$$({}^1/r + \varepsilon)\, u = d^2u \,/\, dr^2,$$

it is not hard to show that this equality is satisfied identically, and that, therefore, it does not amount to some supplementary condition imposed on the function $Y(r)$.

An analogous simplification may be obtained by setting $\alpha_2 = 0$. But one must keep in mind that the iteration process may not converge equally rapidly in the two cases.

The functions $f^+(r)$ and $f^-(r)$ were calculated in the first approximation from the formulas (5)-(7), (9) and (10) and the scattering phases were found from them. The results are given in Figs. 1 and 2. For comparison, the results of a numerical integration[1] of Eq. (2) and the results a variational calculation[2] are also given there. In the antisymmetric case all three curves coincide within the scale of the drawing. In the symmetric case one obtains a somewhat higher value of the phase for $k > 0.5$; at lower energies in this case also a complete agreement is obtained in the first approximation with the result of the numerical integration.

In conclusion, we express our gratitude to G. F. Drukarev for his interest in this work and for a number of valuable suggestions.

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On the Relation between the Distribution of a Quasi-Monochromatic Stationary Process and the Distribution of its Envelope

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NOT long ago Blanc-Lapierre and others[1] showed that if the probability density $w_A(A)$ of the envelope of a quasi-monochromatic stationany stochastic process ξ is known, the characteristic function $f_\xi(u)$ of the process is

$$f_\xi(u) = \int_0^\infty w_A(A)\, J_0(Au)\, dA. \quad (1)$$

Rytov[2] continued the calculation and obtained the following formula:

$$w_\xi(\xi) = \frac{1}{\pi} \int_{|\xi|}^\infty \frac{w_A(A)}{\sqrt{A^2 - \xi^2}}\, dA, \quad (2)$$

connecting the probability density $w_\xi(\xi)$ of a stationary stochastic process with $w_A(A)$. In the present note I wish to show another way of deriving Eqs. (1) and (2), whereby they are obtained as the zero-order approximation in neglecting the narrow passband width of an amplifier. The use in this derivation of the time average, correct in the case of a stationary stochastic process, gives the possibility of obtaining in a natural manner the correction terms accounting for the finite width of the passband.

A quasi-monochromatic stationary process can be written in the form $\xi(t) = A(t) \cos[\omega_0 t + \varphi(t)]$, where $A(t)$ and $\varphi(t)$ are functions of time varying slowly in comparison with $\cos \omega_0 t$. Then the characteristic function $f_\xi(u)$ is

$$f_\xi(u) = \lim_{T\to\infty} \frac{1}{T} \times \int_0^T \exp\{iuA(t)\cos[\omega_0 t + \varphi(t)]\}\, dt. \quad (3)$$

Let us break up the interval $0, T$ into N small intervals, each of length $\tau = 2\pi/\omega_0$. Then

$$f_\xi(u) = \lim_{N\to\infty} \frac{1}{N} \sum_{m=1}^{N} \frac{1}{\tau} \times \int_{(m-1)\tau}^{m\tau} \exp\{iuA(t)\cos[\omega_0 t + \varphi(t)]\}\, dt. \quad (4)$$

Taking into account the smallness of the change of $A(t)$ and $\varphi(t)$ in the time τ, we expand the expression under the integral sign in a series in A and φ and limit ourselves to terms of the first order of smallness.

Then after simple transformations we obtain in place of Eq. (4),

$$f_\xi(u)=\lim_{N\to\infty}\frac{1}{N}\sum_{m=1}^{N}\frac{1}{\tau}\times\int_{(m-1)\tau}^{m\tau}\exp\{iuA_m\cos(\omega_0 t+\varphi_m)\}\,dt$$
$$+\lim_{N\to\infty}\frac{1}{N}\sum_{m=1}^{N}\frac{1}{\tau}\int_0^{\tau} iu\dot{A}_m\theta\cos(\omega_0\theta+\varphi_m)\times\exp\{iuA_m\cos(\omega_0\theta+\varphi_m)\}\,d\theta$$
$$-\lim_{N\to\infty}\frac{1}{N}\sum_{m=1}^{N}\frac{1}{\tau}\int_0^{\tau} iuA_m\dot{\varphi}\,\theta\sin(\omega_0\theta+\varphi_m)\times\exp\{iuA_m\cos(\omega_0\theta+\varphi_m)\}\,d\theta. \quad (5)$$

Here A_m, φ_m, $\dot{A}_m$, and $\dot{\varphi}_m$ are the values of $A(t)$, $\varphi(t)$, and their derivatives at the beginning of the m th interval. It is easy to see that the first of the sums in Eq. (5) gives Eq. (1); the other two sums give the correction depending on the finiteness of the rate of change of A and φ.

If we use the well-known expression determining the probability density of a stochastic variable in terms of its characteristic function, we obtain from Eq. (5),

$$w_\xi(\xi)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} f_\xi(u)\,e^{-iu\xi}\,du \quad (6)$$
$$=\overline{\frac{1}{\pi\sqrt{A^2-\xi^2}}\left(1+\frac{\dot{\varphi}}{\omega_0}\right)}$$

(In this transformation the second sum in Eq. (5) gives zero.)

In this way Eq. (6) agrees with Eq. (2) to the accuracy of the correction term

$$\overline{\dot{\varphi}/(\pi\omega_0\sqrt{A^2-\xi^2})}. \quad (7)$$

We note that although for a stationary process $\dot{\varphi}$ must be equal to zero, the average value, Eq. (7), may differ from zero, since in increasing the passband, A and $\dot{\varphi}$ in the general case are no longer independent random variables. It is interesting that the rate of change of the amplitude does not influence the probability density $w_\xi(\xi)$ in the first approximation.

We note in closing that in the case in which ξ is obtained by use of a narrow-band filter from a quasi-monochromatic stationary stochastic process, obeying the normal probability distribution law, the correction term in Eq. (6) disappears, and we are left with the correct formula, Eq. (2). This agrees with a well-known property of the normal distribution relating to the linear transformation of the spectral components.

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Atomic Magnetic Moments of Ferromagnetic Metals and Alloys

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IN contemporary physics it is well known that on account of the difficulties associated with the many-electron problem, the relation for the quantitative calculation of the atomic magnetic moments m_φ and m_π, for the ferromagnetic and paramagnetic states of ferromagnetics, respectively, has not thus far been found. The relation proposed by us was obtained empirically.

1. *Pure metals.* It is well known that m is expressed as a fractional number of Bohr magnetons M_B (see Table I), and that this fraction is due to the exchange interaction between the s-electrons of the metal and the d- (or f-) electrons of the atoms, as a consequence of which m must somehow depend on the lattice parameter. On this basis we propose that in the equation for m, as well as in Eqs. (3)-(6), there enters the term

$$E=0.641\,[n_1(r_1-R)+n_2(r_2-R)]=E_0+\Delta E, \quad (1)$$

where n_1 is the number of nearest neighbors of an atom in the lattice, n_2 the number of next-nearest neighbors, r_1 and r_2 the corresponding interatomic distances, R an empirical constant characterizing the given transition element, E_0 the integral part of E, and ΔE the fractional remainder, $|\Delta E|<1$.

Let us first examine the first transition series of the periodic table of elements. For it*,

$$R=0.0325\,Z^2+\begin{matrix}30.6182-1.9175\,Z & \text{for} & Z\geqslant 26\\ 27.2382-1.7875\,Z & \text{for} & Z\leqslant 26,\end{matrix} \quad (2)$$