Dependence of the Life Time of Nonequilibrium Charge Carriers in Germanium on Temperature and Composition

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(Submitted to JETP editor May 10, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 261-272 (August, 1956)

The analytic dependence of the position of the Fermi level on the temperature and on the composition of a semiconductor in the case of low trap density has been found in the form of approximate formulas. With the aid of these expressions an investigation is made of the dependence on the above factors of the lifetime of nonequilibrium charge carriers in a semiconductor for small deviations from thermodynamic equilibrium which permits one to improve the formula for the determination of the binding energy of electrons and of holes in the traps.

1. INTRODUCTION

THE lifetime τ of the nonequilibrium charge carriers is one of the fundamental electrophysical characteristics of a semiconductor. For a low density of recombination centers (traps) and for small deviations from thermodynamic equilibrium, τ is expressed by the formula of Shockley and Read[1]

$$\tau = \tau_{p0}(n_0 + n_1)/(n_0 + p_0) + \tau_{n0}(p_0 + p_1)/(p_0 + n_0). \quad (1)$$

Here $n_0 = N_2 \exp\{-(E_u - F)/kT\}$, $p_0 = N_1 \exp\{-(F - E_l)/kT\}$ are the electron and hole densities in thermodynamic equilibrium;

$$n_1 = N_2 \exp\{-(E_u - E_t)/kT\}, \quad p_1 = N_1 \exp\{-(E_t - E_l)/kT\} \quad (2)$$

are quantities which have the dimensions of density and which are numerically equal to the equilibrium electron and hole densities when the Fermi level F coincides with the energy levels of the traps;

$$\tau_{p0} = 1/A_h N_t, \quad \tau_{n0} = 1/A_e N_t, \quad (3)$$

where N_t is the density of traps, A_e and A_h are respectively the probabilities of capture by the trap of an electron and of a hole.

The dependence of the lifetime, and also of other electrophysical characteristics (conductivity, Hall coefficient) on the temperature and on the composition of the semiconductor is expressed not only explicitly through T, E_u, E_l, E_t, A_e, A_h, etc., but also implicitly through the Fermi level F. Consequently it is first of all necessary to determine the law for the variation with temperature of the position of the Fermi level in semiconductors of different compositions. We shall take a definite composition of the semiconductor to mean that we are given the properties of the basic material, and also the densities and the characteristics of all the impurities and structural defects which it contains. In contrast to Refs. 2–5*, where the problem of the Fermi level is investigated by special graphical methods which require the construction for each new composition of its own family of curves, the present article gives an analytic solution of the problem and presents the result in the form of sufficiently simple approximate expressions useful for calculations in specific cases.

2. CALCULATION OF THE POSITION OF THE FERMI LEVEL

The position of the Fermi level under conditions of thermodynamic equilibrium is obtained from the normalization condition

$$\int_{-\infty}^{\infty} N(E) f(E)\, dE = N_0, \quad f(E) = [e^{(E-F)/kT} + 1]^{-1}, \quad (4)$$

where $N(E)$ is the level density , and N_0 is the total electron density at a given point of the semiconductor.

Assuming that in the forbidden band there exist in the general case local levels of S different types with binding energies equal respectively to E_i ($i = 1,2,3, \ldots, S$) for which $N(E) = N_i \delta(E - E_i)$,

*The formulas for F, obtained in Refs. 5-8, refer to a very special case which is realized, as is shown below, only within a very small range of temperatures.

we obtain from (4)

$$\int_1 N(E)\, f(E)\, dE + \int_2 N(E)\, f(E)\, dE + \sum_1^S N_i f(E_i) = N_0, \quad (5)$$

where the integrals 1 and 2 are taken respectively over the lower and the upper bands. In terms of densities, Eq. (5) takes the form:

$$n + N_{10} - p + \sum_{i=1}^{S} n_i = N_0, \quad (6)$$

where N_{10} is the total number of levels in the lower band.

If of the S types of local levels S_1 are of the donor type (i.e., they are occupied by electrons in the electrically neutral condition), then

$$N_0 = N_{10} + \sum_{i=1}^{S_1} N_i - \rho/q, \quad (7)$$

where N_i is the density of donor levels of the i th type, and ρ is the electric charge density at the given point. Substituting (7) into (6) and restricting ourselves to the electrically neutral case we obtain the equation

$$n + \sum_1^S n_i - p = \sum_1^{S_1} N_i, \quad (8)$$

which determines F.

For non-degenerate semiconductors defined by the conditions

$$(E_u - F)/kT \gg 1, \quad (F - E_l)/kT \gg 1, \quad (9)$$

the distribution of electrons in the upper band and the distribution of holes in the lower band correspond to Boltzmann statistics

$$n = \int_{E_u}^{\infty} N(E)\, f(E)\, dE \approx \int_{F_u}^{\infty} N(E) \exp\{-(E-F)/kT\}\, dE = N_2 \exp\{-(E_u - F)/kT\},$$
$$p = \int_{-\infty}^{E_l} N(E)\, f(E)\, dE \approx \int_{-\infty}^{E_l} N(E)\, e^{-(F-E)/kT}\, dE = N_1 e^{-(F-E_l)/kT}. \quad (10)$$

Here E_l and E_u are respectively the top of the lower band and the bottom of the upper band (Fig. 1);

$$N_1 = \int_{-\infty}^{E_l} N(E) \exp\{-(E_l - E)/kT\}\, dE = 2(2\pi m_h kT/h^2)^{3/2},$$

$$N_2 = \int_{F_u}^{\infty} N(E) \exp\{-(E - E_u)/kT\}\, dE = 2(2\pi m_e kT/h^2)^{3/2} \quad (11)$$

are the reduced values of the level densities in the bands; m_e and m_h are the effective masses of the electron and of the hole.

