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Theory of Classical Systems of Interacting Particles Obeying a Noncentral Interaction Law. I.

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The theory of systems of interacting particles with noncentral interaction law is considered on the basis of Bogoliubov's method.[1] Successive approximations are obtained for the distribution functions in the two simplest cases: for a gas consisting of axially symmetric diatomic neutral molecules and for a dipole crystal [provided the condition (32) holds].

THE theory of systems with a noncentral interaction law between the particles is characterized by a number of special difficulties; however, in certain cases it can be constructed rather easily on the basis of Bogoliubov's method.

Let us consider the simplest possible case of a classical system, in which the interaction law between the particles differs only slightly from the law of central short range forces. Such is the case for systems consisting of neutral diatomic molecules when their weak orientation interaction with each other is taken into account.[2]

We introduce the distribution function F_s so normalized that

$$(4\pi V)^{-s} F_s(\mathbf{q}_1, \ldots, \mathbf{q}_s; \vartheta_1, \varphi_1, \ldots, \vartheta_s \varphi_s) \times d\mathbf{q}_1 \ldots d\mathbf{q}_s d\Omega_1 \ldots d\Omega_s \tag{1}$$

determines the probability of the location of the centers of mass of a chosen group of s molecules in the respective volumes $d\mathbf{q}_1, \ldots, d\mathbf{q}_s$ for orientations of these molecules determined by the elements of solid angle $d\Omega_1, \ldots, d\Omega_s$, where $\mathbf{q}_1, \ldots, \mathbf{q}_s$ are vectors determining the position of the centers of mass of the molecules and the angles $\vartheta_1, \varphi_1 \ldots \vartheta_s, \varphi_s$ determine the orientation of the axes of the molecules relative to the chosen system of coordinates. The distribution functions thus determined characterize a system consisting of identical, symmetric diatomic molecules and satisfy the equality

$$F_s(\mathbf{q}_1 \ldots \mathbf{q}_s; \vartheta_1, \varphi_1 \ldots \vartheta_s, \varphi_s) = (4\pi V)^s \int D_N d\mathbf{q}_{s+1} \ldots d\mathbf{q}_N d\Omega_{s+1} \ldots d\Omega_N;$$

$$D_N(\mathbf{q}_1, \ldots \mathbf{q}_N; \vartheta_1, \varphi_1, \ldots \vartheta_N, \varphi_N) = Q_N^{-1} \exp(-U_N/\Theta)$$

is a Gibb's configuration function and

$$U_N = \sum_{1 \leqslant i < j \leqslant N} L_{ij}(\mathbf{q}_i, \mathbf{q}_j; \vartheta_i, \varphi_i; \vartheta_j, \varphi_j),$$

where L_{ij} is the interaction potential and

$$Q_N = \int \exp(-U_N/\Theta) d\mathbf{q}_1 \ldots d\Omega_N; \quad \Theta = kT.$$

Beginning with the identities

$$\partial D_N/\partial q_1^\alpha + (D_N/\Theta)\, \partial U_N/\partial q_1^\alpha = 0, \quad (\alpha = 1, 2, 3),$$

$$\partial D_N/\partial \vartheta_1^\alpha + (D_N/\Theta)\, \partial U_N/\partial \vartheta_1^\alpha = 0, \quad (\alpha = 1, 2),$$

in the usual way, considering equalities of the type

$$\int \frac{\partial L_{1,s+1}}{\partial q_1^\alpha} F_{s+1} (\mathbf{q}_1 \ldots \mathbf{q}_s, \mathbf{q}_{s+1}; \vartheta_1, \varphi_1 \ldots \vartheta_s, \varphi_s, \vartheta_{s+1}, \varphi_{s+1}) \times d\mathbf{q}_{s+1} d\Omega_{s+1}$$

$$= \int \frac{\partial L_{1,s+2}}{\partial q_1^\alpha} F_{s+1} (\mathbf{q}_1 \ldots \mathbf{q}_s, \mathbf{q}_{s+2}; \vartheta_1, \varphi_1 \ldots \vartheta_s, \varphi_s, \vartheta_{s+2}, \varphi_{s+2}) \times d\mathbf{q}_{s+2} d\Omega_{s+2},$$

we get to the equations

$$\frac{\partial F_s}{\partial q_1^\alpha} + \frac{1}{\Theta} \frac{\partial U_s}{\partial q_1^\alpha} F_s + \frac{1}{4\pi\Theta v} \iint \frac{\partial L_{1,s+1}}{\partial q_1^\alpha} F_{s+1} d\mathbf{q}_{s+1} d\Omega_{s+1} = 0, \quad \alpha = 1, 2, 3.$$

$$\frac{\partial F_s}{\partial \vartheta_1^\alpha} + \frac{1}{\Theta} \frac{\partial U_s}{\partial \vartheta_1^\alpha} F_s + \frac{1}{4\pi\Theta v} \iint \frac{\partial L_{1,s+1}}{\partial \vartheta_1^\alpha} F_{s+1} d\mathbf{q}_{s+1} d\Omega_{s+1} = 0, \quad \alpha = 1, 2 \tag{2}$$

