$$dw = \frac{e^2 g^2}{2\pi} \frac{p^2 dp}{E E_\pi} \left\{ \frac{(E_\pi - E)^2 - p^2}{p E_\pi} \right.$$

$$\times \ln \frac{(E+p)(E_\pi - E + p)}{(E-p)(E_\pi - E - p)} + \frac{4}{p} \frac{E_\pi^2 - M^2}{(E_\pi - E)^2 - p^2} \left(-2p \right.$$

$$\left. + E \ln \frac{E+p}{E-p} \right) - 4 \left(-\frac{\mu'}{2e} \right)$$

$$\times M \frac{(E_\pi - E)^2 - p^2}{p E_\pi} \ln \frac{(E+p)(E_\pi - E + p)}{(E-p)(E_\pi - E - p)}$$

$$\left. - 8 \left(\frac{\mu'}{2e} \right)^2 M^2 \left(1 - \frac{E E_\pi}{M^2} \right) \right\}.$$

In the non-relativistic case, $p \gg M, E_\pi$; $E = M + p^2/2M$. Assuming [2] that the mean free path R of the μ-meson is proportional to p^4 we get for the number of μ-mesons with mean free paths less than R

$$w = \int\limits_0^{p = p_0 (R/R_0)^{1/4}} dw = [1 + \tau] \, w_{\mu+\nu+\gamma}$$

$$+ \frac{e^2 g^2}{2\pi} \frac{M^2}{E_\pi} \int\limits_0^{(p_0/M)(R/R_0)^{1/4}} \left[\frac{\tau^2}{2} \left(\frac{E_\pi}{M} - 1 \right) x^2 \right.$$

$$\left. + \frac{8\tau}{3} \left(-\frac{M}{E_\pi} + \frac{E_\pi}{M} \right) \frac{x^4}{(E_\pi / M - 1)^2 - x^2} \right] dx.$$

Here p_0 and R_0 are the momentum and the mean free path of the meson in the decay $\pi \to \mu + \nu$; $\tau = \mu'/(e/2M)$; $w_{\mu+\nu+\gamma}$ is the decay probability for $\tau = 0$ derived by Ioffe and Rudik[1]; $w_{\mu+\nu} = (g^2/2)(1 - M^2/E_\pi^2)\, P_0$ is the probability of the decay $\pi \to \mu + \nu$.

The comparison withthe results of Ioffe and Rudik[1] shows that the μ-meson having an anomalous magnetic moment can lead to an increase of the number of mesons especially of those with short mean free paths. Similar results should be expected in the case of mesons with spin greater than ½.

I wish to thank B. L. Ioffe for the suggestion of this problem and its discussion.

[1] B. L. Ioffe and A. P. Rudik, Dokl. Akad. Nauk SSSR 82, 359 (1952).

[2] B. Rossi and K. Greisen, *Interaction of cosmic rays with matter*, page 17, IIL (1948) (Russian translation).

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Production of π-Meson Pairs on Nuclei by High Energy γ-Quanta

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THE production of π-meson pairs on nuclei by high energy γ-quanta has been discussed by Pomeranchuk.[1,2] In the case of high energy γ-quanta, only small angles between the momenta of the π-mesons and the γ-quanta are important. The range of the process is found to be greater than the dimensions of the nucleus. Therefore, the knowledge of the wave function outside the nucleus is sufficient to determine the cross section of the process. In Refs. 1 and 2, the wave function was taken as a plane wave plus a wave scattered by a perfectly black sphere of radius R (radius of the nucleus). In this paper we take into account the influence of the Coulomb interaction between the π-mesons and the charge of the nucleus on the pair formation.