Selecting for the zero of energies the position of the Fermi level in a pure semiconductor (see Fig. 1)

$$F_0 = {}^1/_2(E_u + E_l) + {}^1/_2 kT \ln(N_1/N_2) = {}^1/_2(E_u + E_l) + {}^3/_4 kT \ln(m_h/m_e), \quad (12)$$

which is determined by the condition n=p (all $N_i = 0$) and introducing the notation

$$\varphi = (F - F_0)/kT_0; \quad \varepsilon_l = (E_l - F_0)/kT_0; \quad \varepsilon_u = (E_u - F_0)kT_0;$$
$$\varepsilon_i = (E_i - F_0)/kT_0; \quad \alpha_u = e^{\varepsilon_u/\Theta}; \quad \alpha_l = e^{\varepsilon_l/\Theta};$$
$$\alpha_i = e^{\varepsilon_i/\Theta}; \quad y = e^{\varphi/\Theta}; \quad \nu_i = N_i/N_0; \quad \Theta = T/T_0;$$
$$\varkappa = (N_2/N_1)^{1/2} = (m_e/m_h)^{3/4};$$
$$N_0 = \sqrt{N_1(T_0)\, N_2(T_0)}; \quad (13)$$

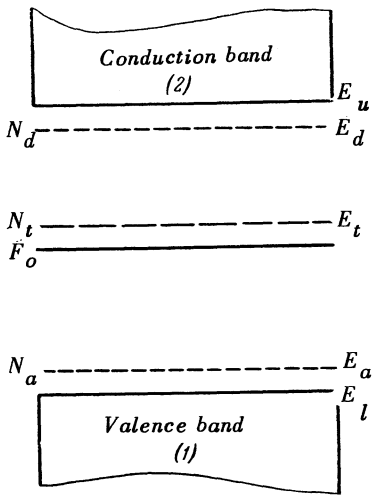

FIG. 1. Energy diagram of a semiconductor.

$$\mathcal{E}_i = E_i - F_0;\ \mathcal{E}_l = E_l - F_0;\ \mathcal{E}_u = E_u - F_0, \quad (14)$$

where $T_0 = 300°$ K we shall bring Eq. (8) to the following form:

$$\varkappa\Theta^{3/2}\frac{y}{\alpha_u} + \sum_1^S \frac{\nu_i}{1+\alpha_i/y} - \frac{1}{\varkappa}\Theta^{3/2}\alpha_l/y = \sum_1^{S_1}\nu_i. \quad (15)$$

In the general case this equation is of order $S+2$ with respect to y. We note, however, that

$$\frac{\alpha_i}{y}\begin{cases}\gg 1 & \text{for } (E_i - F)/kT \gg 1, \\ \ll 1 & \text{for } (F - E_i)/kT \ll 1.\end{cases} \quad (16)$$

Consequently, if the difference in the binding energies of the electrons on local levels of different types is much larger than kT, then (15) represents an equation of order not higher than the third.

The quantity y introduced in place of the Fermi level has a simple physical meaning

$$y = e^{(F-F_0)/kT} \quad (17)$$

$$= (N_2/N_1)^{1/2}\exp\{(2F - E_u - E_l)/2kT\} = \sqrt{n/p}.$$

For a semiconductor whose energy diagram is given in Fig. 1 equation (15) may be written as follows:

$$\varkappa\Theta^{3/2}\frac{y}{\alpha_u} + \frac{\nu_a}{1+\alpha_a/y} - \frac{\nu_d}{1+y/\alpha_d} - \frac{1}{\varkappa}\Theta^{3/2}\frac{\alpha_l}{y} = 0. \quad (18)$$

The number of electrons in traps is assumed to be negligibly small because of the low density of traps.

In the following we shall restrict ourselves to an investigation of semiconductors with a small activation energy for both donors and acceptors. A typical example of such semiconductors is germanium[8]:

$$|E_u - E_d| \sim 0.04 \text{ ev}, \quad |E_a - E_l| \sim 0.04 \text{ ev}, \quad (19)$$

for which all the following calculations have been carried out.

Let us consider the n-type of germanium $\nu_d - \nu_a > 0$. Since the position of the Fermi level in pure germanium has been chosen as the zero of energy, the Fermi level in the case under investigation is positive at all temperatures ($\varphi \geq 0$) and tends to zero for $\Theta >> 1$.

Let us first of all show that in n-type germanium one may take $\alpha_a/y << 1$ at all experimental temperatures, i. e., one may neglect the density of holes on acceptor levels. The ratio α_a/y is determined by

$$\alpha_a/y = \exp\{(\varepsilon_a - \varphi)/\Theta\} < \exp\{\varepsilon_a/\Theta\} \quad (20)$$
$$= (m_e/m_h)^{3/4}\exp\{[E_a - {}^1/_2(E_u + E_l)]/kT\}.$$

The distance of acceptor levels from the middle of the band is ~ 0.3 ev; consequently α_a/y is certainly less than 0.01 up to temperatures $T \approx 500°$ C.

