calculation of nuclear deformations together with the decay scheme of Ref. 4 significantly improves the agreement with experiment,[10] which can be considered as an indirect confirmation of the collective (generalized) model of the nucleus.

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On the Probabilities of Σ-Particle Decay

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(Submitted to JETP editor March 31, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 333-335 (August, 1956)

RECENTLY Gatto[1] indicated the interest in applying arguments developed first by Fermi[2] for the photoproduction of π-mesons to the problem of the decay of "strange" particles. As is well known, Fermi showed, on the basis of the requirements of unitarity and symmetry of the S-matrix, that the phases of strongly interacting particles which are produced in weak processes are defined by their mutual scattering. Gatto considered the decay of the Λ-particle from this point of view and came to the conclusion that the restrictions imposed by the unitarity and symmetry of the S-matrix on the ratio of probabilities of the two possible modes of decay of the Λ-hyperon ($\Lambda \to p + \pi^-$ and $\Lambda \to n + \pi^0$) are very weak. This conclusion, as will be evident from the following, depends to a large extent on the fact that the scattering of π-mesons by nucleons at energies ~ 40 mev is still small and the corresponding phases are small.

We consider in analogous fashion the decay of the Σ-hyperon. In this decay, $\sim$ 115 mev is given off, corresponding to the scattering of π-mesons of $\sim$ 140 mev in the laboratory system by nucleons. It is well known that the scattering of π-mesons by nucleons is already considerable (according to Orear[3] the phases $\alpha_{33} \sim 40°$, $\alpha_3 \sim -10°$, $\alpha_1 \sim 15°$, $\alpha_{31} = \alpha_{13} = \alpha_{11} = 0$; the d-phases do not exceed 5°).

The π-meson-nucleon system formed as a result of Σ-particle decay, has the following values J of total and L of orbital angular momentum, depending on the spin S and parity P of the hyperon:

Spin and parity of the Σ-hyperon		State of the $\pi + N$ system		
S	P	J	L	Phases
1/2	+	1/2	1	α_{31}, α_{11}
1/2	—	1/2	0	α_3, α_1
3/2	+	3/2	1	α_{33}, α_{13}
3/2	—	3/2	2	

(P designates the parity of the Σ-hyperon relative to the nucleon). Thus, L and J of the π-meson-nucleon system are determined unambiguously by the spin and parity of the Σ-hyperon.

We consider two possible decay modes for the Σ-hyperon:

$$\Sigma^+ \to p + \pi^0 \text{ and } \Sigma^+ \to n + \pi^+.$$

If weak interactions are excluded, then the S-matrix has only diagonal elements, transforming Σ^+ into Σ^+ and the π-meson-nucleon system with a given T into a state with the same T. These diagonal elements are equal to 1, $e^{i2\alpha_3}$ and $e^{i2\alpha_1}$ (the second index of the phases α_3 and α_1 is omitted).

If weak interactions are now included, nondiagonal elements arise which transform the Σ-particle into states of the π-meson-nucleon system with $T = 3/2$ and $1/2$. It follows from the unitarity and symmetry of the S-matrix [2] that these diagonal elements have the form $i\rho_3 e^{i\alpha_3}$ and $i\rho_1 e^{i\alpha_1}$, where ρ_3 and ρ_1 are real. The transition amplitude for the Σ^+-hyperon going into the states $\pi^+ n$ and $\pi^0 p$ can be expressed in the following way in terms of the isotopic amplitudes

$$a_+ = \sqrt{\frac{1}{3}}\, i\rho_3 e^{i\alpha_3} + \sqrt{\frac{2}{3}}\, i\rho_1 e^{i\alpha_1},$$

$$a_0 = \sqrt{\frac{2}{3}}\, i\rho_3 e^{i\alpha_3} - \sqrt{\frac{1}{3}}\, i\rho_1 e^{i\alpha_1}.$$

The ratio of corresponding decay probabilities is

$$X = \frac{w_0}{w_+} = \frac{2 + 2z^2 - 4z\cos(\alpha_1 - \alpha_3)}{1 + 4z^2 + 4z\cos(\alpha_1 - \alpha_3)},$$

where

$$z = \rho_1 / \rho_3 \sqrt{2}.$$

The phase differences for various spins and parities of the Σ-hyperon are: $(1/2+)\ \alpha_{11} - \alpha_{31} \approx 0$; $(1/2-)\ \alpha_1 - \alpha_3 \approx 25°$; $(3/2+)\ \alpha_{13} - \alpha_{33} \approx -40°$.

