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An Investigation of the Elastic Scattering of 590 MEV Neutrons by Neutrons

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The differential scattering cross section for the elastic scattering of neutrons by neutrons has been determined using a neutron telescope. The effective energy of the neutrons was 590 mev. A striking anisotropy of the $(n-n)$ scattering has been established:

$$\sigma_{nn}(30^\circ)/\sigma_{nn}(90^\circ) = 2.3.$$

It has been found that the differential $(n-n)$ scattering cross section in the investigated angular region $(30^\circ \leq \vartheta \leq 90^\circ)$ is equal to the proton-proton cross section at the same energy within experimental error. This fact, together with the results of our earlier work[1] (on neutron-neutron scattering at 300 mev) definitely points to the charge symmetry of nuclear forces at high energy.

1. INTRODUCTION

AT low energies information about the interaction of neutrons with neutrons is obtained, for example, in the analysis of the binding energies of mirror nuclei and in the studies of nuclear reactions which give rise to two neutrons moving with small relative velocity. A survey of this experimental material obtained at low energies leads to the conclusion that the forces acting between two neutrons are the same as those acting between two protons to an accuracy up to electromagnetic effects[2].

At high energies, data on this neutron-neutron interaction can be obtained primarily from experiments in which the angular distribution of neutrons scattered by deuterons and protons is compared under identical conditions. Such experiments were carried out by Dzhelepov, Golovin and Satarov[1], using 300 mev neutrons. These experiments showed that the neutron-neutron differential scattering cross section in the investigated angular region of $40^\circ \leq \vartheta \leq 90^\circ$ (ϑ being the center-of-mass angle) was equal to the corresponding proton-proton scattering cross sections within experimental error. This definitely pointed toward the validity of the hypothesis of the charge symmetric character of nuclear forces.

There is, however, a special interest in getting similar evidence at neutron energies where meson production processes are appreciable.

The present work, therefore, was directed at studying the elastic neutron-neutron scattering at 590 mev. The experiments were carried out at the synchrocyclotron of the Institute for Nuclear Studies of the Academy of Sciences of the USSR.

In order to establish the possibility of determining the elastic neutron-neutron scattering using experiments on neutron-deuteron scattering, we first carried out a theoretical analysis (using a nonrelativistic momentum approximation) of the processes by which high energy neutrons could interact with deuterons. This yielded formulas connecting the probability of emission of neutrons into an angle θ (θ being the laboratory scattering angle) relative to the incoming beam, with the scattering cross section of neutrons by free neutrons and protons[3].

A series of such studies have already appeared in the literature[4-7], but the majority of them have the drawback that they use a central force between the nucleons. Moreover, in several cases, there is used a radial nucleon potential even though it is well known that a unique selection of such an interaction cannot be made at the present time. This reduces the generality of the results obtained and makes desirable a similar calculation under the most general assumptions possible concerning the interactions between nucleons.

We present below calculations carried out using the same method as was used by Pomeranchuk[8] and Shmushkevich[9], but keeping the complete expressions for the nucleon-nucleon scattering amplitudes. This permits us to hope that the results obtained should be applicable over a wider angular region than was the case in previous work.

2. CALCULATION OF $(n-d)$ SCATTERING

Let $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$ be the coordinates (in the lab. system) of the incoming nucleon, of the nucleon in the deuteron with the same isotopic spin, and of the nucleon in the deuteron with the opposite isotopic spin. Let us take as a wave function (not antisymmetrized with respect to coordinates of identical particles) for the initial state the expression

$$\Psi_i = \Omega^{-1} e^{i\mathbf{k}_0\mathbf{r}_1}\Phi_0(|\mathbf{r}_2 - \mathbf{r}_3|)\chi^i_{1,23}, \qquad (1)$$

and for the final state

$$\Psi_f = \Omega^{-3/2}\exp\{i\mathbf{k}'\mathbf{r}_1\} \qquad (2)$$
$$\times \exp\{i(\mathbf{k}_2 + \mathbf{k}_3)(\mathbf{r}_2 + \mathbf{r}_3)/2\}\,\Phi_f(\mathbf{r}_2 - \mathbf{r}_3)\chi^f_{1,23}.$$