We shall denote by the term low temperature region that range of temperatures within which one can neglect the density of holes in the valence band in comparison with the density of electrons in the conduction band. We shall determine the upper limit Θ_I of the temperatures in this region from the condition $p/n = 0.01$, i.e., $y = 10$ [see (17)].

In the low temperature region Eq. (18) takes on the following form:

$$\varkappa\Theta^{3/2}\frac{y}{\alpha_u} + \nu_a - \frac{\nu_d}{(y/\alpha_d)+1} = 0. \quad (21)$$

Its solution is

$$y = (\alpha_u/2\varkappa\Theta^{3/2})\{[(\nu_a + \varkappa\Theta^{3/2}\alpha_d/\alpha_u)^2 \quad (22)$$
$$+ 4\varkappa(\alpha_d/\alpha_u)\Theta^{3/2}(\nu_d - \nu_a)]^{1/2} - (\nu_a + \varkappa\Theta^{3/2}\alpha_d/\alpha_u)\}.$$

The low temperature region $0 \leq \Theta < \Theta_1$ may be divided into three temperature intervals in which different approximate expressions for y are possible.

Near absolute zero where one may neglect in (21) the density of free electrons $\nu = \varkappa\Theta^{3/2} y/\alpha_u$,

$$y = \alpha_d \frac{\nu_d - \nu_a}{\nu_a}\left\{1 - \frac{\varkappa}{4(\nu_d - \nu_a)}\frac{\alpha_d}{\alpha_u}\Theta^{3/2} \times \left[4\left(\frac{\nu_d}{\nu_a} - \frac{1}{2}\right)^2 - 1\right]\right\} \approx \alpha_d \frac{\nu_d - \nu_a}{\nu_a}. \quad (23)$$

The boundary of this temperature interval Θ_1 is determined from the equation

$$\Theta_1^{3/2} \exp\{-(\varepsilon_u - \varepsilon_d)/\Theta_1\} = \begin{cases} 0.01\,\nu_a^2/(\nu_d - \nu_a)\varkappa, & \text{if } \nu_d > 2\nu_a, \\ 0.01\,\nu_a/\varkappa, & \text{if } \nu_d < 2\nu_a. \end{cases} \quad (24)$$

Let us now assume in Eq. (21)

$$\nu_d/(y/\alpha_d + 1) \approx \nu_d(1 - y/\alpha_d), \quad (25)$$

which corresponds to assuming a low degree of occupancy of donor levels. In such a case,

$$y = \alpha_d(\nu_d - \nu_a)/(\nu_d + \varkappa\Theta^{3/2}\alpha_d/\alpha_u). \quad (26)$$

Turning to Eq. (18) and demanding that the error introduced by the approximation into the free electron density should not exceed 1% we obtain the formula

$$(\nu_d - \nu_a)/2\varkappa\Theta_2^{3/2}\alpha_d/\alpha_u = 0.01, \quad (27)$$

which determines the lower boundary of the temperatures in the interval under consideration.

Now substituting $y = 10$ into expression (26) we determine Θ_I – the common upper limit of the whole low temperature region:

$$\alpha_d \frac{\nu_d - \nu_a}{\nu_d + \varkappa\Theta_I^{3/2}(\alpha_d/\alpha_u)} \approx \alpha_u(\nu_d - \nu_a)/\varkappa\Theta_I^{3/2} = 10. \quad (28)$$

ν_d has been neglected in the denominator because already at $\Theta = 0.4$ $N_0 \varkappa \Theta^{3/2} \alpha_d/\alpha_u >> 10^{17}\text{cm}^{-3}$ and from (28) it follows that $\Theta_I \sim 1$. Consequently at the usual impurity densities Θ_I does not depend on α_d, i.e., on the magnitude of the binding energy of the electron on impurity levels.

We see that $\Theta_2 < \Theta_I$ in all cases for actually occurring values of the binding energy of the electron on traps. Consequently there exists a finite temperature interval in the low temperature region in which expression (22) for y can be approximated by the expression (26). A more exact form for y in this interval is given by the following:

$$y = \{\alpha_d(\nu_d - \nu_a)/(\nu_d + \varkappa\Theta^{3/2}\alpha_d/\alpha_u)\}\{1 + (\nu_d - \nu_a)/(4\varkappa\Theta^{3/2}\alpha_d/\alpha_u)\}. \quad (29)$$

At temperatures $\Theta_1 < \Theta < \Theta_2$ the general formula (22) should be used.

We now proceed to investigate the high temperature region, i.e., the region $\Theta \geq \Theta_I$. In doing so we must take into account in Eq. (18) the density of holes in the valence band.