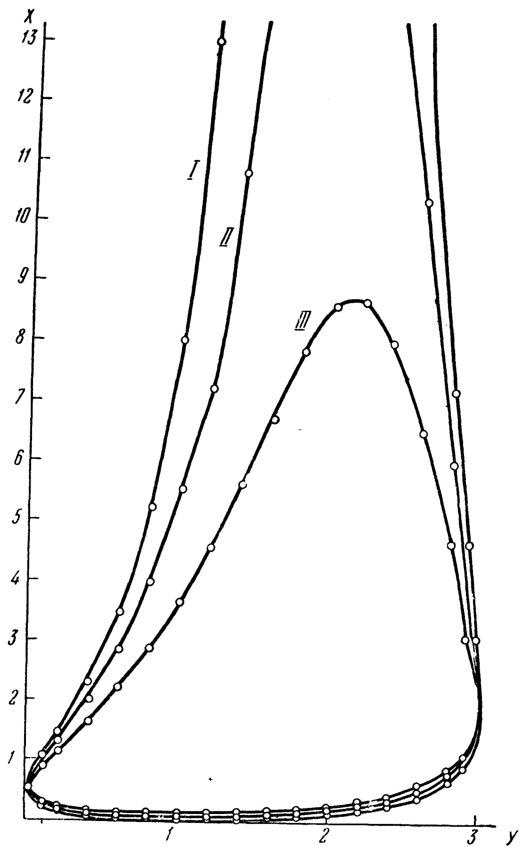

Figure 1

For all other values of the spins and parities the differences of the corresponding phases (and the phases themselves) are close to zero. The limits of X depend on the phase differences in the following way

Difference in phase	0	~25°	~40°
Upper limit of X	∞	~20	~10
Lower limit of X	0	~1/20	~1/10

The restrictions obtained on the magnitude of X are very weak. None the less, if it turns out that $X > 10$ or $X > 1/10$, this will mean that the Σ-particle definitely does not belong to the class $3/2+$, and if $X > 20$ or $X < 1/20$, then the possibility $1/2-$ is unambiguously excluded.

The ratio of probabilities of different Σ^+-hyperon decays can be related to the decay probability of the Σ^--hyperon if it is assumed, as Gatto[1] has done, that the interaction leading to the decay is a tensor of rank 1/2 in isotopic space. It is easy to show that in this case the decay amplitude for $\Sigma^- \to n + \pi^-$ is equal to

$$a_- = i\sqrt{3}\,\rho_3 e^{i\alpha_3};$$

where ρ_3 is the same quantity as in the expressions for a_+ and a_0.

The ratio of the decay probabilities for the Σ^--particle and Σ^+-particle is, in this case, equal to

$$Y = \frac{w_-}{w_+ + w_0} = \frac{3}{1 + 2z^2}.$$

We find in this way that for every possible value of the spin and parity of the Σ-particle, the points (X, Y) can lie only on a well-defined curve in the XY plane. Gatto, who did not use the unitarity and symmetry of the S-matrix in considering the Σ-decay, obtained in this case a considerably less restrictive result; he found that the points (X, Y) can cover a certain allowed region — which is one and the same for all values of the spin and parity of the Σ-particle. If the phase difference is close to zero (that is, in all cases except those in which the spin and parity of the Σ-particle are equal to $1/2-$ or $3/2+$) the curve $Y(X)$ will be close to the curve bounding the allowed region of Gatto's work [1] (see Fig. 1). The curves for the phase differences of 25 and 40° are shown also on this Figure (curves II and III, respectively).

