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Scattering of π^+-Mesons by Hydrogen. II. Discussion and Interpretation of the Results

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(Submitted to JETP editor June 7, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 550-559 (October, 1956)

A phase analysis is made of the data obtained on scattering by hydrogen of π^+-mesons of different energies up to 307 mev. The analysis was carried out, using a high-speed electronic computer, on the assumption that the scattering process can be sufficiently accurately described by *S*- and *P*-waves [(*S-P*)-analysis], as well as on the assumption that the scattering process must be described by five parameters [(*S-P-D*)-analysis]. The energy dependence of the various phase shifts obtained for the (*S-P*)- and (*S-P-D*)-analyses are shown in the Figures. It follows from the measurements that the radius of meson-nucleon interaction is about 7×10^{-14} cm.

As is known, Fermi and others[1] analyzed the aggregation of data on scattering of π^+-mesons up to 200 mev on the assumption that only *S*- and *P*-waves are involved in the scattering and therefore the scattering processes are described by six phase shifts. In the case of positive pions and in the absence of *D*-waves the scattering is described by only three phase shifts α_3, α_{31} and α_{33} which determine the corresponding interaction in the *S*-, $P_{1/2}$- and $P_{3/2}$-states with isotopic spin 3/2.

In the present work, which is a continuation of the work described in Ref. 2, the data on scattering of positive pions are analyzed on the assumption that the contribution of *D*-states to the scattering process is neglibly small, i.e., the interaction takes place only in the *S*- and *P*-states [(*S-P*)-analysis] as well as on the assumption that the contribution of the *D*-states cannot be neglected [(*S-P-D*)-analysis]. The latter assumption is quite reasonable for such high energies as 300 mev. Besides, the data of Ref. 2 presented in Table 10 confirm this assumption to a certain extent.

In the case when *S*-, *P*- and *D*-waves contribute to the scattering of π^+-mesons, beside the above-mentioned three phase shifts α_3, α_{31} and α_{33}, the phase shifts corresponding to the *D*-states with total angular momenta 3/2 and 5/2 are different from zero and will be subsequently designated by δ_{33} and δ_{35}.

1. (S-P)-ANALYSIS

A. Phase Shifts

In the case when the interaction between the pions and hydrogen takes place only in the S- and P-states, the analysis of scattering is sufficiently simple, and the phase shifts can be determined, in the first approximation, by the graphical method[3].

Expressions for the coefficients of angular distribution in terms of the scattering phases have, as known, the following form

$$A = \sin^2\alpha_3 + \sin^2(\alpha_{33} - \alpha_{31}), \quad (1)$$

$$B = 2\sin\alpha_3\,[2\sin\alpha_{33}\cos(\alpha_{33} - \alpha_3) + \sin\alpha_{31}\cos(\alpha_{31} - \alpha_3)], \quad (2)$$

$$C = 3[\sin^2\alpha_{33} + 2\sin\alpha_{31}\sin\alpha_{33}\cos(\alpha_{33} - \alpha_{31})]. \quad (3)$$

Results of graphical analysis are given in Table I. In the same Table are given the "optimized" phase shifts[4] obtainedwith a high-speed electronic computer. In the latter case total cross sections were also used in this analysis as independent measurements.

In Table I, M denotes the quantity

$$\sum_i \left\{ \frac{f_i(\alpha_3, \alpha_{31}, \alpha_{33}) - \sigma_i}{\Delta\sigma_i} \right\}^2 .$$

Only the "resonant" solution was examined, i.e., the Fermi type solution, in which α_{33} passes through 90° in the energy region 170 to 200 mev. The basis for this procedure was discussed in the paper by Anderson *et al.*[5]. Data for 307 mev agree well with the results of Ref. 6.

TABLE I.

Phases of π^+-meson scattering by protons (in degrees)

Meson Energy, mev	Graphical Method			High-Speed Electronic Computer				
	α_3	α_{31}	α_{33}	α_3	α_{31}	α_{33}	M	Number of independent measurements
176	—11	—16	67	—10.6	—19.5	69.0	3.1	10
200	— 9	4	96	— 9.1	10.0	102.0	7.0	7
240	—14	— 8	113	—18.1	— 2.6	114.7	4.2	8
270	—21	— 6	128	—20.2	— 6.7	129.3	5.8	7
307	—24	— 8	134	—23.2	— 8.4	133.2	12.0	9