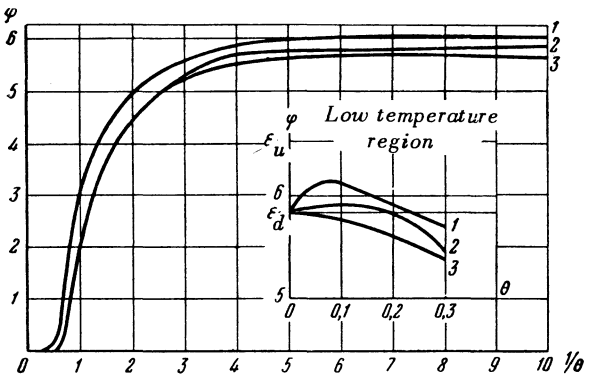

FIG. 2. Dependence of the position of the Fermi level on the temperature. *1*– $N_d = 7.5\times10^{15}$ cm^{-3}, $N_a = 7.5\times10^{10}$ cm^{-3}; *2*– $N_d = 7.5\times10^{14}$ cm^{-3}, $N_a = 7.5\times10^{13}$ cm^{-3}; *3*– $N_d = 7.5\times10^{15}$cm^{-3}, $N_a = 6.75\times10^{15}$cm^{-3}

As a consequence of the fact that $\Theta_I > \Theta_2$, Eq. (18) can be written as follows:

$$\varkappa\Theta^{3/2}(y/\alpha_u) + \nu_d(y/\alpha_d) - \Theta^{3/2}\alpha_l/\varkappa y = \nu_d - \nu_a. \quad (30)$$

Its solution has the following form:

$$y = \frac{\alpha_u}{2\varkappa\Theta^{3/2}}\left\{\left[4\Theta^3\frac{\alpha_l}{\alpha_u} + (\nu_d - \nu_a)^2\right]^{1/2} + (\nu_d - \nu_a)\right\}, \quad (31)$$

where we have taken into account that for $\Theta \geq \Theta_I$, $\varkappa\Theta^{3/2}\alpha_d/\alpha_u >> \nu_d$. For $\Theta = \Theta_I$ Eq. (31) is the same as (26).

In the high temperature region we shall pick out two temperatures: Θ_3, which corresponds to the impurity conductivity being equal to the intrinsic conductivity [$\nu=2(\nu_d - \nu_a)$], and Θ_4, which corresponds to the intrinsic conductivity [$\nu \approx 100(\nu_d - \nu_a)$]; Θ_3 and Θ_4 are determined by the formulas:

$$\Theta_3^3 \alpha_l/\alpha_u = 2(\nu_d - \nu_a)^2,$$

$$[(\nu_d - \nu_a)/2\Theta_4^{3/2}] \exp\{\varepsilon_u/\Theta_4\} = 0.01. \quad (32)$$

These temperatures characterize the transition from the impurity to the intrinsic conductivity.

Figures 2 and 3 show the result of the calculation of the position of the Fermi level and of the density ν according to the above formulas. The boundaries of the temperature intervals are shown there also.

For a p-type semiconductor all the results obtained above also hold, only N_d and N_a should be interchanged. $\mathcal{E}_u$ should be replaced by $-\mathcal{E}_l$, $-\mathcal{E}_l$ by $\mathcal{E}_u$, f by $-f$, $\mathcal{E}_a$ by $-\mathcal{E}_d$, $\mathcal{E}_d$ by $-\mathcal{E}_a$, and $\varkappa$ by $1/\varkappa$.

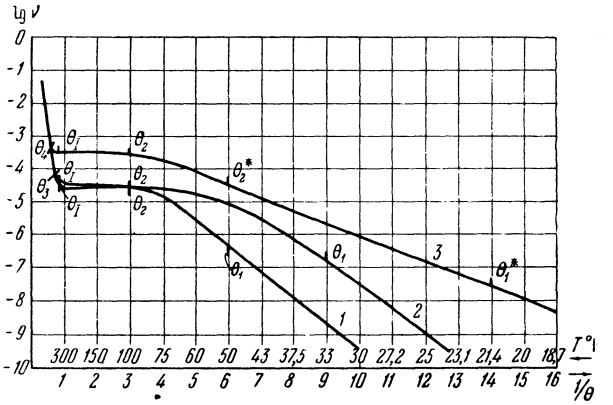

FIG. 3. Temperature dependence of the density of charge carriers. *1*– $N_d=7.5\times10^{15}$cm^{-3}, $N_a=6.75\times10^{15}$cm^{-3}; *2*–$N_d = 7.5\times10^{14}$cm^{-3}, $N_a = 7.5\times10^{13}$ cm^{-3}; *3*– $N_d = 7.5\times10^{15}$cm^{-3}, $N_a=6.75\times10^{10}$cm^{-3}.

In concluding this section we note that the direct use of the formula for extrinsic semiconductors [5,8,10,11]

$$\sigma = A \exp\{-(E_u - E_d)/2kT\} \quad (33)$$

as applied to germanium may lead to quite incorrect values of the ionization levels of the impurities $E_u - E_d$ in the interpretation of experimental data. Formula (33) follows from (21) if the term ν_a is neglected, and if $y / \alpha_d >> 1$, which leads to.

$$y = \sqrt{\nu_d/\varkappa}\,\Theta^{-3/4} \exp\{(\varepsilon_u + \varepsilon_d)/2\Theta\};$$

$$\varphi = \Theta \ln y \approx 1/2(\varepsilon_d + \varepsilon_u). \quad (34)$$

Conditions under which these neglections will hold with an accuracy to δ% for concentrations

$$100/\delta \leqslant y/\alpha_d \leqslant 0.01\delta\nu_d/\nu_a \quad (35)$$

show that (33) may hold only in the case of very strongly extrinsic germanium: $\nu_d/\nu_a >> 10^4/\delta^2$, i.e., at least for $\nu_d \sim 10^3\,\nu_a$.

Moreover, it follows from (35) that in this case also, the approximation (34), which leads to (33), holds only within a very limited temperature interval. For $n_d = 7.5\times10^{15}$ cm^{-3}, $N_a = 7.5\times10^{12}$ cm^{-3} and $\delta = 10\%$ the region in which (33) is applicable is limited to the interval $34°\,\mathrm{K} \leq T \leq 48°\,\mathrm{K}$.