In these equations, Ω is the normalizing volume, Φ_0 is the wave function for the ground state of the deuteron, Φ_f is the wave function for the relative motion of particles 2 and 3 after collision, $\mathbf{k}_2$ and $\mathbf{k}_3$ are the wave vectors of these particles in the final state and $\chi^i_{1,23}$ and $\chi^f_{1,23}$ are the spin functions corresponding to the initial and final states.

We will write the pseudo-potential of the interaction between the incoming nucleon and the deuteron in the form:

$$V = (4\pi\hbar^2/m)\{A_{12}\delta(\mathbf{r}_1 - \mathbf{r}_2) + A_{13}\delta(\mathbf{r}_1 - \mathbf{r}_3)\}, \qquad (3)$$

where A_{12} and A_{13} are the amplitudes for the scattering of the nucleon by the nucleons (in the center-of-mass system of the colliding nucleons),

$$A_{12} = \alpha_{12} + \beta\sigma_1\sigma_2 + \gamma_{12}(\sigma_1 + \sigma_2)\,\mathbf{n} \qquad (4)$$
$$+ \delta_{12}(\sigma_1\mathbf{e}_+)(\sigma_2\mathbf{e}_+) + \varepsilon_{12}(\sigma_1\mathbf{e}_-)(\sigma_2\mathbf{e}_-);$$
$$A_{13} = \alpha_{13} + \beta_{13}\sigma_1\sigma_3 + \gamma_{13}(\sigma_1 + \sigma_3)\,\mathbf{n}$$
$$+ \lambda_{13}(\sigma_1 - \sigma_3)\,\mathbf{n} + \delta_{13}(\sigma_1\mathbf{e}_+)(\sigma_3\mathbf{e}_+)$$
$$+ \varepsilon_{13}(\sigma_1\mathbf{e}_-)(\sigma_3\mathbf{e}_-);$$
$$\mathbf{e}_+ = (\mathbf{k}_i + \mathbf{k}_f)/|\mathbf{k}_i + \mathbf{k}_f|;$$
$$\mathbf{e}_- = (\mathbf{k}_i - \mathbf{k}_f)/|\mathbf{k}_i - \mathbf{k}_f|;\quad \mathbf{n} = [\mathbf{k}_i\mathbf{k}_f]/|[\mathbf{k}_i\mathbf{k}_f]|.$$

The matrix element for the transition of the system from the state in which the deuteron is at rest and the incoming nucleon has the momentum $\hbar\mathbf{k}_0$, to the state where the scattered nucleon has momentum $\hbar\mathbf{k}'$, and the two nucleons move with relative momentum $\hbar f = \hbar(\mathbf{k}_2 - \mathbf{k}_3)/2$ can be written in the form:

$$V_{k'k_0} = (4\pi\hbar^2/m\Omega^{3/2})\{(\chi^{f*}_{1,23}A_{12}\chi^i_{1,23})\,T_1 \qquad (5)$$
$$+ (\chi^{f*}_{1,23}A_{13}\chi^i_{1,23})\,T_2 - (\chi^f_{1,23}A_{23}\chi^i_{2,13})\,T_3\}$$
$$= (4\pi\hbar^2/m\Omega^{3/2})\{a_{12}T_1 + a_{13}T_2 - a_{23}T_3\},$$
$$T_1 = \int \Phi^*_f(\rho)\,e^{-i\Delta\mathbf{k}\rho/2}\Phi_0(\rho)\,d\rho;$$
$$T_2 = \int \Phi^*_f(\rho)\,e^{i\Delta\mathbf{k}\rho/2}\Phi_0(\rho)\,d\rho;$$
$$T_3 = \int \Phi^*_f(0)\,e^{i\mathbf{k}'\rho}\Phi_0(\rho)\,d\rho;\quad \Delta\mathbf{k} = \mathbf{k}' - \mathbf{k}_0. \qquad (6)$$