The matrix element of the process of formation of a π^+, π^- — pair is given by

$$M = -ie \sqrt{\frac{2\pi}{\omega}} \int [\psi_+^* \, \mathbf{j} \cdot \nabla \, \psi_-^* - \psi_-^* (\mathbf{j} \nabla) \psi_+^*] \, e^{i \mathbf{k} \cdot \mathbf{r}} d\mathbf{r}, \quad (1)$$

where $\mathbf{k}$, ω and $\mathbf{j}$ are the wave vector, the frequency and the polarization of the incoming quantum. The wave functions ψ_+ and ψ_- of the created mesons are the sums of plane and converging waves:

$$\psi_+ = \frac{1}{\sqrt{2E_+}} \left\{ e^{i \mathbf{p}_+ \mathbf{r}} \right. \quad (2)$$

$$\left. + \frac{p_+}{2\pi i} \int \frac{\exp\{-i p_+ |\mathbf{r} - \boldsymbol{\rho}|\}}{|\mathbf{r} - \boldsymbol{\rho}|} \{1 - \Omega^* (\rho)\} \, d\boldsymbol{\rho} \right\},$$

where ρ is the radius in a plane orthogonal to the momentum $\mathbf{p}_+$ of the created meson and passing through the center of the nucleus:

$$\Omega(\rho) = \begin{cases} 0 & \rho \leqslant R, \\ e^{2i\eta_+(\rho)}, & \rho > R; \end{cases}$$

$\eta_+(\rho) \approx n_+ \log p + \rho$ is the Coulomb scattering phase, with $n_+ = ze^2 E_+ / p_+$ and E_+ is the energy of the π^+-meson. We break the wave function into three parts:

$$\psi_+ = \frac{1}{\sqrt{2E_+}}\{e^{i\mathbf{p}_+\mathbf{r}} + \Phi_+ + \Phi'_+\}; \qquad (2')$$

$$\Phi_+ = \frac{p_+}{2\pi i}\int\limits_{\rho<R} \frac{\exp\{-ip_+|\mathbf{r}-\rho|\}}{|\mathbf{r}-\rho|}\,d\rho;$$

$$\Phi'_+ = \frac{p_+}{2\pi i}\int\limits_{p>R} \frac{\exp\{-ip_+|\mathbf{r}-\rho|\}}{|\mathbf{r}-\rho|}\{1-e^{-2i\eta_+(\rho)}\}\,d\rho;$$

similarly for π^- :

$$\psi_- = (2E_-)^{1/2}\{e^{i\mathbf{p}_-\mathbf{r}} + \Phi_- + \Phi'_-\}; \qquad (3)$$

$$n_- = -Ze^2E_-/p_-.$$

In the evaluation of the matrix element (1), let us note that the integrals containing the products of the plane waves involved in ψ_+ and ψ_- vanish because of the conservation laws; all the integrals containing the products of non-overlapping wave functions (ψ_+ and ϕ_- , ψ_- and ϕ_+, ϕ_+ and ϕ'_- and finally ϕ_- and ϕ'_+ do also vanish. We get

$$M = -ie\sqrt{\frac{\pi}{2\omega E_+E_-}} \qquad (4)$$

$$\times\int\{(\Phi^*_+ + \Phi'^*_+)\,\mathbf{j}\cdot\nabla e^{-i\mathbf{p}\cdot\mathbf{r}} + e^{-i\mathbf{p}_+\cdot\mathbf{r}}\,\mathbf{j}\cdot\nabla\Phi'^*$$

$$-(\Phi^*_- + \Phi'^*_-)\,\mathbf{j}\cdot\nabla e^{-i\mathbf{p}_+\mathbf{r}} - e^{-i\mathbf{p}_-\cdot\mathbf{r}}\,\mathbf{j}\cdot\nabla\Phi'^*_+\}\,e^{i\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}.$$

In the case of relativistic energies (E_+ and $E_- \gg \mu$) and of small angles between $\mathbf{p}_+$, $\mathbf{p}_-$ and k (which is the only case where the diffraction treatment is adequate), and assuming that $n \ll 1$ we have:

$$M = ie\sqrt{8\pi^3 p_+p_-/\omega^3} \qquad (5)$$

$$\times\left[\frac{\mathbf{j}\xi}{1+\xi^2} - \frac{\mathbf{j}\eta}{1+\eta^2}\right]\frac{1}{2\mu^2}\frac{R}{q}J_1(2\mu Rq)$$