($v = V/N$). We limit ourselves here to a consideration of an interaction potential of the following type:

$$L_{1,2} = \Phi(|\mathbf{q}_1 - \mathbf{q}_2|) \times [1 + \lambda \psi_{1,2}(\vartheta_1, \varphi_1, \vartheta_2, \varphi_2)], \tag{3}$$

where Φ is the interaction potential which corresponds to short range central forces and $\psi_{1,2}$ are terms taking into account the dependence of the interaction on the orientation of the particles. Although we have written explicitly only the dependence of $\psi_{1,2}$ on the pairs of variables ϑ_1, φ_1 and ϑ_2, φ_2 which determine the orientation of the axes of the molecules in space, this function actually depends on the pairs of variables $\vartheta_{12}, \varphi_{12}$ also. These latter determine the orientation of the line of centers of the pair of particles under consideration; λ denotes the parameter whose smallness determines the weakness of the orienting effects in the system under consideration.[2]

Limiting ourselves to a consideration of systems with sufficiently low density, we employ an expansion in powers of the latter:

$$F_s = F_s^0 + v^{-1}F_s^1 + v^{-2}F_s^2 + \dots \tag{4}$$

The corresponding groups of the systems of equations will have the form

$$\frac{\partial F_s^0}{\partial q_1^\alpha} + \frac{1}{\Theta}\frac{\partial U_s}{\partial q_1^\alpha} F_s^0 = 0 \quad (\alpha = 1,\ 2,\ 3), \tag{5}$$

$$\frac{\partial F_s^0}{\partial \vartheta_1^\alpha} + \frac{1}{\Theta}\frac{\partial U_s}{\partial \vartheta_1^\alpha} F_s^0 = 0 \quad (\alpha = 1,\ 2);$$

$$\frac{\partial F_s^1}{\partial q_1^\alpha} + \frac{1}{\Theta}\frac{\partial U_s}{\partial q_1^\alpha} F_s^1 + \frac{1}{4\pi\Theta} \tag{6}$$

$$\times \iint \frac{\partial}{\partial q_1^\alpha}\Phi_{1,s+1}(1+\lambda\psi_{1,s+1})F_{s+1}^0 d\mathbf{q}_{s+1} d\Omega_{s+1} = 0,$$

$$\frac{\partial F_s^1}{\partial \vartheta_1^\alpha} + \frac{1}{\Theta}\frac{\partial U_s}{\partial \vartheta_1^\alpha} F_s' + \frac{1}{4\pi\Theta}$$

$$\times \iint \frac{\partial}{\partial \vartheta_1^\alpha}\Phi_{1,s+1}(1+\lambda\psi_{1,s+1})F_{s+1}^0 d\mathbf{q}_{s+1} d\Omega_{s+1} = 0$$

and so forth. Here,

$$U_s = \sum_{1\leqslant i<j\leqslant s}\Phi_{ij}(|\mathbf{q}_i - \mathbf{q}_j|) \tag{7}$$

$$\times (1+\lambda\psi_{ij}(\vartheta_i,\ \varphi_i,\ \vartheta_j,\ \varphi_j))$$

can be written in the form of a sum $U_s = U_s^0 + \lambda U_s^1$, where

$$U_s^0 = \sum_{1\leqslant i<j\leqslant s}\Phi(|\mathbf{q}_i - \mathbf{q}_j|),$$

$$U_s^1 = \sum_{1\leqslant i<j\leqslant s}\Phi(|\mathbf{q}_i - \mathbf{q}_j|)\,\psi_{ij}(\vartheta_i,\ \varphi_i,\ \vartheta_j,\ \varphi_j).$$