The cross section corresponding to this transition is equal to:

$$d\sigma(\theta) = \frac{1}{(2J_d + 1)(2S + 1)} \qquad (7)$$
$$\times \frac{2\pi\Omega}{\hbar v}\sum_{\chi^i}\sum_{\chi^f}|V_{k'k_0}|^2\rho_E,$$
$$\rho_E = \frac{\Omega^2}{(2\pi)^3}k'^2\left(\frac{\partial k'}{\partial E}\right)do'\,\frac{d\mathbf{f}}{(2\pi)^3}$$
$$= \frac{\Omega^2}{(2\pi)^6}\,d\mathbf{f}\,\frac{m}{\hbar^2}\,\frac{k'\,do'}{1 + {}^1/_2(k' - k\cos\theta)/k'};$$

where J_d is the spin of the deuteron, S is the spin of the incoming nucleon, v is the velocity of the incoming nucleon and $do' = 2\pi\sin\theta\,d\theta$.

In order to carry out further calculations it is necessary to know the actual form of the wave function $\Phi_f(\mathbf{r}_2 - \mathbf{r}_3)$ of the nucleons 2 and 3 in the final state. These functions are not known at the present time and the calculations, therefore, involve some additional assumptions concerning their properties. The problem is somewhat simplified because we are primarily interested in the total intensity of neutrons scattered at a given angle; their energy spectrum, which depends on the motion and initial state of the nucleons in the deuteron before collision, has only secondary interest. For this reason, Eq. (7) must be integrated over all possible values of the relative momentum $\hbar f$ consistent with conservation laws. The integration is simplified if one keeps in mind the experimental fact that in $(p-d)$ collisions the momenta of the scattered particles in the majority of cases is little different from that which they would have in the scattering from free nucleons into the same angle[10]. For this reason we will not make a big mistake if in Eq. (7) we place $k' = k\cos\theta$, which corresponds to this situation in most cases. In this case, k', and consequently also Δk, can be considered as constant (for a given angle of scattering) independent of f.

Expanding the integral in terms with T_1 and T_2 to infinity (which makes possible the use of the completeness theorem) and evaluating the result of the integration in the region in which $|\mathbf{f}| > f_m$, where $\hbar f_m$ is the largest value of the momentum of relative motion $\hbar\mathbf{f}$ (consistent with conservation laws), we arrive at the formula for the total cross section of $(n-d)$ scattering into an angle θ:

$$\frac{d\sigma(\theta)}{do'} = \frac{4k'}{6k_0}\{|a_{12}|^2(1-T_1') + |a_{13}|^2(1-T_2') + 2\mathrm{Re}\,(a_{12}^* a_{13})(J(\theta)-J'(\theta)) + \int \frac{d\mathbf{f}}{(2\pi)^3}[|a_{23}|^2|T_3|^2 - 2\mathrm{Re}\,(a_{12}^* a_{23}^* T_1^* T_3) - 2\mathrm{Re}\,(a_{13}^* a'_{23} T_2^* T_3)]\}. \tag{8}$$