3. LIFETIME

Introducing dimensionless quantities (13) and adopting as the unit of time τ_{p0} ($\tau^* = \tau/\tau_{p0}$), we can reduce formula (1) to the following form:

$$\tau^* = \frac{(1+\alpha_t/y)(1+\gamma/\alpha_t\nu)}{1+y^{-2}}, \qquad (36)$$

where

$$\gamma = \frac{\tau_{n0}}{\tau_{p0}}; \quad \alpha_t = \exp\{\varepsilon_t/\Theta\}.$$

The system of equations (36) and (18) represents a parametric expression of the dependence of the lifetime on the temperature and on the composition of a semiconductor. In the low temperature region ($\Theta \leq \Theta_1$) in place of (18) one should use (22), while in the high temperature region ($\Theta \geq \Theta_1$) one should use formula (31).

Figure 4 shows the result of eliminating y from the system of equations (36) and (18) for $N_a = 10^{13}$ cm^{-3}, $N_d = 10^{14}$ cm^{-3}, ν=1, γ=1 and for the other parameters determined by (19). The curve *I* shows the dependence of τ^* on the temperature in the case that the recombination levels are situated at the same height as the donors $\mathcal{E}_u - \mathcal{E}_t = \mathcal{E}_u - \mathcal{E}_d = 0.04$ ev. Curve *II* corresponds to

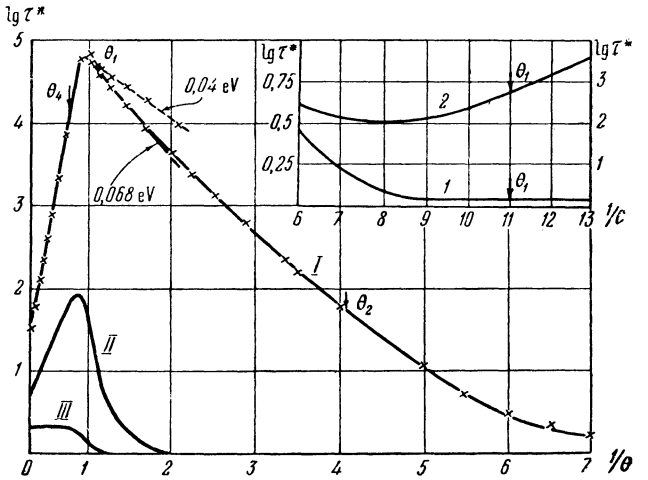

FIG. 4. Temperature dependence of the lifetime for different positions of the traps.

$\mathcal{E}_u - \mathcal{E}_t = 0.22$ ev, while curve *III* corresponds to $\mathcal{E}_u - \mathcal{E}_t = 0.375$ ev. In the latter case the trap levels are situated in the middle of the forbidden band.*

We have shown separately the behavior of lg τ^* at temperatures close to absolute zero ($\Theta \leq \Theta_1$). In addition to the continuation of the graph for $\mathcal{E}_u - \mathcal{E}_t = 0.04$ ev (curve 1, left hand scale) we have also shown here the graph for the case when the trap levels are situated closer to the conduction band than the donor levels (curve 2, right hand scale, which corresponds to $\mathcal{E}_u - \mathcal{E}_t = 0.02$ ev).

As may be seen from Fig. 4, the lifetime of nonequilibrium carriers does not vary monotonically with temperature. As the temperature is increased τ^* increases at first, then reaches a maximum, then decreases. If $|\epsilon_t| > \epsilon_d$, then τ^* increases also as $\Theta \to 0$ (see curve 2). An exception occurs in the case when the recombination levels are situated in the middle of the forbidden band (more cccurately when $\epsilon_t = 0$, i.e., when the traps coincide with the position of the Fermi level in pure germanium). For such a position of the traps τ^* increases monotonically, and then stays at a constant value (see curve *III*).

Let us elucidate the physical meaning of the results obtained above. The relaxation of the nonequilibrium electron density is determined by two processes: 1) the recombination of excess electrons with equilibrium empty traps, and 2) the recombination of equilibrium electrons with excess empty traps. For small deviations from thermodynamic equilibrium we can neglect the quadratic effect—the recombination of excess electrons with excess traps. The mathematical formulation of the above appears as follows:

$$-dn'/dt = A_e n' N_{et}' + (A_e n + B_e) N_{et}', \qquad (37)$$

*We note that ν=1, i.e., the middle of the forbidden band coincides with the position of the Fermi level in pure germanium.

where A_e and B_e are respectively the probabilties per unit time for the recombination and the thermal release of an electron, n and n' are the equilibrium and excess electron densities, N_{et} and N'_{et} are the equilibrium and excess empty trap densities. The second term on the right hand side of (37) contains the expression $B_e N'_{et}$ because the appearance of the excess density of empty traps N'_{et} not only speeds up the recombination of electrons, but also slows down their ejection from the traps back into the conduction band. In a similar way, for the hole we have

$$-dp'/dt = A_h v' N_{ft^3} + (A_h p + B_h) N'_{ft}. \quad (38)$$

Evidently the excess density of filled traps is given by

$$N'_{ft^3} = -N'_{et^n}. \quad (39)$$

Under the conditions of stationary pair formation and of low trap density under which formula (1) was obtained, the lifetime of nonequilibrium carriers represents essentially the average time for the disappearance of an electron-hole pair, and not for the disappearance of either one of these two particles. The act of the recombination of the electron-hole pair is made up of two elementary transitions 1) of an electron into an empty trap and 2) of a hole into an occupied trap. As a result of such a process the electron-hole pair disappears, while the degree of occupancy of the traps is not changed. The lifetime is determined fundamentally by the slower one of the two transitions. However, a simple addition of the characteristic times of the elementary acts cannot be used, as this would correspond to such conditions under which the recombination of one of the particles, for example a hole, would become possible only after the recombination of the other particle, for example, an electron, had already occurred.