B. Behavior of S-Phase α_3 on the Assumption that Scattering is Determined only by Three Phase Shifts

It should be emphasized that in the "resonance" region the values of all three phases cannot be determined with good accuracy. As seen from Eqs. (1)-(3), in the region where the phase α_{33} approaches 90°, it is very sensitive to small changes in the coefficients A, B and C. The difficulty in determining the S-phase α_3 is due to the fact that the interference coefficient B, which primarily determines this phase, is close to zero and is not accurately known in the "resonance" region. region. At energies which considerably exceed the "resonance" value there is a possibility of determining α_{33} as well as α_3 with comparatively high accuracy; therefore, the large values of α_3 phase for 240, 270 and 307 mev deserve special study. The magnitudes of this phase shift exceed considerably those which could be expected if the linear dependence of the α_3 phase on the meson momentum as proposed by Orear[7], and which well describes experimental data for small energies, were applicable up to the energies under consideration. As known, Orear proposed the following phase equations:

$$\alpha_3 = -0.11\eta; \quad (4)$$

$$(\eta^3/\omega)\,\mathrm{ctg}\,\alpha_{33} = 8.05 - 3.8\omega; \quad (5)$$

$$\alpha_{31} = 0, \quad (6)$$

where η denotes the meson momentum in the center-of-mass system, in units μc and ω---the total energy, exclusive of the proton rest energy. It is true that not all the values of the "optimized" phase shifts satisfy Eqs. (4)-(6), but this can possibly be connected with the inaccuracies in the determination of "optimized" phases, especially phase α_3 at energies near "resonance". In particular, Orear showed that all known differential cross sections up to 220 mev are not contrary to these equations.

In view of the considerable theoretical interest in the behavior of the S-phase, it is desirable to investigate the reasons for the observed deviation, at energies 240, 270 and 307 mev, of α_3 from the linear relationship observed in investigations of a whole series of processes in the region of small meson energies. It is reasonable to ask the question, whether or not this deviation can be explained only by the inaccuracies in the obtained values of phase shifts α_3. To answer this question, the magnitudes of the differential cross sections (experimental values) given in Tables 2-6 (Ref. 2) were compared with the cross sections computed on the basis of Eqs. (4)-(6). Results of the comparison are presented in Table II.

TABLE II.

Meson Energy, mev	Number of experimental points	$M_{(4)-(6)}$
176	9	4.6
200	7	8.5
240	7	31
270	6	260
307	8	240

In the last column are given the values:

$$M_{(4)-(6)} = \sum_i \left\{ \frac{f(\alpha_3, \alpha_{33}, \theta_i) - \sigma(\theta_i)}{\Delta\sigma(\theta_i)} \right\}^2,$$

which give the degree of deviation of the curve $f(\alpha_3, \alpha_{33}, \theta_i)$, computed according to Eqs. (4)-(6), from the experimentally measured values of differential cross sections $\sigma(\theta_i)$. It can be seen from the Table that at energies 176 and 200 mev the experimental data do not contradict Eqs. (4) and (6), which is also confirmed by analyzing other data in this energy range, obtained by Orear. At higher energies, however, it is completely impossible to reconcile the experimental results with the values of phase shifts predicted by Eqs. (4) and (6).

If it is assumed that up to energies of the order of 300 mev scattering is sufficiently well described by S- and P-waves only; it must then be definitely concluded from the above that the linear dependence of the phase α_3 on the meson momentum does not hold for energies higher than 200-240 mev. On the other hand, in view of the attractiveness of such a simple dependence, it is necessary to investigate carefully whether the linear dependence of the S-phase on the meson momentum is preserved with the introduction of higher angular moments ($l = 2$), especially since our experimental data for 307 mev mesons point to the difficulty of approximating the observed angular distribution by a function of the type $a + b\cos\theta + c\cos^2\theta$ (see Table 7 of Ref. 2).

2. (S-P-D)-ANALYSIS

In the case when S-, P- and D-waves contribute to the scattering, the angular distribution can be represented in the form

$$\lambda\!\!\!^{-}{}^{-2}\frac{d\bar{\sigma}}{d\Omega} = \mathcal{A} + \mathcal{B}\cos\theta + C\cos^2\theta + \mathcal{D}\cos^3\theta + \mathcal{E}\cos^4\theta.$$

The phase shifts and the above-indicated coefficients of angular distribution are connected by the relationships:

$$\mathcal{A} = |S|^2 + |P_+ - P_-|^2 + |{}^3/{}_2 D_+ + D_-|^2 - 2I\left(S/{}^3/{}_2 D_+ + D_-\right); \tag{7}$$

$$\mathcal{B} = 2I\,(S/2P_+ + P_-) - 2I\left({}^3/{}_2 D_+ + D_-/2P_+ + P_-\right) + 6I\,(P_+ - P_-/D_+ - D_-); \tag{8}$$

$$C = |2P_+ + P_-|^2 - |P_+ - P_-|^2 + 9|D_+ - D_-|^2 - 6\left|{}^3/{}_2 D_+ + D_-\right|^2 + 6I\left(S/{}^3/{}_2 D_+ + D_-\right); \tag{9}$$

$$\mathcal{D} = 6I\left(2P_+ + P_-/{}^3/{}_2 D_+ + D_-\right) - 6I\,(P_+ - P_-/D_+ - D_-); \tag{10}$$

$$\mathscr{E} = 9\left|{}^3/_2 D_+ + D_-\right|^2 - 9\left|D^+ - D_-\right|^2. \quad (11)$$

