on the oscilloscope screen. The duration of the sweep was 1.3×10^{-7} sec, and the minimum rise in the amplifier 2.5×10^{-9} sec. The precision of measuring the time between impulses was determined basically by the time dispersion of the photomultiplier belonging to counter C. To reduce this dispersion, we used a specially selected multiplier, type FEU-19. The exposed central part of the photocathode measured 5×12 mm, and the overall voltage was 4500 v.

The experimental error, connected with the time dispersion of a given FEU-19 tube, did not exceed 1.6×10^{-9} sec. In the photomultiplier there sometimes occurred a secondary spurious impulse following the basic impulse, but not connected with the passage of a particle through C. Such cases could imitate the decay of a K-meson when the set-up was triggered by shower particles. In view of this, it was necessary to reduce to a minimum the number of times the apparatus was triggered by showers.

In the first phase of this work, this was accomplished by including C_4 and C_4' in anticoincidence with $C_1 + C_2 + C_3$ or $C_1 + C_2 + C_3'$. The efficiency of this method was 96%. The presence of a group of Geiger-Muller counters covering C_4 and C_4' further reduced the number of times the system was triggered by showers. It is necessary to note that such a system excludes K-mesons accompanied by wide showers. Later on, the anticoincidence counters were replaced by a system of delayed coincidences, by introducing into channels C_1 and C_2 additional delay cables (1.4×10^{-8} sec).

From the resolution curve of Fig. 2 it can be seen that the probability of triggering the system by the simultaneous passage of particles through C_1, C_2, C_3 (C_3') did not exceed 0.02.

In order to take account of the secondary photomultiplier impulses, and the time displacement between pulses which resulted from the different flight times of two related particles, we measured the distribution of time intervals between pulses in counter C. In phase I of the work, this was done by including C_4 and C_4' in anticoincidence, while in phase II, we disconnected the additional delay cables. In such control investigations the number of delays in counter C was negligibly small. The results of these control experiments were included in the interpretation of the results.

The smallest energy of μ-meson decay which could also trigger the set-up was 25 mev. Therefore, we excluded cases of $\pi \rightarrow \mu + \nu$. The $\mu \rightarrow e + 2\nu$ decay could trigger the set-up, but in view of the fact that the resolution of the coincidence circuit was 4×10^{-8} sec, the probability of such an event was sufficiently small.

From among a total of 1600 cases, 64 were observed with a decay in an interval $10^{-8} - 4 \times 10^{-8}$ sec. The integral distribution of decay times is

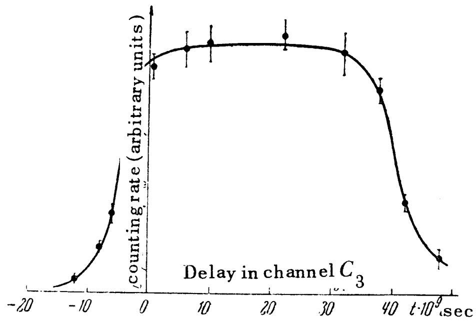

FIG. 3. Integral spectrum of K-meson decay time.

drawn in Fig. 3. It yields a mean lifetime of K-mesons of $(9.5 \pm 2.0) \times 10^{-9}$ sec, assuming a single-exponent decay. This result is in accord with Refs. 3-5.

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A Method of Investigation of Radial-Phase Oscillations of Electrons in a Synchrotron

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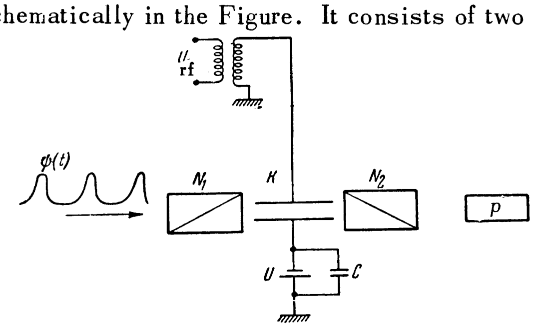
IT is well known that in electron accelerators of synchrotron type, the accelerating electrons fill part of the orbit in sort of bunch, whose azimuthal dimensions are determined by the radial-phase (synchrotron) oscillations.