If the characteristic times of the elementary transitions of an electron and of a hole are quantities of different orders of magnitude, then the lifetime τ is equal to the larger one of these two characteristic times. In the general case the value of the lifetime of the electron-hole pair is influenced by the fact that the transitions of both particles to traps may occur not only one following another, but also simultaneously. Instead of a singlevalued dependence, a correlation exists between the transitions (the transition of the electron may occur either simultaneously with or after the transition of the hole, etc.). For the stationary case, when the rate of recombination is equal to the rate of pair formation (illumination or injection) this correlation finds its expression in Eqs. (1) and (36).

We now proceed to the examination of individual specific cases. We shall restrict ourselves to an investigation of n-type germanium. The results which we shall obtain can be easily adapted also to the case of p-type germanium.

EXAMPLE 1. The traps are situated in the upper half of the forbidden band below the donors (Fig. 5a). Near absolute zero all the traps are filled, and the lifetime of the hole is equal to τ_{p0} or to unity in dimensionless units. Since the Fermi level $\varphi \approx \epsilon_d$, i.e., since it is close to the conduction band, the trap which becomes vacant after the recombination of a hole is filled almost immediately (in a time smaller than τ_{p0}) by some electrons. The lifetime is determined by the recombination time of the hole, i.e., $\tau^* = 1$. As the temperature is increased, a partial emptying of the traps takes place. The density of filled traps is decreased, and consequently the probability of recombination for holes is also decreased. Consequently the lifetime is increased. An increase in the temperature leads to an increase in the free electron density and for $\epsilon_t > 0$ to a considerable increase in the number of vacant traps. Therefore the characteristic time for an elementary electron transition is very small and has no effect on the value of τ.

Calculations using formulas (36) and (22) show that over a sufficiently wide interval to the right of the maximum

$$\tau^* = [\varkappa \Theta^{3/2}/(\nu_d - \nu_a)] \exp\{-(\varepsilon_u - \varepsilon_t)\Theta\}. \quad (40)$$

From (40) it follows that

$$d \ln \tau^*/d(1/\Theta) = -(\varepsilon_u - \varepsilon_t) - {}^3/_2\Theta. \quad (41)$$

Under the condition that

$$(\varepsilon_u - \varepsilon_t)/\Theta \gg {}^3/_2 \quad (42)$$

the slope of the curve of the dependence of the lifetime on the temperature plotted in terms of $1/\Theta$, $\ln \tau^*$ is simply equal to the distance of the

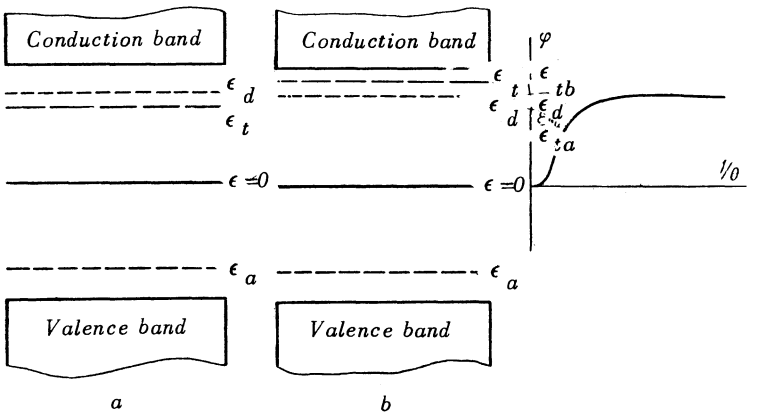

FIG. 5. Energy diagram of n-type germanium for $\epsilon_t > 0$. On the right-hand side of the diagram is shown the dependence on the temperature of the Fermi level, with the energies of the traps in both cases a and b indicated on the vertical axis.

recombination levels from the bottom of the conduction band. This gives the possibility of a direct experimental determination of the binding energy of the electron on recombination levels.

It should be noted that such a method of determining $\epsilon_u - \epsilon_t$ is by no means always possible. Indeed, in (42) one may take $\Theta_1 \sim 1$ ($T \sim 300°$ K), since the temperature range under investigation is adjacent to the temperature Θ_1, beyond which an appreciable intrinsic conductivity appears in germanium which then leads to a decrease of τ^*. Noting that the unit of energy is $kT_0 = 0.026$ ev, we obtain from (42)

$$\Delta\mathcal{E}_e = \mathcal{E}_u - \mathcal{E}_t \geqslant 0.04 \text{ ev}. \tag{43}$$

Consequently, only in the case when the traps are situated in the immediate vicinity of the middle of the forbidden band ($\Delta\mathcal{E}_e \sim 0.37$ ev) may the binding energy of an electron on a trap be determined sufficiently accurately from the slope of the curve of $\ln\tau$ plotted against $1/T$. In the general case the neglect of the term $3/2\,\Theta$ in (41) may lead to a considerable error in the determination of $\Delta\mathcal{E}_e$.