Now, for each of the systems, we consider an expansion in powers of the small parameter λ :

$$F_s^i = F_s^{i0} + \lambda F_s^{i1} + \lambda^2 F_s^{i2} + \dots, \tag{8}$$

$$i = 0,\ 1, \dots$$

The corresponding systems of equations in this case are described in the following way:

$$\frac{\partial F_s^{00}}{\partial q_1^\alpha} + \frac{1}{\Theta}\frac{\partial U_s^0}{\partial q_1^\alpha} F_s^{00} = 0, \quad \frac{\partial F_s^{00}}{\partial \vartheta_1^\alpha} = 0 \tag{5a}$$

$$\frac{\partial F_s^{01}}{\partial q_1^\alpha} + \frac{1}{\Theta}\frac{\partial U_s^0}{\partial q_1^\alpha} F_s^{01} + \frac{1}{\Theta}\frac{\partial U_s^1}{\partial q_1^\alpha} F_s^{00} = 0, \tag{5b}$$

$$\frac{\partial F_s^{01}}{\partial \vartheta_1^\alpha} + \frac{1}{\Theta}\frac{\partial U_s^1}{\partial \vartheta_1^\alpha} F_s^{00} = 0 \quad \text{etc.} \tag{6a}$$

$$\frac{\partial F_s^{10}}{\partial q_1^\alpha} + \frac{1}{\Theta}\frac{\partial U_s^0}{\partial q_1^\alpha} F_s^{10} \tag{6b}$$

$$+ \frac{1}{4\pi\Theta}\iint \frac{\partial \Phi_{1,s+1}}{\partial q_1^\alpha} F_{s+1}^{00} d\mathbf{q}_{s+1} d\Omega_{s+1} = 0; \quad \frac{\partial F_s^{10}}{\partial \vartheta_1^\alpha} = 0;$$

$$\frac{\partial F_s^{11}}{\partial q_1^\alpha} + \frac{1}{\Theta}\frac{\partial U_s^0}{\partial q_1^\alpha} F_s^{11} + \frac{1}{\Theta}\frac{\partial U_s^1}{\partial q_1^\alpha} F_s^{10}$$

$$+ \frac{1}{4\pi\Theta}\iint \frac{\partial \Phi_{1,s+1}}{\partial q_1^\alpha} F_{s+1} d\mathbf{q}_{s+1} d\Omega_{s+1}$$

$$+ \frac{1}{4\pi\Theta}\iint \frac{\partial}{\partial q_1^\alpha}(\Phi_{1,s+1}\psi_{1,s+1}) F_{s+1}^{00} d\mathbf{q}_{s+1} d\Omega_{s+1} = 0,$$

$$\frac{\partial F_s^{11}}{\partial \vartheta_1^\alpha} + \frac{1}{\Theta}\frac{\partial U_s^1}{\partial \vartheta_1^\alpha} F_s^{10}$$

$$+ \frac{1}{4\pi\Theta}\iint \frac{\partial}{\partial \vartheta_1^\alpha}(\Phi_{1,s+1}\psi_{1,s+1}) F_{s+1}^{00} d\mathbf{q}_{s+1} d\Omega_{s+1} = 0$$

and so on. The solution of the system (5a) is, evidently,

$$F_s^{00} = C_s \exp(-U_s^0/\Theta), \tag{9}$$

where C_s is a constant which is determined by the condition of the decay of the correlation

$$F_s(\mathbf{q}_1 \dots \mathbf{q}_s;\ \vartheta_1,\ \varphi_1, \dots \vartheta_s,\ \varphi_s) \tag{10}$$

$$- \prod_{1\leqslant i\leqslant s} F_1(\hat{\mathbf{q}}_i,\ \vartheta_i,\ \varphi_i) \to 0$$

for all $|\mathbf{q}_i - \mathbf{q}_j| \to \infty$ and the normalization condition

$$\lim_{V\to\infty} \frac{1}{4\pi V} \iint F_1(\mathbf{q},\vartheta,\varphi)\, d\mathbf{q}\, d\Omega = 1. \tag{11}$$

We find from the second of the equations of (5b) that

$$F_s^{01} = -(U_s^1/\Theta)\, F_s^{00}. \tag{12}$$

This expression automatically satisfies the first equation of (5b).

The general solution of (5b) can be written formally as

$$F_s^{01} = -(U_s^1/\Theta) F_s^{00} + C g_s(\mathbf{q}_1 \ldots \mathbf{q}_s),$$

where g_s is determined upon substitution in the first equation of (5b) and is equal to $g_s = F_s^{00}$. However, it is easy to prove directly that preservation in the solution of terms of the type $Cg_s(\mathbf{q}_1, \ldots \mathbf{q}_s)$, both in (12) and in all subsequent approximations, would reduce to the corresponding determination of the normalization constant; in what follows, we shall neglect these terms.

