(if we neglect the improbable possibility of a chance coincidence) the correctness of Gatto's hypothesis and will make it possible to draw definite conclusions about the spin and parity of the Σ-hyperon.

I wish to express my sincere gratitude to I. Iu. Kobzarev for interesting and useful discussions and to Prof. I. Ia. Pomeranchuk for the interest shown by him in this work.

Note added in proof. After this article was submitted, the author learned of the work in Refs. 4–6 in which the essentials of the above results are contained. In addition to this, preliminary experimental results were announced at the Sixth Rochester Conference which indicated that the magnitude of X was near to 1, and that of Y near to 0.1 –0.2. As is evident from the Figure, these data agree with the assumption that $\Delta T = 1/2$.

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58

Relation between the Parameters of Longitudinal and Transverse Optical Vibrations of Ions in Crystals

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LET us consider vibrations with wavelengths much larger than the lattice constant, but smaller than $c/\nu \sim 10^{-3}$ cm. (ν is the characteristic frequency for infrared dispersion in the crystal). The latter assumption enables us to treat the electromagnetic field created by the vibrating ions as electrostatic, i.e., to neglect the retardation effects as well as the effect of the magnetic field.[1,2] For this range of wavelengths one can also neglect the dispersion of the vibration eigenfrequencies.

In isotropically polarizable ionic (cubic) crystals, the polarization vibrations separate into transverse and longitudinal vibrations. We so normalize the normal coordinates that, for any transverse or longitudinal vibration, the Hamiltonian has the form $\frac{1}{2}(\dot{q}^2 + \Omega_j^2 q^2)$, where Ω_j is the eigenfrequency of the corresponding transverse vibration ($\Omega_{\perp j}$) or longitudinal vibration ($\Omega_{\parallel j}$); j is the branch number of the dispersion. Each normal vibration gives rise, in the crystal, to an inertial polarization, i.e., to a polarization due to the displacement of the ions and to the electron polarization produced by the displacement of the ions in the absence of an external field. The inertial polarization dipole moment density $\mathbf{p}(\mathbf{r}, t)$ varies sinusoidally in space, and its amplitude $\mathbf{p}_0(t)$ is proportional to q. Let $p_0 = \alpha_{\perp j}\, q$ for the transverse vibration and $p_0 = \alpha_{\parallel j}\, q$ for the longitudinal vibration. The relation between the parameters Ω_j, α_j for transverse and longitudinal vibrations is derived below.

Let us consider the forced vibrations of ions produced by an external electric field $\mathfrak{E}(\mathbf{r}, t)$. This field is chosen as a plane sinusoidal standing wave, vibrating harmonically in time with a frequency ω. Assuming that the interaction energy per unit volume of the crystal is equal to $-\mathbf{p}\mathfrak{E}$, we get for the dipole moment desnity due to forced vibrations:

$$\mathbf{p} = \sum_{j=1}^{s} \mathfrak{E}\alpha_j^2 / (\Omega_j^2 - \omega^2), \tag{1}$$

where s is the number of ions in the elementary cell of the crystal, minus one. The total polarization dipole moment due to the external field is $\mathbf{P} = \mathbf{p} + \mathbf{p}_e$ where $\mathbf{p}_e$ is the additional non-inertial polarization dipole moment due to the direct effect of the external field on the electron shells, the positions of the ions being held fixed. Let us consider two cases:

1) The external field is transverse and div $\mathbf{P} = 0$, i.e., the fictitious charges of dielectric polarization and their corresponding fields do not arise. In this case the external field $\mathfrak{E}$ coincides with the field $\mathbf{E}$ of macroscopical electrodynamics, and $\mathbf{p}_e = \mathbf{E}(n^2 - 1)/4\pi$, where n is the index of refraction for light in the crystal, in the plateau region of the dispersion curve— between the region of electronic absorption and the region where the absorption of infrared light by the vibration of the ions takes place. We get

$$\mathbf{P} = \mathbf{p} + \mathbf{p}_e = \left(\sum_{j=1}^{s} \frac{\alpha^2_{\perp j}}{\Omega^2_{\perp j} - \omega^2} + \frac{n^2 - 1}{4\pi} \right) \mathbf{E}. \tag{2}$$

Identifying this formula with the formula of phenomenological electrodynamics: $P = [\epsilon - 1]\, E/4\pi$ we get:

$$\varepsilon(\omega)/n^2 = 1 + \sum_{j=1}^{s} a_{\perp j}/(\Omega^2_{\perp j} - \omega^2), \tag{3}$$

$$a_{\perp j} = 4\pi\alpha^2_{\perp j}/n^2.$$

In particular, if E is the field of the light wave (3) is the usual dispersion formula in the infrared region.