In this equation

$$T_1' = (2\pi)^{-3}\int_{f \geqslant f_m} d\mathbf{f}\left|\int e^{-i\Delta\mathbf{k}\rho/2}\Phi_f^*(\rho)\,\Phi_0(\rho)\,d\rho\right|^2; \tag{9}$$

$$T_2' = (2\pi)^{-3}\int_{f \geqslant f_m} d\mathbf{f}\left|\int e^{i\Delta\mathbf{k}\rho/2}\Phi_f^*(\rho)\,\Phi_0(\rho)\,d\rho\right|^2;$$

$$J(\theta) = \int e^{i\Delta\mathbf{k}\rho}|\Phi_0(\rho)|^2\,d\rho;$$

$$J'(\theta) = (2\pi)^{-3}\int_{f \geqslant f_m} d\mathbf{f}\,T_1^*\,T_2\,.$$

The multipliers in front of the integrals, $|a_{12}|^2$, $|a_{13}|^2$, etc., are various combinations of the coefficients α, β, γ, etc., appearing in the scattering of nucleons by free nucleons. Comparison of these multipliers with the nucleon-nucleon cross sections expressed in terms of the same parameters leads to the following relations:

$$\begin{aligned} &{}^1/_6\,|a_{12}|^2 = [\sigma_{nn}(\vartheta)]_{\text{c.g.}}; \\ &{}^1/_6\,|2\mathrm{Re}\,(a_{12}^* a_{13})| \leqslant [\sigma_{nn}(\vartheta) + \sigma_{np}(\vartheta)]_{\text{c.g.}}; \\ &{}^1/_6\,|a_{13}|^2 = [\sigma_{np}(\vartheta)]_{\text{c.g.}}; \\ &{}^1/_6\,|2\mathrm{Re}\,(a_{12}^* a_{23})| \leqslant 3\,[\sigma_{nn}(\vartheta) + \sigma_{np}(\vartheta)]_{\text{c.g.}}; \\ &{}^1/_6\,|a_{23}|^2 = [\sigma_{np}(\vartheta)]_{\text{c.g.}}; \\ &{}^1/_6\,|2\mathrm{Re}\,(a_{13}^* a_{23})| \leqslant 3\,[\sigma_{np}(\vartheta)]_{\text{c.g.}}. \end{aligned} \tag{10}$$

The completeness theorem cannot be used for terms containing T_3; however, it is easy to see that these contribute significantly only at scattering angles close to $\pi/2$. In fact, numerical integration of the integrals $(2\pi)^3\int d\mathbf{f}\,|T_3|^3$ and $(2\pi)^3\int d\mathbf{f} \times T_{1(2)}^* T_3$ with the function $\Phi_f(\mathbf{r}_2-\mathbf{r}_3)$, taken either as a plane wave or as a function taking into account the interaction between nucleons 2 and 3 only in the state $l=0$ [11], shows that these integrals are sufficiently small for scattering into angles $\theta < 60°$.

A knowledge of the exact form of the function Φ_f is also necessary to evaluate the functions T_1', T_2' and $J'(\theta)$; however, in these cases the integration is carried out only in the region of large relative momenta and therefore the use of Φ_f as a plane wave appears more justified.

Thus we can write down the total $(n-d)$ scattering cross section for angles $\theta < 60°$ in the form:

$$d\sigma(\theta)/do' = 4\cos\theta\,\{[\sigma_{nn}(\vartheta)]_{\text{c.g.}}(1-T_1') + [\sigma_{np}(\vartheta)]_{\text{c.g.}}(1-T_2) + {}^1/_6\,[2\mathrm{Re}(a_{12}^* a_{13})]\,I(\theta)\}, \tag{11}$$
$$I(\theta) = J(\theta) - J'(\theta).$$

The last term in the formula represents the scattering due to interference of waves scattered by the different nucleons of the deuteron. From Eq. (10), the upper limit of this scattering is given by

$$[d\sigma_{\text{int}}(\theta)/do']_{\max} = 4\cos\theta\,\{[\sigma_{nn}(\vartheta)]_{\text{c.g.}} + [\sigma_{np}(\vartheta)]_{\text{c.g.}}\}\,I(\theta). \tag{12}$$

The multipliers of the terms $1-T_1'$ and $1-T_2'$ are not significantly different for angles $\theta \leq 50°$ and they decrease rapidly for larger angles. The numerical values of the functions $I(\theta)$ are given in Fig. 1 for nucleon energies of 300, 450 and 600 mev.