Here we use the symbols:

$$S = e^{i\alpha_3}\cdot \sin\alpha_3;$$

$$P_- = e^{i\alpha_{31}}\cdot \sin\alpha_{31}; \quad P_+ = e^{i\alpha_{33}}\cdot \sin\alpha_{33}.$$

$$D_- = e^{i\delta_{33}}\cdot \sin\delta_{33}; \quad D_+ = e^{i\delta_{35}}\cdot \sin\delta_{35};$$

$$I(a/b) = \frac{1}{2}(ab^* + a^*b).$$

The total cross section is expressed in terms of phase shifts as follows:

$$\lambda^{-2}\frac{\sigma_t(\pi^+, p)}{4\pi} = \sin^2\alpha_3 + \sin^2\alpha_{31} + 2(\sin^2\alpha_{33} + \sin^2\delta_{33}) + 3\sin^2\delta_{35}. \quad (12)$$

From an examination of the coefficients (Table 9, Ref. 2) obtained for 307 mev, computations were made to evaluate the phases of scattering for the D-waves. For simplicity, an approximate solution of the system of Eqs. (10) and (11) was sought by assuming that $\alpha_{31} = 0$ and $\alpha_{33} = 134°$, i.e., the magnitude of the phase shift α_{33} was used as obtained graphically in the (S-P)-analysis. Such an assumption is quite natural since the perturbation introduced by the D-waves has a relatively weak influence on the phase α_{33}. The result was that the obtained values for δ_{33} and δ_{33} can satisfy the equations for all coefficients, the phase shift α_3 obtained thereby being considerably less than 24°, i.e., much less than the value obtained if only the S- and P-waves are considered*. In view of this, the experimental data of Ref. 2 and of Refs. 2-5, 7 cited in Ref. 2, were again analyzed with the aid of the high-speed electronic computer BESM of the Academy of Sciences, USSR*. Computations were made of the optimal phase shifts α_3, α, α_{33}, δ_{33} and δ_{35}, thereby only such solutions were sought in which α_3 is negative and α_{33} lies between the limits ±10° from the values given in Table I.

In the computations, the differential and total cross sections were expressed by the five phase shifts according to Eqs. (7)-(12). A minimum was sought for the quantity

$$M = \sum_i \left\{\frac{f_i(\alpha_3, \alpha_{31}, \alpha_{33}, \delta_{33}, \delta_{35}) - \sigma_i}{\Delta\sigma_i}\right\}^2,$$

where $f_i(\alpha_3, \alpha_{31}, \alpha_{33}, \delta_{33}, \delta_{35})$ denotes a function expressing the cross section in terms of phase shifts, and σ_i is the cross section as measured experimentally with an error $\Delta\sigma_i$. The values of phase shifts obtained by(S-P)-analysis with the aid of Ashkin diagrams served as the initial data. As a first approximation the phases were changed in turn, beginning with α_3, within given limits for each degree. This cycle was repeated several times, as long as the value of M has not ceased to diminish, after which the measurements were made every half degree, etc.

The "optimized" scattering phases for 270 and 307 mev obtained in this manner are presented in Table III.

As seen from this Table, the experimental values for 307 mev, which are not well satisfied by three parameters (see Table 7, Ref. 2 and Table I of this work), are well satisfied by the five phase shifts. At this energy, δ_{33} and δ_{35} are approximately equal in absolute value and have different signs. This property, as follows from Eq. (10) may explain the fact that the coefficient $\mathcal{D}$ determined by the interference of P- and D-waves is not observable in practice. As far as coefficient $\mathcal{B}$ is concerned, it is determined by the interference of (S-P)- as well as by interference of (P-D)-waves, whence it follows that for all energies up to 300 mev it is extremely difficult to determine the separate contributions of S- and D-waves.

* It should be noted that Orear already attempted some time ago to account for the effect of D-waves[8] by interpreting the experimental data on π-meson scattering in hydrogen in the energy range up to 220 mev. However, he took into consideration only the effect of the D-wave with a total angular momentum 5/2. As seen from the equations, phase δ_{35} produces a small "perturbation" in the phase shift α_3 if the value of δ_{35} is taken to be of the order of several degrees at 160-200 mev. The value $\delta_{35} = -0.013\eta^5$ proposed by Orear (which agrees with experimental data in the energy range of 100-200 mev) is definitely excluded by the results of the present work. Besides, as shown in a later work of Orear[7], all experimental data in the energy range up to 220 mev satisfy Eqs. (4)-(6) sufficiently well, a fact also confirmed by our measurements (see Table II).

* The authors are thankful to the director of the Computation Center, of the Academy of Sciences, USSR, Academician A. A. Dorodnitsyn for the opportunity to make computations on the BESM and also to the collaborators of the Computation Center, Academy of Sciences, USSR and of the Institute of Exact mechanics & Computation Techniques, Academy of Sciences, USSR, who rendered assistance in this work.

TABLE III.