2)The external field is longitudinal. In this case the curls of the vectors p and p_e vanish, but their divergences do not. The field of the fictitious charge of dielectric polarization is $-4\pi P$. It follows that the total macroscopic field is equal to $E = \mathfrak{E} - 4\pi P$, i.e., the external field $\mathfrak{E}$ coincides with the displacement of macroscopical electrodynamics. Then $p_e = D(n^2 - 1)/4\pi n^2$ and

$$\mathbf{P} = \left(\sum_{j=1}^{s} \frac{\alpha^2_{\parallel j}}{\Omega^2_{\parallel j} - \omega^2} + \frac{n^2 - 1}{4\pi n^2} \right) \mathbf{D}. \tag{4}$$

Identifying this formula with the formula of pneomenological electrodynamics $P = [\epsilon - 1]\, D/4\pi\epsilon$, we get a "dispersion formula" for the longitudinal waves:

$$\frac{n^2}{\varepsilon(\omega)} = 1 - \sum_{j=1}^{s} a_{\parallel j}/(\Omega^2_{\parallel j} - \omega^2), \tag{5}$$

$$a_{\parallel j} = 4\pi n^2 \alpha^2_{\parallel j}.$$

$\epsilon(\omega)$ should be the same in both cases, because the coefficient of proportionality between P and E should not depend on whether the fictitious dielectric polarization charges affect or do not affect the state of the field E. Identifying the expressions of n^2/ϵ from (5) and (3), we obtain the desired relation between the parameters $\Omega_{\parallel j}$, $\alpha_{\parallel j}$ and $\Omega_{\perp j}$.

In practice this is achieved most easily by considering the reciprocal of (3) as a rational fraction with respect to the variable ω^2 :

$$n^2/\varepsilon(\omega) = \prod(\omega^2) \Big/ \left[\prod(\omega^2) + \sum_{j=1}^{s} a_{\perp j} \prod{}'^{\,j}(\omega^2) \right]; \tag{6}$$

where

$$\prod = \prod_{j=1}^{s} (\Omega^2_{\perp j} - \omega^2); \quad \prod{}'^{\,j} = (\Omega^2_{\perp 1} - \omega^2) \dots$$

$$(\Omega^2_{\perp j-1} - \omega^2)(\Omega^2_{\perp j+1} - \omega^2) \dots (\Omega^2_{\perp s} - \omega^2).$$

After having expressed this fraction in elementary fractions one has to identify each partial fraction with the corresponding term of the sum in the right hand side of (5).

A useful additional relation may be obtained by setting $\omega = 0$ in (3):

$$\varepsilon_0 \;\; n^2 = 1 + \sum_{j=1}^{s} a_{\perp j}/\Omega^2_{\perp j}, \tag{7}$$

where ϵ_0 is the static dielectric constant.

In the simplest case , when the number of ions in the elemenatry cell is two, $s = 1$ and the expansion (6) in partial fractions has the form:

$$n^2/\varepsilon = 1 - a_{\perp}/(\Omega^2_{\perp} - \omega^2 + a_{\perp}).$$

Identifying this expression with (5) we get $a_{\parallel} = a_{\perp}$ and $\Omega^2_{\parallel} = \Omega^2_{\perp} + a_{\perp}$ and it follows from (7) that $a_{\perp} = \Omega^2_{\perp}(\epsilon_0 n^{-2} - 1)$. Therefore:

$$\alpha_{\parallel} = \alpha_{\perp}/n^2, \qquad \Omega^2_{\parallel} = \Omega^2_{\perp}\varepsilon_0/n^2. \tag{8}$$

In the case of a three ion cell, $s = 2$, and the expansion (6) in elementary fractions and the identification of the result with (5) give

$$\begin{aligned} a_{\parallel 1} &= [a_{\perp 1}(\Omega^2_{\perp 2} - \Omega^2_{\parallel 1}) \\ &\quad + a_{\perp 2}(\Omega^2_{\perp 1} - \Omega^2_{\parallel 1})]/(\Omega^2_{\parallel 2} - \Omega^2_{\parallel 1}) \\ &= -(\Omega^2_{\perp 1} - \Omega^2_{\parallel 1})(\Omega^2_{\perp 2} - \Omega^2_{\parallel 1})/(\Omega^2_{\parallel 2} - \Omega^2_{\parallel 1}), \\ a_{\parallel 2} &= -(\Omega^2_{\perp 1} - \Omega^2_{\parallel 2})(\Omega^2_{\perp 2} - \Omega^2_{\parallel 2})/(\Omega^2_{\parallel 1} - \Omega^2_{\parallel 2}), \end{aligned} \tag{9}$$

where $\Omega^2_{\parallel 1}$ and $\Omega^2_{\parallel 2}$ are the roots of the quadratic equation

$$(\Omega^2_{\perp 1} - \omega^2)(\Omega^2_{\perp 2} - \omega^2) + a_{\perp 1}(\Omega^2_{\perp 2} - \omega^2)$$

$$+ a_{\perp 2}(\Omega^2_{\perp 1} - \omega^2) = 0,$$

where the unknown is ω^2 . The quantities $\alpha^2_{\perp j}$ and $\alpha^2_{\parallel j}$ can be expressed easily through $a_{\perp j}$ and $a_{\parallel j}$ using Eqs. (3) and (5).