Let us now turn to Fig. 4. The slope of the curve I to the right of Θ_1 is equal to 1.19. Going over to natural logarithms and to dimensional units we find that the slope of the curve corresponds to 0.068 and not to 0.04 ev. If one determines the electron binding energy from the slope of the curve in this case one will make an error of 72%.

In order to determine the position of the traps from the experimental data on the lifetime one should construct in the region of the density plateau the curve showing the dependence of $\ln(\tau T^{-3/2})$ on $1/T$.

As may be seen from (40), at temperatures to the right of Θ_1 ($\Theta < \Theta_1$), the following exact equality holds:

$$\frac{d\ln(\tau^*/\Theta^{3/2})}{d(1/\Theta)} = -(\varepsilon_u - \varepsilon_t), \text{ or } \frac{d\ln(\tau/T^{3/2})}{d(1/T)} = -(\mathcal{E}_u - \mathcal{E}_t)/k. \tag{44}$$

The corresponding curve is shown dotted in Fig. 4. Its slope in dimensional units is equal to 0.04 ev, which exactly corresponds to the position of the traps.

The considerations outlined above have not been taken into account by Hall.[12] If one introduces the necessary correction for $\Theta^{3/2}$, then the position of the traps determined from Hall's experimental data turns out to be equal to 0.22 and not to 0.18 ev, which corresponds to an error of 20% committed by Hall.

At temperatures higher than Θ_1, the density of thermally generated (equilibrium) holes first appears and rapidly increases. If for $\Theta < \Theta_1$ the lifetime was determined largely by the process of the recombination of excess holes with the equilibrium filled traps

$$-dp'/dt \approx A^{\frac{1}{2}}p'N_{ft}^{3} \tag{45}$$

[see (38)], then for $\Theta > \Theta_1$ the process of the recombination of equilibrium holes with excess filled traps also begins to play an essential role. As long as the lifetime was determined by

the first term in (38), it increased wihh increasing temperature, because the probability of recombination correspondingly decreased due to a decrease of N_{ft} In the region of high temperatures, in which the second term of Eq. (38) predominates, τ decreases with temperature, because the probability of recombination increases with increasing p . This explains the non-monotonic character and the maximum of the curves giving the temperature dependence of τ^*.

The above investigation shows that for $\epsilon_t > 0$ the lifetime of the electron-hole pair is fundamentally determined by the time of a transition to a trap of that one of the components of the pair which belongs to the minority current carriers in germanium of a given conductivity type. An increase in τ by several orders of magnitude in comparison with τ_{p0} * at intermediate temperatures is determined by the fact that in this temperature range the degree of occupancy of traps is small, i.e., the probability of capture of a hole is small. For $\Theta \to 0, N_{ft} \to N_t$, and for $\Theta \to \infty$, $N_{ft} \to 1/2\, N_t$, as a result of which τ becomes of the order of τ_{p0} (or τ_{n0}).

If $\epsilon_t = 0$, then in n-type germanium ($\gamma > 1$) we have for all Θ

$$^1/_2 N_t \leqslant N_{t^3} \leqslant N_t. \qquad (46)$$

The occupancy of traps remains quite high at any temperature and therefore τ^* is small (see curve *III*, Fig. 4). Moreover, in the region of intrinsic conductivity ($\gamma \approx 1$) $N_{ft} = 1/2\, N_t$. Therefore for $\epsilon_t = 0$ the dependence of ln τ^* on $1/\Theta$, instead of having a maximum and a falling off at high temperatures, acquires a plateau.

EXAMPLE 2 . The traps are situated above the donors (Fig. 5b). The temperature dependence of τ^* differs from the one discussed above only for $\Theta < \Theta_1$, where a decrease of temperature is accompanied by an increase of τ^* (see curve 2 in Fig. 4). The reason for this is the following.

If $\epsilon_t > \epsilon_d$, then for $\Theta = 0$ all the traps are vacant, since $\varphi \to \epsilon_d$ as $\Theta \to 0$. Therefore the probability of hole capture at $\Theta = 0$ is equal to zero and correspondingly $\tau = \infty$. In this case as the temperature is raised the degree to which the traps are filled at first increases, and only then begins to decrease, while for $\epsilon_t < \epsilon_d$ N_{ft} decreases beginning with $\Theta = 0$. For $\epsilon_t > \epsilon_d$, in the temperature range up to Θ_1 , N_{ft} increases with increasing temperature and consequently τ decreases.

For $\epsilon_t = \epsilon_d$ the degree to which the traps are filled remains practically constant up to $\sim \Theta_1$, as a result of which

$$\tau^* = \tau_h/\tau_{p0} = \nu_d/(\nu_d - \nu_a) = \text{const}. \qquad (47)$$

The ratio $(\nu_d - \nu_a)/\nu_d$ is equal to the degree to which the donors are filled, and consequently to the degree to which the traps are filled which have the same energy as the donors.