"Optimum" phase shifts (in degrees)

Meson energy, mev	α_3	α_{31}	α_{33}	δ_{33}	δ_{35}	M	Number of experimental points
270	—13.6	—4.3	128.8	4.3	— 6.9	4.03	7
307	—13.0	—4.0	133.7	9.5	—10.0	3.78	9

A. D-Phases δ_{33} and δ_{35}

It is difficult, on the basis of only the "optimum" solution presented in Table III, to assert anything with certainty concerning the size of D-waves contribution at energies of the order 300 mev. Nevertheless, it is of definite interest to examine the results of such a procedure. This will be carried out in the following section, chiefly in the discussion of the S-phase behavior.

It is known that for small values of phase shifts their dependence on the meson momentum η, in the center-of-mass system, can be written in the form const $\times \eta^{2l+1}$. As a working hypothesis, therefore, the values of phase shifts for the D-waves were taken as equal (in degrees) to $\delta_{33} = 0.20\eta^5$ and $\delta_{35} = -0.21\eta^5$, where the coefficients are determined from the "optimum" values of phases δ_{33} and δ_{35} for 307 mev. Of course, such a choice is entirely arbitrary, especially in the case of phase δ_{33}, the "optimum" values of which are not regular. The dependence of phase shift δ_{35} as written above is shown in Fig. 1. In

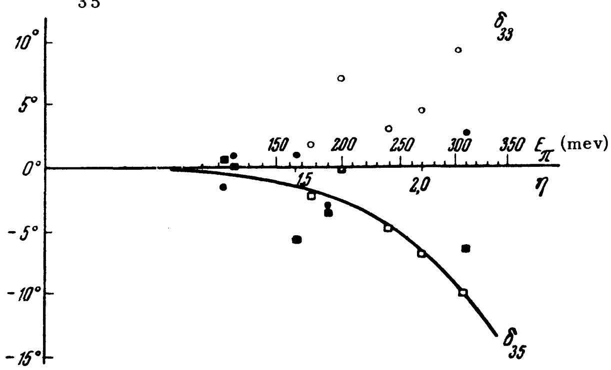

FIG. 1. "Optimized" D-wave phase shifts, obtained by (S-P-D)-analysis. δ_{33} o—data analysis, the present work; ●—data analysis, other papers. δ_{35}: □—data analysis, present work; ■—data analysis, other papers. Solid curve—$\delta_{35} = \eta - 21 \times \eta^5$.

the same Figure, for comparison, are shown the optimum phase shifts δ_{33} and δ_{35} obtained with the aid of the electronic computer. The tendency of the D-phase, particularly phase δ_{35}, to increase with the meson energy is seen from the graph. Considering the accuracy of the initial data and the nature of the mathematical problem, it can be only stated that there is no contradiction between $\delta \sim \eta^5$ dependence and the obtained data.

B. S-Phase α_3

Figure 2 shows the values of phase shifts α_3

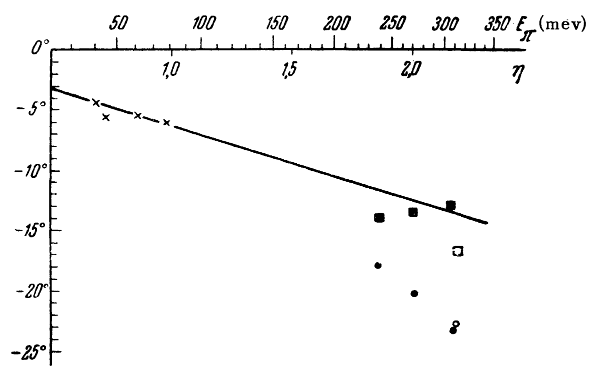

FIG. 2. Phase shift α_3 for low- and high-energy mesons. ×—data cited in Ref. 7; ■—(S-P-D)-data analysis, present work; □—(S-P-D) data analysis, Ref. 6; ●—(S-P)-analysis of data, present work; o—(S-P)-analysis of data in Ref. 6. Solid curve—dependence $\alpha_3 = 6.3°\eta$.

as a function of the meson momentum η in the energy range less than 100 and greater than 240 mev, i.e., in those energy regions where this phase can be determined relatively accurately. The straight line represents the relationship $\alpha_3 = -6.3°\eta$, which best satisfies the experimental data at low energies. The values of α_3 at low energies are indicated by crosses. At energies of

240 mev and higher, the "optimized" values of α_3, obtained on the assumption that only *S*- and *P*-waves figure in the scattering, are shown as small circles.

It is seen from the Figure that, as already pointed out above, the linear dependence of α_3 is incompatible with the values obtained in the region of high energies using the (*S*-*P*)-analysis. This indicates that at energies ~200-240 mev the meson wavelength, in the center-of-mass system, becomes comparable to the interaction radius r_0 ($r_0 \approx ƛ \sim 8 \times 10^{14}$ cm). In the same Figure, small squares indicate "optimized" values of α_3 for energies 240, 270, 307 and 310 mev when the *D*-wave is also considered in the analysis of the data. As can be seen, the values of phase α_3 are in complete agreement with the dependence obtained at low energies. This behavior of the *S*-phase shows indirectly, but, of course, does not prove, that even at energies of the order of 300 mev the contribution of the *D*-waves cannot be neglected in the evaluation of the *S*-phase.