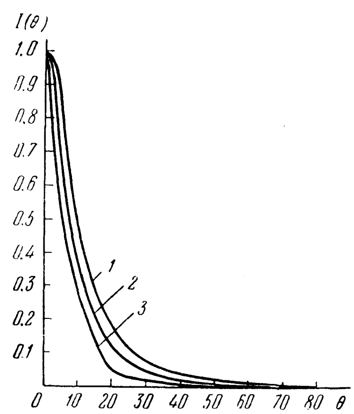

FIG. 1. Calculated values of the integral $I(\theta)$ for various neutron energies: 1 – 300, 2 – 450, 3 – 600 mev.

Even though present-day theory allows only an approximate evaluation of interference terms contributing to scattering, the result of the calculations indicates that there is a region of angles where the total $(n-d)$ scattering cross section is essentially equal to the sum of the scattering of the neutrons by free neutrons and protons. It is evident that the difference in scattering cross sections for $(n-d)$ and $(n-p)$ processes in these angle region will be equal to the scattering of neutrons by neutrons.

As already indicated, the calculation of the functions $I(\theta)$ was carried out nonrelativistically. There existed, therefore, the possibility that the inclusion of relativistic effects would contract significantly the angular region in which the calculations indicate that interference effects are important. However, since the relativistic generalization of the momentum approximation has not yet been proposed, it was decided to examine experimentally the angular region in which the scattering of nucleons by deuterons has an additive character.

3. EXPERIMENTS WITH THE PROTON BEAM

In these experiments a proton beam was scattered by D_2O, H_2O, CH_2 and C. The scattered protons were measured by a telescope consisting of three scintillation counters. In order to minimize the effect of the different energies of protons scattered by deuterons and protons, the measurements were performed with a telescope having a low energetic threshold (E_{thr} was taken close to 50 mev). Under these conditions the relative yield of protons from $(p-d)$ and $(p-p)$ scattering is practically indistinguishable from the ratio of the total cross section of $(p-d)$ scattering into the given angle to the differential $(p-p)$ cross section scattering into the same angle.

The experiments were carried out using 300 and 400 mev protons. From the measurements one obtained the ratio $[\sigma_{pd}(\theta) - \sigma_{pp}(\theta)]/\sigma_{pp}(\theta)$, which, according to the calculations, is equal to $[\sigma_{pn}(\theta) + \sigma_{int}(\theta)]/\sigma_{pp}(\theta)$. Comparison of these relations with the relative cross sections of $(n-p)$ and $(p-p)$ scatterings makes possible the determination of that angular region of scattering where $\sigma_{int}(\theta) \ll \sigma_{np}(\theta)$. The results obtained indicate that at least in the angular region

$$30° \leqslant \vartheta \leqslant 90° \text{ for } E_p = 300 \text{ mev}, \qquad (13)$$
$$25° < \vartheta \leqslant 90° \text{ for } E_p = 400 \text{ mev}$$

the total $(p-d)$ scattering cross section is equal to the sum of the $(p-p)$ and $(p-n)$ scatterings to within a few percent. For completeness, it would have been desirable to carry out similar experiments with protons of even higher energy. Such experiments were not carried out because of difficulties in excluding meson effects. However, the tests we did carry out indicate that the angular region of scattering where the interference processes in $(n-d)$ collisions contribute a significant part to the scattering of nucleons by nucleons decreases with increasing energy. For this reason we conclude that for neutrons having an energy of 590 mev, the region of additive scattering will be at least no narrower than for 400 mev neutrons. Therefore, the $(n-n)$ scattering can be determined for this particular region of angles ($30° \leq \vartheta \leq 90°$) as the difference between the scattering of neutrons by deuterons and by free protons.

