{p}} = A_3' v_{\tilde{p}} / c.$$

At the same time, the cross sections for the effects (1) and (2), which take place at low velocity in states with zero angular momentum (S-wave) have the usual form

$$\sigma_{1,2} = A_{1,2} c / v_{\tilde{p}}.$$

The effects of the annihilation of antiprotons into three π – mesons are not, generally speaking, subject to any strict selection rules. However, in the case of annihilation into three neutral π^0 – mesons

$$4) \quad \tilde{p} + p \rightarrow \pi^0 + \pi^0 + \pi^0$$

the requirement of the conservation of charge parity restricts the possible states of $\tilde{p} - p$ to singlet states with even orbital momentum and to triplet states with odd orbital momentum.

Now going over to the effects of the annihilation of antiprotons into an arbitrary number of π – mesons we shall point out that the following effects are strictly forbidden.

1. Annihilation into an even number of π – mesons in a singlet system with zero (total) orbital angular momentum is strictly forbidden. This selection rule is related to the incompatibility of the requirements of conservation of total angular momentum and of space parity. In view of the fact that in states with zero orbital angular momentum the parities of the initial and final states ($I_{\tilde{N}-N}$ and $I_{(2k)\pi}$) are determined by the intrinsic parities of the particles[3],

$$I_{\tilde{N}-N} = -1, \quad I_{(2k)\pi} = (-1)^{2k} = +1.$$

2. Annihilation into an arbitrary number of neutral π – mesons

$$5) \quad \tilde{p} + p \rightarrow k\pi^0 \quad (k = 4, 5 \ldots)$$

is allowed in triplet states with odd values of the orbital angular momentum and also in singlet states with even values of the orbital angular momentum. Since all the states of the system consisting only of π^0 – mesons are of even charge parity, the initial states of the system of $\tilde{p} - p$ annihilating each other must also be of even charge parity. The states of the system particle – antiparticle noted above are of just that kind (see Ref. 4). The selection rule under discussion is responsible for the smallness of the effect consisting of the annihilation of the triplet nucleon- antinucleon system into an arbitrary number of π^0 – mesons in the nonrelativistic approximation

$$\sigma^{tr}_{k\pi^0} = A_5 v_{\tilde{p}} / c.$$

3. The comparison of the two selection rules given above leads to the annihilation of the $\tilde{p} - p$ system in the state with zero orbital angular momentum into an even number of neutral π – mesons being forbidden. As a result of this in the nonrelativistic approximation the effective cross section for the reaction

$$6) \quad \tilde{p} + p \rightarrow (2k)\pi^0 \quad (k = 2, 3, \ldots)$$

is equal to

$$\sigma_{(2k)\pi^0} \approx \sigma^{tr}_{(2k)\pi^0} \approx A_6 v_{\tilde{p}} / c.$$

In conclusion, I wish to express my sincere thanks to Prof. V. L. Ginzburg for the interest shown in this work and for the discussion of results.

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[1] S. Amati and B. Vitale, Nuovo Cimento 2, 719 (1955).

[2] I. Brabant, Phys. Rev. **101**, 498 (1956).

[3] V. B. Berestetskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **21**, 93 (1951).

[4] A. I. Akhiezer and V. B. Berestetskii, *Quantum Electrodynamics*, GITTL, 1953, p. 153.

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Remarks on the Theory of Scattering of Particles with a Given Total Angular Momentum

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1. WE consider the scattering of two fermions interacting by means of a boson field, but, in contrast to the usual treatment of this problem, the initial and final states of the pair of fermions are specified as states with a certain given total angular momentum. It turns out that this approach to the problem presents a number of peculiarities.

It is convenient to separate the motion of the center-of-mass from the relative motion and then to examine the process in the system of coordinates in which the center-of-mass is at rest. The matrix element which corresponds to the simplest irreducible diagram (which is the only one we shall consider for the sake of brevity) may be written in the form

$$\overline{\psi}_{\sigma_1\sigma_2}(k)\,\Gamma^{(1)}D(k-k_0)\,\Gamma^{(2)}\psi_{\sigma_1^0\sigma_2^0}(k_0), \qquad (1)$$

where D is the retarded interaction function, $\Gamma^{(i)}$ is the vertex part, the matrix of which operates on the indices of the ith particle; $k(\epsilon, \mathbf{k})$ is the 4-momentum of the particle in the system of coordinates under consideration ($\mathbf{k}_1 = -\mathbf{k}_2 = \mathbf{k}$); $\sigma_i = \pm\frac{1}{2}$ define the spin states of the particles. We consider only the process of interaction which does not lead to the radiation or absorption of bosons and also the case of the absence of external fields. The diagrams which take into account the proper mass of the particles and which are not related to the interaction between the particles are also not taken into account.

The matrix elements of interest to us in the energy-angular momentum space may be most conveniently obtained from the matrix elements of type (1) by a suitable transformation which takes into account the spinor nature of the wave functions[1]

$$\mathfrak{M}^{jm\chi}_{j_0m_0\chi_0} = \sum_{\sigma_1,\sigma_2}\sum_{\sigma_1^0,\sigma_2^0}\iint (dk)\,(dk_0)\,A^{\chi}_{jm}A^{\chi^0}_{j_0m_0}M^{\sigma_1\sigma_2 k}_{\sigma_1^0\sigma_2^0 k_0}. \qquad (2)$$

Here j and m are the quantum numbers of the total angular momentum and of its component, and the index χ characterizes the spin state of the pair of particles (singlet and triplet). The coefficients A^{χ}_{jm}, for example, in the simplest case of the singlet state, have the following form:

$$A^{s}_{jm} = \delta(k_4-\varepsilon)\,k^{-2}\delta \qquad (3)$$

$$\times(|\mathbf{k}|-p)\,Y_{jm}(n_k)\,2\sigma_1\delta_{\sigma_1,-\sigma_2}/\sqrt{2}.$$

In the same way, one may define the coefficient for the triplet case which includes suitable spherical vectors and the required symmetric combinations of spin states (ϵ is the energy, p is the absolute magnitude of the momentum of the particles in the energy-angular momentum space).

By transforming in the manner indicated the matrix element of the zero order approximation to (1) (Möller scattering), for example for the transition singlet → singlet ($\chi = \chi_0 = s$), we obtain an expression of the form

$$e^2A(\varepsilon)\,Q_l(\Delta)\,\delta_{jj_0}\delta_{mm_0},$$

where the form of the function $A(\epsilon)$, which arises as a result of the summation in (2) [similar to (6), see below] depends on the explicit form of the interaction and on the kind of Bose particles, but does not depend on j and m. The argument of Q_j (Legendre function of the second kind) is the quantity $\Delta = 1 + \mu^2/2p^2$ where $p = |\mathbf{k}|$ is the absolute magnitude of the momentum of the particles, and μ is the mass of the boson responsible for the interaction.

In the case of electrodynamics the integration in (2) cannot be carried out formally (if one considers the particles in the initial and final states to be free) since $Q_j(1)$ diverges. For bound states $k^2 \neq m^2$ and the function Δ will contain the binding energy of the particles.

For large $j \gg l$ we have

$$Q_j(1+\mu^2/2p^2) \approx K_0(\mu j/p).$$

Therefore, for not too large initial energies