It should be emphasized that if the assumed relationship $\delta_{33} = 0.20\eta^5$ is indeed correct, then the influence of the *D*-wave on the α_3-phase determining coefficient *B* becomes noticeable even at energies 100-150 mev. To illustrate this, the solid curve in Fig. 3 represents the energy dependence

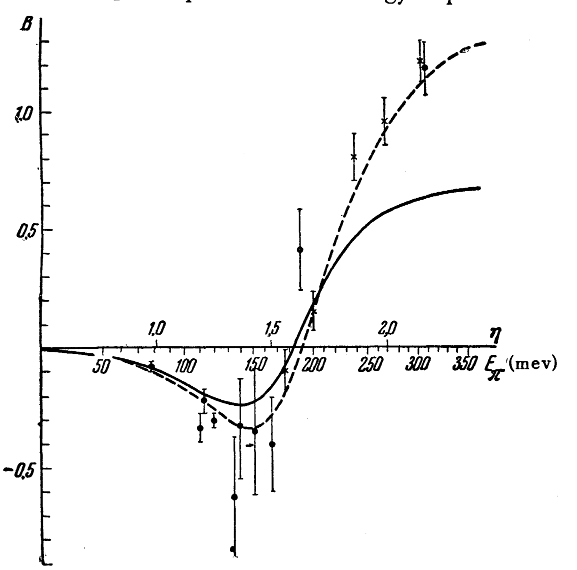

FIG. 3. Dependence on meson energy of coefficient *B*. Experimental points — values of coefficient *B* obtained by the method of least squares using as an approximation of the angular distribution the function $ƛ^{-2}\, d\sigma/d\Omega = A + B \cos\theta + C\cos^2\theta$. × — data, present work; ● — data, works of other authors.

of the coefficient *B* computed by considering the *S*- and *P*-waves only (α_3 taken as equal to 6.3η), while the dotted curve represents the energy dependence of the coefficient *B* computed by considering the *S*-, *P*- and *D*-waves ($\alpha_3 = -6.3\, M$ and $\alpha_{33} = -\alpha_{35} = +0.20\eta^5$) (all values of phase shifts are given in degrees). In the same Figure are given the values of the coefficient *B* for different energies obtained by the method of least squares, if the angular distribution is approximated by the function:

$$ƛ^{-2}\frac{d\sigma}{d\Omega} = A + B\cos\theta + C\cos^2\theta.$$

It is seen from this that the linear dependence in the form proposed by Orear, even at relatively low energies, agrees better with experimental data when the *D*-wave contribution is taken into consideration.

The linear dependence of α_3 which manifests itself when (*S*-*P*-*D*)-analysis is used, deserves consideration. It is known that phase shifts will be proportional to η^{2l+1} under the condition that the radius of the meson-nucleon interaction r_0 is smaller than the wavelength $ƛ$ of the meson in the system of the center-of-mass; therefore, linear or almost linear dependence of α_3 up to 307 mev indicates that r_0 cannot appreciably exceed $ƛ$ (307 mev) $= 6.5 \times 10^{-14}$ cm. Thus, independently of whether or not *S*- and *P*-waves alone are sufficient to describe scattering, it follows from our measurements, that the radius of meson-nucleon interaction is $\sim 7 \times 10^{-14}$cm. Strictly speaking, r_0 is here the radius of interaction between the meson and the nucleon in the *S*-state with an isotopic spin 3/2. However, it is natural to correlate this value with "dimensions" of the proton, if we consider the experimental results on scattering of electrons and protons by protons[9].

C. Phase Shift α_{31}

"Optimized" phase shifts α_{31}, previously obtained by a series of authors, and also those obtained in the present work, are extremely irregular at energies up to 220 mev so that even the sign of this phase cannot be uniquely determined. However, in the region of higher energies phase, α_{31} behaves regularly. It can be definitely concluded, from data obtained in this work, that phase α_{31} is negative and apparently less than 10° up to 310 mev. This is shown in Fig. 4, where the "optimum" values of phase α_{31}, obtained from analysis of data with consideration of S- and P-waves only, are shown as little squares, and the results which include the D-waves as well are shown as little circles. It is seen from the Figure that α_{31} is systematically decreasing when the data are expressed through the five phase shifts.

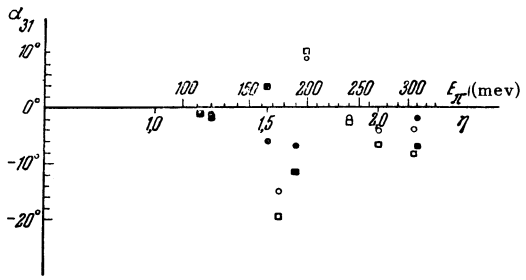

FIG. 4. Phase shift α_{31} at different meson energies. □ – (S-P)-analysis of data, present work; o – (S-P-D)-analysis of data, present work; ■ – (S-P-D)-analysis, other papers; ● – (S-P-D)-analysis of data, other papers.