If the point (X, Y) characterizing the values of the decay probabilities as found by experiment lies on one of these two curves, this will signify (if we neglect the improbable possibility of a chance coincidence) the correctness of Gatto's hypothesis and will make it possible to draw definite conclusions about the spin and parity of the Σ-hyperon.

I wish to express my sincere gratitude to I. Iu. Kobzarev for interesting and useful discussions and to Prof. I. Ia. Pomeranchuk for the interest shown by him in this work.

Note added in proof. After this article was submitted, the author learned of the work in Refs. 4–6 in which the essentials of the above results are contained. In addition to this, preliminary experimental results were announced at the Sixth Rochester Conference which indicated that the magnitude of X was near to 1, and that of Y near to 0.1 –0.2. As is evident from the Figure, these data agree with the assumption that $\Delta T = 1/2$.

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Translated by G. E. Brown
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Relation between the Parameters of Longitudinal and Transverse Optical Vibrations of Ions in Crystals

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(Submitted to JETP editor April 30, 1956)
J. Exptl. Theoret. Phys.(U.S.S.R.) 31, 343-346 (August, 1956)

LET us consider vibrations with wavelengths much larger than the lattice constant, but smaller than $c/\nu \sim 10^{-3}$ cm. (ν is the characteristic frequency for infrared dispersion in the crystal). The latter assumption enables us to treat the electromagnetic field created by the vibrating ions as electrostatic, i.e., to neglect the retardation effects as well as the effect of the magnetic field.[1,2] For this range of wavelengths one can also neglect the dispersion of the vibration eigenfrequencies.

In isotropically polarizable ionic (cubic) crystals, the polarization vibrations separate into transverse and longitudinal vibrations. We so normalize the normal coordinates that, for any transverse or longitudinal vibration, the Hamiltonian has the form $\frac{1}{2}(\dot{q}^2 + \Omega_j^2 q^2)$, where Ω_j is the eigenfrequency of the corresponding transverse vibration ($\Omega_{\perp j}$) or longitudinal vibration ($\Omega_{\parallel j}$); j is the branch number of the dispersion. Each normal vibration gives rise, in the crystal, to an inertial polarization, i.e., to a polarization due to the displacement of the ions and to the electron polarization produced by the displacement of the ions in the absence of an external field. The inertial polarization dipole moment density $\mathbf{p}(\mathbf{r}, t)$ varies sinusoidally in space, and its amplitude $\mathbf{p}_0(t)$ is proportional to q. Let $p_0 = \alpha_{\perp j}\, q$ for the transverse vibration and $p_0 = \alpha_{\parallel j}\, q$ for the longitudinal vibration. The relation between the parameters Ω_j, α_j for transverse and longitudinal vibrations is derived below.

Let us consider the forced vibrations of ions produced by an external electric field $\mathfrak{E}(\mathbf{r}, t)$. This field is chosen as a plane sinusoidal standing wave, vibrating harmonically in time with a frequency ω. Assuming that the interaction energy per unit volume of the crystal is equal to $-\mathbf{p}\mathfrak{E}$, we get for the dipole moment desnity due to forced vibrations:

$$\mathbf{p} = \sum_{j=1}^{s} \mathfrak{E}\alpha_j^2 / (\Omega_j^2 - \omega^2), \qquad (1)$$

where s is the number of ions in the elementary cell of the crystal, minus one. The total polarization dipole moment due to the external field is $\mathbf{P} = \mathbf{p} + \mathbf{p}_e$ where $\mathbf{p}_e$ is the additional non-inertial polarization dipole moment due to the direct effect of the external field on the electron shells, the positions of the ions being held fixed. Let us consider two cases:

1) The external field is transverse and div $\mathbf{P}=0$, i.e., the fictitious charges of dielectric polarization and their corresponding fields do not arise. In this case the external field $\mathfrak{E}$ coincides with the field $\mathbf{E}$ of macroscopical electrodynamics, and $\mathbf{p}_e = \mathbf{E}(n^2 - 1)/4\pi$, where n is the index of refraction for light in the crystal, in the plateau region of the dispersion curve— between the region of electronic absorption and the region where the absorption of infrared light by the vibration of the ions takes place. We get