EXAMPLE 3. The traps are situated in the lower half of the forbidden band (Fig. 6). The character of the temperature dependence is qualitatively the same as for $\epsilon_t > 0$. An analysis of the formulas shows that for $\gamma = 1$, τ depends only on $|\epsilon_t|$

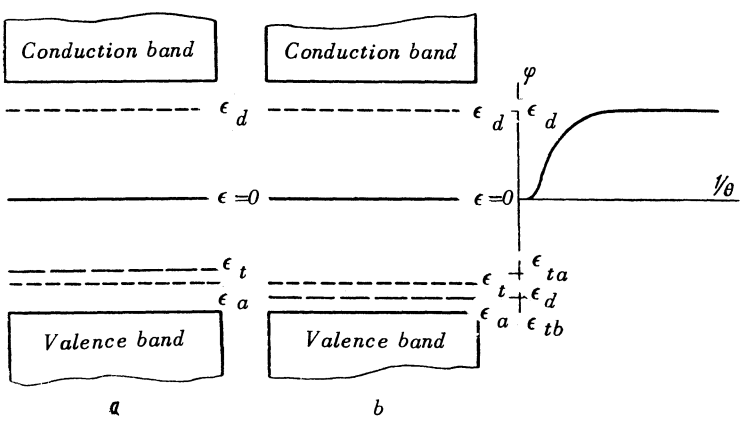

FIG. 6. Energy diagram for n-type germanium for $\epsilon_t < 0$.

*By comparison with τ_{n0} in p-type germanium.

If $-\epsilon_t < \epsilon_d$ (Fig. 6a) then τ^* increases monotonically in the low temperature region, then goes through a maximum, and falls off in the high temperature region. The general form of the function $\tau^*(\Theta)$ is the same as that shown in Fig. 4 (curves *I* and *II*). However, the physical character of the processes which determine the temperature dependence of τ^* is quite different. This may be seen already from the fact that the temperature dependence is contained in terms which include as a factor γ, i.e., the lifetime is determined not by the process of recombination of holes (minority carriers), but by the process of recombination of electrons (majority carriers),

As may be seen from Fig. 6 for $\epsilon_t < 0$ the occupancy of traps in n-type germanium ($\varphi > 0$) remains at all actual temperatures close to unity, and only for $\Theta > \Theta_4$ it tends in the limit to one-half. Therefore, practically at all temperatures the capture of a hole by a trap occurs during a time τ_{p0}. On the other hand, the transition of an electron into a trap requires a considerably longer time than τ_{n0}, since the traps are almost entirely occupied by electrons.

In accordance with the above, the dominant role in Eq. (37) is played by the second term on the right-hand side; in other words the rate at which the process of disappearance of pairs takes place is determined by the recombination of equilibrium electrons n with excess empty traps N'_{et};

$$-dn'/dt \approx A_e n N'_{et} n. \qquad (48)$$

As n increases, the lifetime τ decreases, while when N'_{et} is increased τ decreased.

The temperature dependence of τ is determined by which of these two factors dominates.

In the temperature interval from Θ_2 to Θ_1, which corresponds to the saturation of impurity conductivity, $n \approx$ const, while N'_{et} will decrease as the temperature increases, because of the thermal ejection of electrons from the valence band into empty traps. In other words, here the principal role is played by the thermal barrier,[13] which oonsists of the fact that a hole captured by a trap is ejected back into the valence band before an electron from the conduction band has time to enter this trap. As a result τ in this case increases with temperature.

As the temperature goes through Θ_1 an appreciable density of equilibrium holes appears both in the valence band and in the traps which leads to a decrease in τ just as in the case $\epsilon_t > 0$ which was considered in detail above.

The principal difference of the physical processes taking place in n-type germanium for $\epsilon_t > 0$ and $\epsilon_t < 0$ consists of the following. For $\epsilon_t > 0$ the lifetime is determined primarily by the characteristic time of the elementary act of the transition of a hole into a trap. At intermediate temperatures this may exceed τ_{p0} by a large factor as a result of the fact that a large fraction of the traps is vacant which makes the recombination of the minority carriers more difficult. For $\epsilon_t < 0$, the lifetime is determined mainly by the characteristic time of the elementary act of the transition of an electron into a trap. At intermediate temperatures this may exceed τ_{n0} by a large factor because of the fact that a thermal barrier is created at the traps: the vacant traps are filled by electrons from the valence band which makes the recombination of conduction electrons (majority carriers) more difficult.

In the general case the lifetime is determined primarily by the characteristic time of the elementary transitions of the minority carriers when the traps are situated in the same half of the forbidden band as the levels of the dominant impurity ($\epsilon_t > 0$ in n-type germanium; $\epsilon_t < 0$ in p-type germanium). The lifetime is determined primarily by the characteristic time of the elementary transitions of the majority carriers when the traps are situated in the other half of the forbidden band ($\epsilon_t < 0$ in n-type germanium, $\epsilon_t > 0$ in p-type germanium).

The further are the recombination levels situated from the middle of the forbidden band, the greater is the role played by the effect of the emptying of the traps, or by the thermal barrier, and therefore the larger will be the value of the lifetime τ at intermediate temperatures in comparison with τ_{p0} or τ_{n0}. This can be seen from Fig. 4 where for $\mathcal{E}_u - \mathcal{E}_t = 0.04$ ev (curve *I*) the value of τ approaches $\sim 5\times10^4\ \tau_{p0}$, for $\mathcal{E}_u - \mathcal{E}_t = 0.22$ ev $\tau_{max} \sim 10^2\ \tau_{p0}$, and for $\mathcal{E}_t = 0$ τ does not exceed $2\ \tau_{p0}$.

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