D. Phase Shift α_{33}

In Fig. 5 are shown "optimized" values of phase α_{33} obtained by a series of authors (see, for example, summary 5 in Ref. 2). The "optimized" values of α_{33} obtained in this work by data analysis which includes D-waves is indicated by small circles. It should be noted that, if only S- and P-waves are considered, the phase shifts α_{33} are very little different from those shown in the Figure. Data for higher than 240 mev cannot be represented in the form of a straight line on the Chew and Low diagram[10]. The theoretical significance of this fact is not clear.

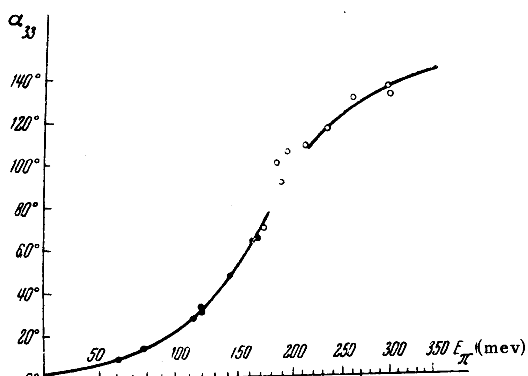

FIG. 5. Dependence of the phase shift α_{33} on π-meson energy. o – data, present work; ● – data, other papers. Solid curve – relation $\eta^3 \times \arctan \alpha_{33} = 4.3 + 0.6\eta^2 - 0.8\eta^4$.

As should be expected, at high energies the values of α_{33} given in Fig. 5 definitely exceed those obtainable from total cross section data (see Fig. 17, Ref. 2, and also the work of Lindenbaum and Yuan[11]) if it is considered that the interaction is completely determined by the state $P_{3/2}$.

Since the values of α_{33} near "resonance" cannot be determined with good accuracy, we have made an attempt to obtain an analytical dependence of this phase in a form that preserves for small energies the dependence $\alpha_{33} = 0.235\eta^3$ (Ref. 7) and conforms to the data obtained in this work for high energies. We have selected the relation[12] of the form $\eta^3 \arctan \alpha_{33} = 4.3 + 0.6\eta^2 - 0.8\eta^4$, which is shown in Fig. 5 as a solid curve. Naturally, the relation obtained in this manner may not reflect the behavior of α_{33} in the resonance region. The behavior of α_{33} in this region of energy is discussed below.

According to the data of Lindenbaum and Yuan[11], the maximum $\sigma_t(\pi^+, p)$ is found at ~ 175 mev, while from our data (see Fig. 17 of Ref. 2) this maximum is found approximately at 185 mev. Considering the experimental errors in our measurements and those in Ref. 11, it can be stated that there is no significant disagreement between the experimental results. However, the statement by Lindenbaum and Yuan that α_{33} can pass through 90° at as low an energy as 175 mev, can, in our opinion, be refuted on the basis of the behavior of the coefficient B. As it is known, considering S- and P-waves only,

$$B = 2\sin\alpha_3\,[2\sin\alpha_{33}\cdot\cos(\alpha_{33} - \alpha_3) + \sin\alpha_{31}\cos(\alpha_{31} - \alpha_3)].$$

Considering that in the energy region near "resonance", α_3 is approximately $-10°$, and α_{31} close to zero and negative, it can be stated that the value of α_{33} does not exceed 80° when $B = 0$. It is seen in Fig. 3 that B cannot be equal to zero at energy less than 176 mev, from which follows that the "resonance" energy can hardly be less than 190 mev. This conclusion is confirmed by the analysis of Bethe and others[13].

3. CAUSALITY

Based on the work of Goldberger, Myazowa and Oehme[14] on the application of the causality principle to the meson-nucleon scattering, Anderson and others[5] and also Sternheimer[15] obtained the energy dependence of the real part of the scattering amplitude below 0° from the combined data on total scattering cross sections of mesons by protons. The real part of the scattering amplitude below 0° obtained in this manner was compared by Anderson, Davidon and Kruse with its expression in terms of phase shifts up to 220 mev. New data obtained in this work permitted the extension of this comparison to 310 mev; here the phase shift values obtained by (*S-P*)-analysis were used as well as those obtained by (*S-P-D*)-analysis.

If we denote by $D_+(\eta)$ the real part of the forward scattering amplitude and by $\lambda\!\!\!^-$ and $\lambda\!\!\!^-_{cm}$ the meson wavelengths in the laboratory system and in the center-of-mass system, respectively, then

$$\frac{2\lambda\!\!\!^-}{\lambda\!\!\!^{-2}_{cm}} D_+(\eta) = \sin 2\alpha_3 + \sin 2\alpha_{31} + 2(\sin 2\alpha_{33} + \sin 2\delta_{33}) + 3\sin 2\delta_{35}.$$

In Fig. 6 are shown results of data analysis of this work and also analysis of certain π^+-p scattering data of other authors (Refs. 2-5, 7 in Ref. 2).