$$+ e\sqrt{8\pi^3 p_+p_-/\omega^3}\,\frac{n^2}{\mu^3q^2}$$

$$\times\left\{\frac{\mathbf{j}\xi}{1+\xi^2}[J_0(2\mu Rq) + 2\mu RqJ_1(2\mu Rq)\ln p_-R]\right.$$

$$\left.+\frac{\mathbf{j}\eta}{1+\eta^2}[J_0(2\mu Rq) + 2\mu RqJ_1(2\mu Rq)\ln p_+R]\right\},$$

where $q = \frac{1}{2}(\xi+\eta)$ and ξ and η are determined by the relations

$$\mathbf{p}_+ = p_+(1-\xi^2\mu^2/2p_+^2)\frac{\mathbf{k}}{\omega} + \mu\xi, \quad \mathbf{k}\cdot\xi = 0,$$

$$\mathbf{p}_- = p_-(1-\eta^2\mu^2/2p_-^2)\frac{\mathbf{k}}{\omega} + \mu\eta, \quad \mathbf{k}\cdot\eta = 0.$$

The differential cross section for pair formation is equal to

$$d\sigma_j(E_+, \xi, \eta) \qquad (6)$$

$$= 2\pi|M^2||F|^2\mu^4(2\pi)^{-6}\,d\xi\,d\eta\,dE_+,$$

where F is a form factor taking into account the finite size of the particles and their interaction between themselves. Substituting (5) into (6), and averaging over polarization of the quanta, we get:

$$d\sigma = \frac{e^2p_+p_-}{32\pi^2\omega^3}\left\{\frac{R^2J_1^2(2\mu Rq)}{q^2}\left[\frac{\xi}{1+\xi^2} - \frac{\eta}{1+\eta^2}\right]^2\right. \qquad (7)$$

$$+\frac{4n^2}{\mu^2q^4}\left[\frac{\xi}{1+\xi^2}(J_0(2\mu Rq) + 2\mu RqJ_1(2\mu Rq)\ln p_-R)\right.$$

$$+\frac{\eta}{1+\eta^2}(J_0(2\mu Rq)$$

$$\left.\left.+ 2\mu RqJ_1(2\mu Rq)\ln p_+R)\right]^2\right\}|F|^2\,dE_+\,d\xi\,d\eta.$$

In (5) the most important lengths are the $r_{\text{eff.}} \sim 2p_+p_-/\omega\mu^2$; the effective value of q is therefore approximately equal to $1/\mu r_{\text{eff.}}$ Furthermore $r_{\text{eff.}} \gg R$; hence $2\mu Rq_{\text{eff.}} \ll 1$ and

$$d\sigma = \frac{e^2E_+(\omega - E_+)}{32\pi^2\omega^3} \qquad (8)$$

$$\times\left\{\frac{R^2J_1^2(2\mu Rq)}{q^2}\left(\frac{\xi}{1+\xi^2} - \frac{\eta}{1+\eta^2}\right)^2\right.$$

$$\left.+\frac{4n^2J_0^2(2\mu Rq)}{\mu^2q^4}\left(\frac{\xi}{1+\xi^2} + \frac{\eta}{1+\eta^2}\right)^2\right\}|F|^2\,dE_+\,d\xi\,d\eta.$$

The first term in the { } brackets corresponds to the diffractional $\pi^+\pi^-$ pair formation, without excitation of the nucleus[2] ; the second term takes into account the pair formation produced by the Coulomb interaction. For $n \ll 1$, the interference between the diffractional and Coulomb pair formation does not appear. Setting $F = 1$ and integrating (8) over ξ and η , we get

$$d\sigma = d\sigma^d + d\sigma^c; \qquad (9)$$

$$d\sigma^d = \frac{e^2R^2}{2}\varepsilon(1-\varepsilon)\,d\varepsilon\left[\ln\frac{1+\xi^2_{\max}}{e} + \frac{1}{1+\xi^2_{\max}}\right];$$

$$d\sigma^c = \frac{2e^2n^2}{\mu^2}\varepsilon(1-\varepsilon)\,d\varepsilon\int\limits_{q_{\min}}^{\infty}\frac{J_0^2(2\mu Rq)}{q^3}\varphi(q)\,dq;\ \varepsilon = \frac{E_+}{\omega}, q_{\min}$$