4. DETERMINATION OF THE NEUTRON-NEUTRON SCATTERING CROSS SECTION

The determination of the $(n-n)$ scattering cross section was carried out by a comparison yield of high energy neutrons from $(n-d)$ and $(n-p)$ scatterings at the same angle relative to the neutron beam. For this purpose, scatterers of D_2O, H_2O, CH_2 amd C were successively interposed in the beam. The replacement of one scatterer by another was accomplished by use of a specially constructed sample changer operable from a distance, so that the accelerator did not have to be shut off during the changing process. The scattered neutrons were counted by means of a neutron telescope. This consisted of an aluminum converter, four scintillators in coincidence to register exchange protons in back of the converter, and one scintillator in front of it. The counter in front of the converter was in anti-coincidence with the others and was used to exclude charged particles arising from the scatterer. The energetic threshold of the neutron telescope was established by placing absorbers in the path of the exchange protons, and corresponded to neutrons having an energy of 470 mev. The effective energy of the incident neutrons in these experiments was 590 mev*. The monitoring of the neutron beam was carried out via a telescope T_M consisting of three scintillators placed in an auxiliary beam. The general lay-out of the experiment is given in Fig. 2.

* The energy spectrum of the neutron beam had been established earlier[12].

Establishment of the absorber thickness of the neutron telescope. In order to determine the thickness of absorber corresponding to the selected energetic threshold for the neutrons, it is necessary to know the energy spectrum of the exchange protons from the converter. The broad energy distribution of the primary neutrons and the low in-intensity of the beam complicate the investigation of this exchange proton spectrum and thus the comparison of it with the spectrum of the neutrons. For this reason we carried out measurements which made it possible to decide on the absorber thicknesses without a detailed knowledge of the form of the spectrum of the protons. In these experiments the telescope was placed at a certain angle relative to the neutron beam and the dependence of the counting rate of the neutrons scattered from free protons was determined as a function of the absorber thickness. The statistical accuracy of these experiments was insufficient to establish the differential spectrum of the exchange protons, but was sufficient to establish the absorber thickness needed to cut out the fastest protons, and therefore to determine their energy. On the assumption that such protons arise from the charge exchange of the highest energy neutrons we can conclude that this absorber thickness corresponds to an energetic threshold of the telescope equal to the maximum energy of neutrons scattered into this angle.

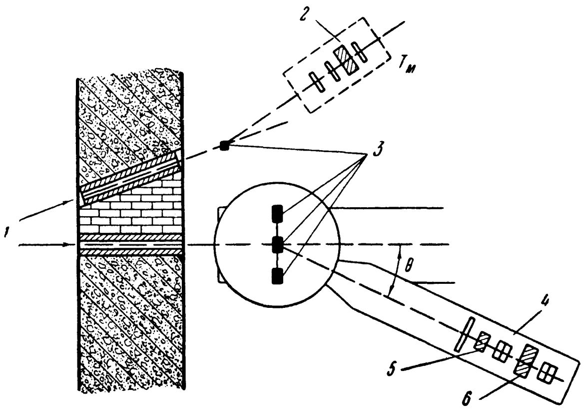

FIG. 2. Measurement of elastic ($n-n$) scattering (plan of the experiment). 1 – neutrons, 2 – monitor, 3 – scatterers, 4 – neutron telescope, 5 – converter, 6 – absorber.

Experiments carried out at scattering angles of 30, 40 and 50° showed that under the conditions of these experiments (a polychromatic spectrum of primary neutrons, the geometrics of the converter, telescope, etc.) the magnitude of the effective relative energy loss of the neutrons upon charge exchange, Δ_{eff}, was equal to

$$(E_n(\theta)_{\max} - E_p(\theta)_{\max}^{\mathrm{eff}}) / E_n(\theta)_{\max} = 0.3 \pm 0.02$$

and was practically independent of the energy in the interval from 230 to 460 mev.

The value of Δ_{eff} that was found made it possible to determine, for various scattering angles, the absorber thickness corresponding to a given energy threshold for the neutrons.