In the case when the elementary cell consists of a large number of ions, one has to solve a higher order equation in ω^2. This is possible when the parameters of the transverse (longitudinal) vibrations are given numerically. The experimental determination of the parameters for the transverse vibrations can be made, for instance, by approximating through (3) the experimental dispersion curve for infrared light.

The relation between $\Omega_{\parallel}$ and $\Omega_{\perp}$ in the simplest particular case of two ions per cell, has been considered in earlier publications. It seems that Ref. 3 reports wrong relations. The relation (8) was derived for the first time in Ref. 4; it has later been derived by Tolpygo[1] and Callen[5] on the basis of the microtheory of ionic crystals; Huang[2] has derived it using the macroscopical electromagnetic theory.

In the case of an anisotropic crystal, if the orientation of one of its principal axes of polarization does not depend on the frequency (because of the symmetry of the crystal) the relation derived above between the parameters of longitudinal and transverse waves still hold for a longitudinal wave travelling along the said axis, and for a transverse wave travelling in a perpendicular direction and polarized along the principal axis. If the wave travels in an arbitrary direction it is, generally speaking, neither longitudinal nor transverse, and the relations derived above need a basic generalization.

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65

Hyperfine Structure of the Paramagnetic Resonance Spectrum of $^{53}Cr^{3+}$ in Al_2O_3

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In Ref. 1 the fine structure of the paramagnetic resonance spectrum of Cr^{3+} in monocrystalline Al_2O_3 was investigated. More detailed study of this spectrum showed that with concentrations of Cr $\sim 10^{-4}$ the fine structure lines, corresponding to various electronic transitions, have identical widths, equal to 13 G. Further decrease in the Cr concentration and also a lowering of the temperature of the crystal to the temperature of liquid nitrogen did not cause a narrowing of the lines. These data led to the conclusion, that the line width for small Cr concentrations depends mainly on the magnetic interaction of the spins of Cr^{3+} with the Al^{27} nuclei surrounding the Cr^{3+} ions in monocrystalline corundum.

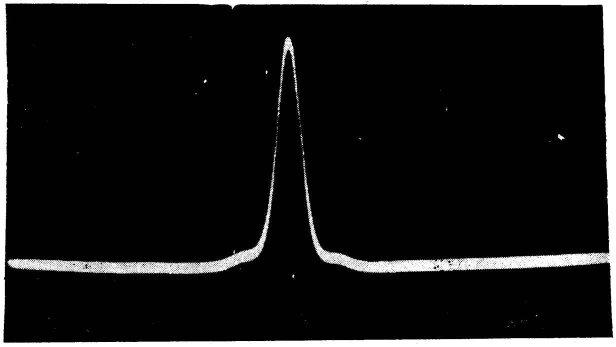

FIG. 1

For Cr concentrations greater than 10^{-4}, the line widths for various electronic transitions are not identical: the lines, corresponding to the transition $M = 3/2 \longleftrightarrow 1/2$ and $M = -1/2 \longleftrightarrow -3/2$ are wider than the line $M = 1/2 \longleftrightarrow -1/2$. This is connected with the fact that in this case the line width depends primarily on the magnetic interaction between the Cr^{3+} ions.

Relatively small line widths for small chromium concentrations permit one to observe hyperfine structure caused by the single stable odd isotope Cr^{53} whose concentration in the natural mixture of isotopes equals 9.4%. Figure 1 shows the oscillogram of the line corresponding to the electronic transition $M = 1/2 \leftrightarrow -1/2$. Two components of the hyperfine structure caused by the Cr^{53} isotope may be clearly seen on the wings of the central line from the even chromium isotopes, whose nuclear spins are zero. The relative intensities of the central line and the two supplementary lines are in agreement with the values of the nuclear spin of Cr^{53}, $I = 3/2$[2], and the concentration of the Cr^{53} isotope in the natural mixture, 9.4%.

In order to resolve the hyperfine structure completely, we investigated the paramagnetic resonance in monocrystalline corundum, containing chromium,