Proceeding stepwise from one system of equations to the other, we find

$$F_s^{02} = {}^1\!/_2 (U_s^1/\Theta)^2 F_s^{00}. \tag{13}$$

Further,

$$F_s^{10} = \exp(-U_s^0/\Theta) \tag{14}$$

$$\times \int \Big\{ \prod_{1\leqslant i\leqslant s} (1 + f(|\mathbf{q}_i - \mathbf{q}_{s+1}|)) - 1$$

$$- \sum_{1\leqslant i\leqslant s} f(|\mathbf{q}_i - \mathbf{q}_{s+1}|)\Big\}\, d\mathbf{q}_{s+1},$$

$$F_s^{11} = -\frac{U_s^1}{\Theta} F_s^{10} \tag{15}$$

$$-\frac{1}{4\pi\Theta} \sum_i \iint \Phi_{i,s+1} \psi_{i,s+1} F_{s+1}^{00}\, dq_{s+1}\, d\Omega_{s+1}$$

$$+ a_s^{11} \exp(-U_s^0/\Theta),$$

$$F_s^{12} = -\frac{U_s^1}{\Theta} F_s^{11} - \frac{1}{2}\left(\frac{U_s^1}{\Theta.}\right)^2 F_s^{10} \tag{16}$$

$$+ \frac{1}{4\pi\Theta^2} \iint \frac{1}{2}\Big(\sum_i \Phi_{i,s+1}\psi_{i,s+1}\Big)^2 F_{s+1}^{00}\, d\mathbf{q}_{s+1}\, d\Omega_{s+1}$$

$$+ a_s^{12} \exp(-U_s^0/\Theta),$$

where

$$f(|\mathbf{q}_i - \mathbf{q}_{s+1}|) = \exp(-\Phi(|\mathbf{q}_i - \mathbf{q}_{s+1}|)/\Theta) - 1;$$

a_s^{11} and a_s^{12} are constants. Carrying forward the computation, it is easy to determine a much higher approximation. Knowledge of the first terms of the series described here is sufficient that we can, using unitary and binary distribution functions, estimate the effect of noncentral forces on the equilibrium properties of the system under consideration.

Calculation of the mean and free energies of the system can be carried out in the usual fashion and can be compared with the results of the corresponding calculation for purely central interaction on the one hand, and with experimental data for diatomic gases on the other.

As a second problem of known simplicity, we consider a molecular crystal consisting of molecules which possess a permanent dipole moment.*

Let us consider a dipole crystal of cubic structure, at the lattice points of which are located dipole molecules with moments $\mathbf{p}_i$ ($i = 1, \ldots, N$).

The set of angular coordinates which determine the orientation of each dipole (ϑ_i, φ_i) will be denoted by θ_i, the interaction potential of two dipoles $\Phi a_i a_k$ is written in the form

$$\Phi_{a_i a_k} = \mathbf{p}_{a_i} \mathbf{p}_{a_k} r_{a_i a_k}^{-3} \tag{17}$$

$$-3(\mathbf{p}_{a_i} \mathbf{r}_{a_i a_k})(\mathbf{p}_{a_k} \mathbf{r}_{a_i a_k})\, r_{a_i a_k}^{-5},$$

where the indices a_i denote the types of molecules. The type characteristics are introduced in the following way. The dipole crystal, in which the centers of mass of the dipole molecules are rigidly fixed at the lattice points, will be considered as a system of particles of different types. The type of particle is determined by the lattice vector of that point at which the center of mass of the molecule is located. In Eq. (17) the type characteristics appear in the form of numerical factors in connection with the equality

$$r_{a_i a_k} = |\mathbf{r}_{a_i} - \mathbf{r}_{a_k}| = d l_{a_i a_k}^{1/2}, \tag{18}$$

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*The case of a gas consisting of molecules with constant quadrupole moments can easily be considered if the law of interaction of two quadrupoles is taken in the form

$$\Phi_{ab} = (3Q_a Q_b/16r^5)$$

$$\times [1 - 5\cos^2\vartheta_a - 5\cos^2\vartheta_b - 15\cos^2\vartheta_a \cos^2\vartheta_b$$

$$+ 2(\sin\vartheta_a \sin\vartheta_b \cos(\varphi_a - \varphi_b) - 4\cos\vartheta_a \cos\vartheta_b)^2],$$

where Q is the quadrupole moment of a cylindrically symmetrical molecule.

where d is the lattice constant, l is an integer. They also follow from the cosines of the angles which determine the direction of the lines of centers of the two dipoles under consideration. In such a model the dipole crystal is an "orientation" gas or liquid at the temperature which exceeds the temperature of orientation flow.