Measurements. During the measurements of the differential ($n-d$) and ($n-p$) scattering, the neutron telescope was placed at a given angle relative to a neutron beam and for each of the scatterers the difference in counting rate with and without the converter in place was determined. The difference of the effects produced by the polyethylene and by the C is proportional to the scattering cross section of neutrons by free protons, i.e.,

$$N_{\mathrm{CH_2}}(\theta) - N_{\mathrm{C}}(\theta) = B(\theta)\,\sigma_{np}(\vartheta). \tag{14}$$

The corresponding difference obtained by using D_2O and H_2O is proportional to the difference between the scattering cross section of neutrons by deuterons and by protons:

$$N_{\mathrm{D_2O}}(\theta) - N_{\mathrm{H_2O}}(\theta) = B(\theta)\,[\sigma_{nn}(\vartheta) + \sigma_{\mathrm{int}}(\vartheta)]. \tag{15}$$

The proportionality constants in the two cases are equal and depend on the geometry of the experiment, the efficiency of the neutron telescope and the neutron beam intensity.

In order to avoid the difficulty of determining the absolute values of these coefficients we measured merely the ratio of the values of Eqs. (14) and (15):

$$A(\theta) = [\sigma_{nn}(\vartheta) + \sigma_{\mathrm{int}}(\vartheta)] / \sigma_{np}(\vartheta) = S(\vartheta) / \sigma_{np}(\vartheta), \tag{16}$$

From this, using the differential cross sections for elastic ($n-p$) scattering determined by Kazarinova and Simonova[13], at the same neutron energy, we deduce the desired quantities $S(\vartheta)$.

5. RESULTS AND DISCUSSION

The measurements were carried out for scattering angles between 30 and 90° in the center-of-mass. The results obtained are listed in the Table and presented in Fig. 3.

TABLE

$\vartheta°$	$10^{27}\sigma_{nn}$ (ϑ, cm^2/sterad^{-1}
30	5.8±0.8
49	4.7±0.5
55	3.8±0.4
67	2.9±0.35
78	2.3±0.30
89	2.5±0.25

In view of the fact that the results of the experiments carried out in the proton beam (Sec. 3), and likewise the results of our calculations (Sec. 2) point to the additive character of the $(n-d)$ scattering at neutron energies larger than 400 mev for angles between 30 and 90°, we can conclude that the cross sections, $S(\theta)$, that we have determined at 590 mev, correspond to the elastic scattering of neutrons by free neutrons.

The most striking feature of the elastic neutron-neutron scattering which appears on going from 300 mev to 590 mev is that, like $(p-p)$ scattering, it gets to be strongly anisotropic. If the relative scattering $\sigma_{nn}(30°)/\sigma_{nn}(90°)$ is close to unity for 300 mev neutrons[1], then at 590 mev the $(n-n)$ scattering at 30° is 2.3 times higher that at 90°.

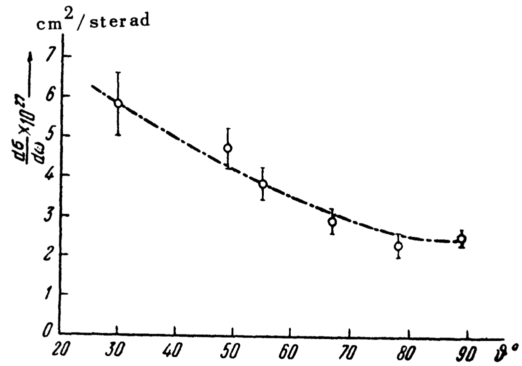

FIG. 3. Differential cross section for elastic scattering of neutrons by neutrons at 590 mev. Curve—$\sigma_{pp}(\vartheta)$ at E_p = 590 mev [14]; Points—$\sigma_{nn}(\vartheta)$ for E_n = 590 mev.