$$= \frac{\mu}{4\varepsilon(1-\varepsilon)\omega};\ \varphi(q) = \frac{2q^2+1}{q\sqrt{1+q^2}}\ln(q+\sqrt{1+q^2}) - 1;$$

the maximum value $\xi_{max} = \eta_{max}$ is determined by the properties of the π-particles interactions,[2] and q_{min} is determined from the conservation laws. Noting that the most important quantity in $d\sigma^c$ is $q << 1$ and setting $\varphi(q) \approx 4/3\, q^2$, we get a known result[3] for the cross section for pair formation in the Coulomb field:

$$d\sigma^c = (8e^2n^2/3\mu^2)\,\varepsilon(1-\varepsilon) \times d\varepsilon \ln[(2\varepsilon(1-\varepsilon)\,\omega/\mu^2R];\quad 2\mu R q_{min} \ll 1. \tag{10}$$

The integral cross sections are equal to

$$\sigma^d = \frac{e^2R^2}{12}\left[\ln\frac{1+\xi^2_{max}}{e} + \frac{1}{1+\xi^2_{max}}\right]; \tag{11}$$

$$\sigma^c = \frac{4}{9}\frac{e^2n^2}{\mu^2}\ln\frac{2\omega}{\mu^2R}.$$

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1. I. Ia. Pomeranchuk, Dokl. Akad. Nauk SSSR 96, 265 (1954).

2. I. Ia. Pomeranchuk, Dokl. Akad. Nauk SSSR 96, 481 (1954).

3. R. Christy and S. Kusaka, Phys. Rev. 59, 414 (1941)

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Invalidity of the Fermi-Dirac Distribution for Electrons of Semiconductor and Crystal Phosphor Impurity Centers

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AMONG the original assumptions in introducing the Fermi-Dirac distribution we have the following: 1) The electrons must not interact with each other, in particular, the energy of a given single electronic state must not depend on the distribution of electrons in the states ; 2) subject to the well known limitations, interaction of electrons is permitted with other subsystems (for example, if the state of this subsystem follows the electronic motion adiabatically). But it is necessary that the entropy of this subsystem not depend on the electronic distribution of the state.

It is simple to demonstrate that this assumption is not usually realized for electrons of semiconductor impurity centers. For example, if the donor is an atom of a monovalent metal, then the essential interaction of the electrons (other than the valence electrons) with one another and with the valence electrons, is still not a difficulty since the state of strongly bound electrons of the ion core follows the motion of the valence electrons adiabatically (and thus the motion of the conduction electrons). Therefore the ion core can be considered as the above-mentioned subsystem and the statistical distribution can be introduced only for the valence electrons of the donor and the conduction electrons. But the essential difficulty in obtaining a Fermi distribution is the following circumstance : there exist two equilibrium states of the valence electrons corresponding to the two possible orientations of their spin; hence, if one is occupied by an electron the energy level of the second state is raised considerably and even gets into the conduction band (this is consistent with the instability of the negative ions of the alkali elements, when introduced into crystals). Hence this violates the original assumption 1).

If the donor is an atom of a divalent element, in which the valence electrons have opposite spin orientations, then the state of the atom is nondegenerate. But after a single ionization of the donor, the state of the remaining electrons shows a twofold degeneracy corresponding tothe two spin orientations of a single valence electron. Thus the degeee of degeneracy of the states of the subsystem consisting of ionic core is equal to 2^{N_1}, where N_1 is the number of singly ionized donors. In this case the entropy of the subsystem depends essentially on the electron distribution, i.e., in violation of assumption 2).

In consequence of the violation of the original assumptions 1) and 2), the Fermi-Dirac distribution, generally speaking, is not applicable to the donor electrons. Instead, we are led to employ the more general Gibbs distribution for a system with a variable number of particles, and we regard the donor as a system capable of losing electrons to the surrounding medium, and absorbing electrons from the medium. The probability that a donor will contain N electrons and be found in a quantum state n, is equal to

$$w_{nN} = \exp\{(\Omega + \mu N - E_{nN})/kT\}. \tag{1}$$