We introduce the distribution function $F_{a_1 \ldots a_s}$ so normalized that

$$(4\pi)^{-s} F_{a_1 \ldots a_s}(\theta_1, \ldots \theta_s)\, d\Omega_1 \ldots d\Omega_s,$$

$$d\Omega_i = \sin\vartheta_i d\vartheta_i d\varphi_i,$$

is the probability that the direction of the dipole moments in the chosen group of s-dipoles lies within elements of solid angles $d\Omega_1 \ldots d\Omega_s$ for arbitrary orientations of the other dipoles.

Considering the functional defined on a class of arbitrary regular functions $U_1(\theta_1) \ldots U_N(\theta_N)$ given on the surface of the unit sphere

$$L_N(U_1, \ldots U_N) = \int D_N \prod_{1 \leqslant a_i \leqslant N} (1 + 4\pi U_{a_i}(\theta_i))\, d\Omega_1 \ldots d\Omega_N, \tag{20}$$

and making use of the identity

$$\frac{\partial D_N}{\partial \theta_i^\alpha} + \frac{\partial U_N}{\partial \theta_i^\alpha} \frac{D_N}{\mu} = 0, \quad U_N = \frac{1}{2} \sum_{a_i, a_j}^{N} \Phi_{a_i a_j},$$

(where $\mu = kT$, $\alpha = 1, 2$, $\theta_i^1 = \vartheta_i$ and $\theta_i^2 = \varphi_1$) it is easy to express the equations in functional derivatives

$$\frac{\partial}{\partial \theta_i^\alpha} \frac{\delta L}{\delta U_{a_1 \theta_1}} + \frac{1}{4\pi\mu} \int \sum_{1 \leqslant a_2 \leqslant N} \frac{\partial \Phi_{a_1 a_2}}{\partial \theta_1^\alpha} \frac{\delta^2 L}{\delta U_{a_1 \theta_1} \delta U_{a_2 \theta_2}} (1 + 4\pi U_{a_2})\, d\Omega_2 = 0. \tag{21}$$

Carrying out the functional differentiation and replacing in the final calculation the derivative functions U_a by zero, we obtain a system of equations for the distribution functions introduced above [taking into account the relations

$$(\delta^s L / \delta U_{a_1 \theta_1} \ldots \delta U_{a_s \theta_s})_0 = F_{a_1 \ldots a_s}(\theta_1 \ldots \theta_s)]. \tag{22}$$

this system can be written in the form

$$\frac{\partial F_{a_1 \ldots a_s}}{\partial \theta_1^\alpha} + \frac{1}{\mu} \frac{\partial U_{a_1 \ldots a_s}}{\partial \theta_1^\alpha} F_{a_1 \ldots a_s} + \frac{1}{4\pi\mu} \int \sum_{\substack{1 \leqslant a_{s+1} \leqslant N \\ (a_{s+1} \neq a_1, \ldots a_s)}} \frac{\partial \Phi_{a_1, a_{s+1}}}{\partial \theta_1^\alpha} F_{a_1 \ldots a_{s+1}} d\Omega_{s+1} = 0, \tag{23}$$

where $U_{a_1 \ldots a_s}$ is the potential energy of the system of s chosen dipoles. All the calculations in this case are entirely analogous to those used by Bogoliubov for the distribution function of the positions of molecules in a gas or liquid.[1]

We write the interaction potential in the form

$$\Phi_{a_1 a_2} / \mu = \gamma \psi_{a_1 a_2} \tag{24}$$

and will consider the quantity $\gamma = p^2 / d^3 \mu$ (p is the dipole moment of the molecule; for simplicity we assume that the magnitudes of the dipole moments of all the molecules are identical) as a parameter which is characteristic for the system under consideration.