The significant anisotropy of $(n-n)$ scattering at this energy presumably reflects the increased importance of nuclear interactions in states of higher angular momentum. However, it is also possible that the anisotropy arises from the significant rise in the inelastic processes occurring when the energy is raised from 300 to 590 mev.

As is seen from Fig. 3 the differential cross section for elastic $(n-n)$ scattering at 590 mev in the angular region investigated agrees within experimental error with the corresponding $(p-p)$ scattering cross section measured by Smith *et al.* at the same energy*[14]. This fact, together with the earlier results of analogous experiments carried out with 300 mev neutrons is direct evidence for the charge symmetry of nuclear forces at high energies.

The identity of nuclear forces between two neutrons and two protons is likewise supported (within the accuracy of the experiments and electromagnetic effects) by the equality of total cross sections of $(n-d)$ and $(p-d)$ interactions at high energies[12,16]. It is obvious that, to the extent that charge symmetry of nuclear forces has been established, all known conclusions concerning the purely nuclear interactions between two protons become equally valid when applied to the interaction of neutrons with neutrons.

The comparison of the differential $(n-p)$ elastic scattering at 580 mev with the cross sections for proton-proton scattering, has been shown not to contradict the hypothesis of charge independence of nuclear forces[13]. The results of the experiments described here with neutrons make it possible to extend this conclusion to neutron-neutron scattering also.

The authors take this opportunity to express their appreciatoin to R. N. Rindin for discussion of a series of questions in the theory of $(n-d)$ scattering and to G. N. Tentiukov, I. V. Popov and L. A. Kuliukin for performing calculations.

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* Our measured σ_{nn} for ϑ = 30° and 90° agree likewise within experimental error with the corresponding $\sigma_{pp}(\vartheta)$ obtained by interpolating the curves for $\sigma_{pp}30°(E)$ and $\sigma_{pp}90°(E)$ presented in Ref. 15.

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The Theory of Collisions of Electrons with Atoms

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The collision between an electron and an arbitrary atom is considered. The wave function of the system "atom + electron" with given spin is described by the coordinate and spin functions of this system. The coordinate function of the system is constructed from the atomic and one-electron functions in such a way that it possesses in explicit form the correct symmetry properties relative to a transposition of the arguments. A system of integro-differential equations, similar to the Fock self-consistent field equations have been obtained for the one-electron coordinate functions. These equations can be transformed into integral equations. The angular variables have been separated and integral equations have been obtained for the radial one-electron functions. The integral equations can be simplified if approximate atomic functions are used. The problem is reduced to a system of Volterra integral equations which are suitable both for general investigations and for computations. An analysis of the asymptotic expressions is carried out and formulas are derived for effective cross sections.

INTRODUCTION

RECENT investigations (see, for example, Ref. 1) have revealed the unsuitability of the Born approximation (and its modification which takes exchange into account --- the method of Born-Oppenheimer) for calculation of collisions of slow electrons with atoms. Especially poor results are obtained in the calculation of the effective cross section for excitation near the threshold in those cases in which exchange effects play a role.

In a number of cases the effective excitation cross section calculated according to the Born-Oppenheimer method exceeds the theoretical limit imposed by conservation of the number of particles. For example, the effective excitation cross section of the $2S$ level of the hydrogen atom in the antisymmetric case (parallel spins) for 13.5 ev is twice the theoretical limit. For the excitation of the 2^3S level of helium at 22.5 ev the cross section computed according to Born-Oppenheimer is 1.1 times the theoretical limit and exceeds experimental values by a factor of 20.

In the methods of Born and Born-Oppenheimer, the interaction of the electron with the atom is considered weak, and therefore the wave function of the electron is taken in the form of a plane wave. Refinement of the methods of calculation is obtained by consideration of the perturbation of the electronic wave function by the strong field of the atom and by exchange interaction.

Representing the wave function of the system "atom + electron" in the form of a properly symmetrized sum of products of atomic and one-electron functions, and carrying out the computa-