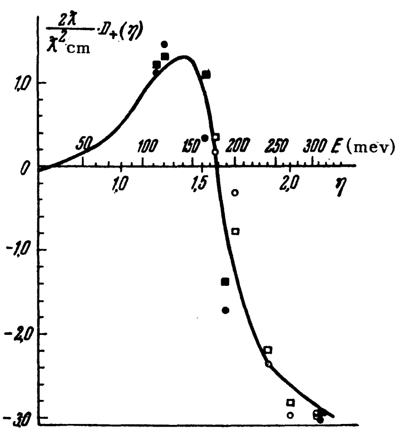

FIG. 6. Comparison of the value $(2\lambda/\lambda^2_{cm})D_+(\eta)$, computed on the basis of causality conditions with the values: $\sin 2\alpha_3 + \sin 2\alpha_{31} + 2(\sin 2\alpha_{33} + \sin \times 2\delta_{33}) + 3\sin 2\delta_{35}$ {o – present work; • – result of data analysis, other papers} $2\sin 2\alpha_3 + \sin 2\alpha_{31} + 2\sin 2\alpha_{33}$ {□ – present work; ■ – other papers}.

It is noted from the Figure that agreement between the real parts of the forward scattering amplitudes computed on the basis of the causality principle with the corresponding data obtained from the angular distribution in π^+-p scattering continues to be good up to meson energies of 310 mev. The question whether better agreement is obtained by using (*S-P*)-or (*S-P-D*)-analysis for the computed curve cannot be answered. This was to be expected since the basic characteristics of the solid curve shown in the Figure, even at energies ~300 mev, is determined in the final count by the phase shift α_{33} which is practically equal for both (*S-P*)- and (*S-P-D*)-data analysis.

CONCLUSION

Below is a summary of the contents of the preceding[2] and present papers.

1. Angular distribution of π^+-meson scattering by hydrogen at 176, 200, 240, 270 and 307 mev, as measured by the scintillation counting method, are presented in Figs. 12-16 of Ref. 2.
2. Energy dependence of the total cross section $\sigma_t(\pi^+, p)$ is shown in Fig. 17, Ref. 2.
3. With the aid of a high-speed electronic computer, a phase analysis was made of the data on the assumption that the scattering process is described by *S*- and *P*-waves only[(*S-P*)-analysis, Table I], as well as on the assumption that *S*-, *P*- and *D*-waves contribute to the scattering [(*S-P-D*)-analysis, Table III; Figs. 1-5]. Among the possible solutions only the phase group in which α_3 is negative and α_{33} passes through 90° in the energy region 170-200 mev, was examined.
4. Phase shifts α_{33} are shown in Fig. 5. They are practically the same for (*S-P*)- and (*S-P-D*)-analysis. Some evidence is presented indicating that α_{33} passes through 90° at an energy > 176 mev.
5. It is impossible to reconcile the obtained values of α_3 for 240, 270 and 307 mev with Orear's equation $\alpha_3 = -0.11\eta$, which approximates well the data for low energies, if the data is described only by the three phase shifts of *S*- and *P*-waves, α_3, α_{31} and α_{33}.
6. There is no direct indication of the need for using (*S-P-D*)-analysis for the data presentation if we discount the measurements made at 307 mev, which are approximated only with difficulty by a function of the type $a + b\cos\theta + c\cos^2\theta$.
7. The "optimized" values of phase α_{31}, which are, as known, highly irregular up to ~200 mev, become regular at higher meson energies (see Fig. 4). It can be stated that α_{31} is negative and is apparently less than 10° up to 310 mev. Phase α_{31} is definitely decreasing when the data are expressed by five phase shifts.
8. "Optimized" *D*-wave phase shifts δ_{33} and

δ_{35} are positive and negative, respectively. The phases are represented in Fig. 1 where their tendency to grow with energy can be seen.

9. With (*S*-*P*-*D*)-analysis the values of *S*-phase α_3 are considerably smaller compared with those obtained by (*S*-*P*)-analysis (see Fig. 2). If we consider the possible *D*-wave contribution, α_3 depends linearly or almost linearly on the meson momentum up to 310 mev, according to the equation determined for low energies $\alpha_3 = -0.11\eta$.

10. At any rate, independent of whether or not *S*- and *P*-waves only are sufficient to describe scattering, it follows from 5 and 9 that the radius of meson-nucleon interaction is $r_0 \sim 7 \times 10^{-14}$ cm.

11. The real part of the forward scattering amplitude, obtained from the causality condition, agrees well with its representation by phase shifts up to 310 mev.

The authors deem it a pleasure to thank I. E. Tamm and L. D. Landau for valuable advice, L. I. Lapidus for numerous discussions, and also I. B. Popov and G. N. Tentiukov for their considerable assistance in making the computations.