If at first we do not consider the effect of the external electric field, and assume the inequality $\gamma << 1$ to be satisfied, then the equation

$$\frac{\partial}{\partial \theta_1^\alpha} F_{a_1 \ldots a_s} + \gamma \frac{\partial U^*_{a_1 \ldots a_s}}{\partial \theta_1^\alpha} F_{a_1 \ldots a_s} + \frac{\gamma}{4\pi} \int \sum_{1 \leqslant a_{s+1} \leqslant N} \frac{\partial \psi_{a_1, a_{s+1}}}{\partial \theta_1^\alpha} F_{a_1 \ldots a_{s+1}} d\Omega_{s+1} = 0 \tag{25}$$

$$U a_1 \ldots a_s = \sum \varphi a_i a_k$$

can be solved with the help of an expansion in powers of the small parameter γ. Writing $F_{a_1 \ldots a_s}$ in the form

$$F_{a_1 \ldots a_s} = F^0_{a_1 \ldots a_s} + \gamma F^1_{a_1 \ldots a_s} + \ldots, \tag{26}$$

we find

$$F^0_{a_1 \ldots a_s} = \text{const.}$$

If the normalization condition is written in the form

$$(4\pi)^{-s} \int F^i_{a_1 \ldots a_s} d\Omega_1 \ldots d\Omega_s = 0, \tag{27}$$

$$i = 0, 1, \ldots,$$

then $F^0_{a_1 \ldots a_s} = 1$ follows from this equality.

The equation for $F^1_{a_1 \ldots a_s}$ has the form

$$\frac{\partial}{\partial\theta_1^\alpha} F^1_{a_1\ldots a_s} + \frac{\partial U^*_{a_1\ldots a_s}}{\partial\theta_1^\alpha} + \frac{1}{4\pi}\int \sum_{1\leqslant a_{s+1}\leqslant N} \frac{\partial\psi_{a,a_{s+1}}}{\partial\theta_1^\alpha}\, d\Omega_{s+1} = 0, \tag{28}$$

and since, for the dipole potential,

$$\int \psi_{a_1,\, a_{s+1}} d\Omega_{s+1} = 0, \qquad \text{identically,}$$

we get

$$F^1_{a_1\ldots a_s} = -U^*_{a_1\ldots a_s} \tag{29}$$

in agreement with the normalization condition. The second approximation is found in a similar way and has the form

$$F^2_{a_1\ldots a_s} = \frac{1}{2}\Big(U^{*2}_{a_1\ldots a_s} + \frac{1}{4\pi}\sum_{a_{s+1}}^{N}\int\Big(\sum_{a_i}^{s}\psi_{a_i a_{s+1}}\Big)^2 d\Omega_{s+1}) + K^{(2)}_{a_1\ldots a_s}\Big), \tag{30}$$

where, in accord with the normalization condition, the constant K must be given by

$$K^{(2)}_{ai\ldots a_s} = -\frac{1}{2}\frac{1}{(4\pi)^s}\Big\{\int\sum_{a_i a_k}^{s}\psi^2_{a_i a_k} d\Omega_1\ldots d\Omega_s + \frac{1}{4\pi}\int\sum_{a_{s+1}}^{N}\sum_{a_i}^{s}\psi^2_{a_i a_{s+1}} d\Omega_1\ldots d\Omega_{s+1}\Big\}. \tag{31}$$

Here we have studied the properties of the dipole potential ψ. In similar fashion the constants for further approximations can be determined, but their calculation is not so simple.

In the presence of an external homogeneous electric field E and upon fulfillment of the condition

$$\gamma = p^2/d^3\mu \ll 1 \tag{32}$$

the computation scheme does not undergo any change.

These expressions for the distribution function permit us to construct a theory of the equilibrium properties of the dipole crystals under consideration. Numerical calculations and comparison with experiment, and also comparison with other theories which apply to the two very simple cases considered by us will be given in a subsequent paper. Separate consideration is necessary for the construction of a theory in the case of violation of the relation (32), while this case has the greatest interest. We hope in the near future to publish results which apply to this variant.

For the second of the problems considered by us the considerations developed above can without essential change be applied for the purpose of constructing a semiclassical theory of ferromagnetic bodies without use of the "sphericalizing" approximation.

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[1] N. N. Bogliubov, *Problems of dynamical theory in statistical physics*, Moscow, 1946.

[2] C. Zener, Phys. Rev. 37, 556(1931).

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42

SOVIET PHYSICS JETP VOLUME 4, NUMBER 2 MARCH, 1957

Relaxation between Electrons and the Crystalline Lattice*

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The relaxation between the electrons of a metal and the crystalline lattice (phonons) is considered. The state of the electrons and the lattice is described by equilibrium Fermi and Bose functions with different temperatures. The heat transfer coefficient connected with the "Cerenkov" radiation of sound waves by the electrons has been determined.

1. THERE have appeared recently several experimental[1,2] and theoretical[3,5] papers illuminating the investigation of deviations from Ohm's law in metals. The departures from a linear relation between the current J and the field E, noted by

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