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ERRATA TO VOLUME 4

	reads	should read
P. 218, column 2, Eq. (10)	$\ldots \xi(\sqrt{3}+2)(2-\sqrt{3})$	$\ldots \xi(\sqrt{2}+2)/(2-\sqrt{3})\ldots$
P. 219, column 1, Eq. (11)	$\ldots (t\xi)^{\sqrt{3/2}}\ldots$	$\ldots (t\xi)]^{\sqrt{3/2}}\ldots$
P. 219, column 1, Eq. (12)	$y^2=\rho^{2/3}$	$y^2-\rho^{2/3}\gg 1$
P. 223, column 1, Eq. (45)	$\ldots (E_0\mu^{3/4})^{\sqrt{3/4}}$	$\ldots (E_0\,\mu^{3/4})^{\sqrt{3}/4}$
P. 223, column 2, Eq. (46)	$\ldots \mu^{3\sqrt{3/4}}\ldots$	$\ldots \mu^{3\sqrt{3/4}}\ldots$
P. 225, column 1, 3 lines above Eq. (1.1)	transversality	cross section
P. 225, column 1, 3 lines above Eq. (1.2)	transversality	cross section
P. 256, column 1, Eq. (37)	$\ldots \frac{55\sqrt{3}}{48}\ldots$	$\ldots \frac{55}{\sqrt{3}\,48}\ldots$
P. 289, column 2, Eq. (2)		$I=\sum_n \frac{1}{2n+1}A_n\sum_{\nu=-n}^{n}\frac{1}{1+i\omega\tau}Y_{n\nu}(n_1)Y_{n\nu}(n_2)$
P. 377, column 1, last line	$\delta_{35}=\eta-21\times\eta^5$	$\delta_{35}-21\,\eta^5$
P. 436–7	Figures 2 and 3 should be exchanged.	
P. 449, column 1, last Eq.	$\ldots Y_{lm}\,\varphi\sigma\,\alpha$	$\ldots Y_{lm}\,\varphi_{\sigma\,\alpha}$
P. 449, column 2, Eq. (12)	$\ldots W(l,j,\sigma\,1;\,j)\ldots$	$\ldots W(l,j,\sigma\,1;\,\sigma\,j)\ldots$
P. 451, column 1, Eq. (7)	$\ldots D^{(1)}_{\alpha\beta}(p,0,\lambda'\lambda)=\ldots$	$\ldots D^{(1)}_{\alpha\beta}(p,\omega_0,\lambda',\lambda)=\ldots$
P. 541, column 1, Eq. (28)	$M^{*\,\mathrm{monex}}_{++}$	$M^{*\,\mathrm{monex}}_{+}$
P. 543, column 2, Eq. (35)	$\ldots\int_{\rho^2-\tau^2+l_0^2}$	$\ldots\int_{\rho^2<\tau^2+l_0^2}\ldots$

Other Errata

Page	Column	Line	Reads	Should Read
		Volume 4		
38	1	Eq. (3)	$\ldots\frac{\pi r^2\rho^2\rho^2{}_n}{\rho_s^2},$	$\ldots\frac{\pi r^2\rho^2\rho_n}{\rho_s^2},$
196		Date of submittal	May 7, 1956	May 7, 1955
377	1	Caption for Fig. 1	$\delta_{35} = \eta - 21\cdot\eta^5$	$\delta_{35} = -21^\circ\eta^5.$
377	2	Caption for Fig. 2	$\alpha_3 = 6.3^\circ\ \eta$	$\alpha_3 = -6.3^\circ\eta$
516	1	Eq. (29)	$s^2/c^2\ldots$	s/c
516	2	Eqs. (31) and (32)	Replace $A_1 s^2/c^2$ by A_1	
497		Date of submittal	July 26, 1956	July 26, 1955
900	1	Eq. (7)	$\ldots\frac{i}{4\pi}\sum_{c,\alpha}\frac{\partial w_a(t, P)}{\partial P^\alpha}\ldots$	$\ldots 2\pi^2 i\sum_{c,\alpha}\ldots$
			(This causes a corresponding change in the numerical coefficients in the expressions that result from the calculation of the effects of the plasma particles on each other).	
804	2	Eq. (1)	$\ldots\exp\{-(\overline{T} - V')\}$	$\ldots\exp\{-(\overline{T} - V')\tau^{-1}\}$
		Volume 5		
59	1	Eq. (6)	$v_l(l\partial F_0/\partial x)+\ldots$ where E_l is the projection of the electric field E on the direction l	$\overline{(v\partial F_0/\partial x)}+\ldots$ where the bar indicates averaging over the angle θ and E_l is the projection of the electric field E along the direction l
91	2	Eq. (26)	$\Lambda = 0.84\,(1+22/A)$	$\Lambda = 0.84/(1+22/A)$
253		First line of summary	$Tl^{204,\ 206}$	$Tl^{203,\ 205}$
318	1	Figure caption	$\ldots e^2mc^2 = 2.8\cdot10^{-23}$ cm,	$\ldots e^2/mc^2 = 2.8\cdot10^{-13}$ cm,
398		Figure caption	... to a cubic relation. A series of points etc.	... to a cubic relation, and in the region 10 – 20°K to a quadratic relation. A series of points ●, coinciding with points ○, have been omitted in the region above 10°K.