At the start of the synchrotron acceleration cycle, the amplitude of radial-phase oscillation is appreciable and reaches 180°. As the electron energy E increases, the amplitude damps out as $E^{-1/4}$. (Generally speaking, this is true only for small oscillations. The larger ones decay somewhat faster[1].) However, after the electrons acquire sufficient energy there occurs intense electromagnetic radiation[2] which should, according to theory, cause noticeable synchrotron oscillations due to the quantum nature of the rediation[3]. Therefore, in the electron accelerators of high energy ($\sim$1000 mev) one should expect a diminution of the azimuthal dimensions of the electron bunch at the beginning of the acceleration cycle, and an increase later.

In this communication we propose an experimental method for investigating these radial-phase oscillations of electrons in the process of being accelerated to high energies. The method is based on the use of optical radiation from the electrons. Inasmuch as the radiation is sharply directed along the tangents to the orbit, it will be found in the form of short light pulses (thin radiation beams) whose length and shape are determined by the electron distribution, according to the amplitude of radial phase oscillation. Thus the problem of investigating the synchrotron oscillations is reduced to one of examining the light pulses. In the actual work we used a fast-acting optical shutter based on the Kerr effect in nitrobenzene (Kerr cell). Let us note that while the relaxation time in nitrobenzene is $10^{-9} - 10^{-12}$ sec (see, e.g., Refs. 4 and 5), we do not have to take it into account for the frequency of operation of the shutter, which is at best 100 *mc*.

The Kerr cell used in this work is represented schematically in the Figure. It consists of two



Schematic diagram of Kerr cell. p – radiant energy detector.

crossed Nicols, N_1 and N_2, between which is located a condenser K immersed in nitrobenzene. A steady voltage U is applied to the condenser, and at the same time, an alternating voltage U_{rf} from the generator which excites the synchrotron resonator. This makes the shutter operation frequency coincide with the frequency of appearance of the light pulses. When light rays from an electron bunch are transmitted through such a Kerr cell, the radiation detector p, with a sufficiently long time constant, will register a light beam J averaged in time. The magnitude of J depends on the time displacement θ between the shutter operation and the appearance of the light pulse. Designating by $\psi(t)$ and $f(t)$ the functions which describe the form of light pulse and the light transmission curve of the Kerr cell, $J(\theta)$ can be written as follows:

$$J(\theta) = \frac{1}{T}\int_0^T f(t-\theta)\,\psi(t)\,dt, \qquad (1)$$

where T is the period of circulation of electrons in synchrotron orbit.

The function $J(\theta)$ is found experimentally by measuring J at different values of θ between the limits 0 and T. A change in θ at high frequencies ($\sim$60 *mc*) can be accomplished by inserting delay lines between the generator of the voltage U_{rf}, and the condenser K. The function $f(t)$ is also experimentally determined. In form it coincides with the light intensity distribution, after modulation by the Kerr cell of the uninterrupted light beam. It is easy to find, using low-frequency modulation (the time t must be measured within a period T of the alternating voltage). It is necessary to make the low-frequency alternating voltage amplitude equal to the amplitude of U_{rf}. The unknown function $\psi(t)$ is found by solving integral equation (1).

Since the frequency of electron circulation in the synchrotron is known, the time in the function $\psi(t)$ can be replaced by the azimuthal angle α. Then $\psi(\alpha)$ will determine the distribution of electrons along the orbit. The alternating voltage U_{rf} should be impressed on the condenser K in the form of pulses, short with respect to the acceleration time and synchronized with the accelerating cycle. A shift of these pulses with respect to zero magnetic field of the accelerator allows a measurement of the azimuthal spread of the electron bunch at different stages of acceleration.

This method was applied to the synchrotron of the P. N. Lebedev Physical Institute, Academy of Sciences, USSR, having a maximum energy of 260 mev, an orbit radius of 81 cm, a circulation frequency of 58 mc and acceleration time of 10,000 μ-sec. Preliminary investigation shows the azimuthal spread of the electron bunch at the end of the cycle is $100 \pm 10°$.

A detailed description of the method and results of this experiment will be published at a later date.

The author expresses his sincere gratitude to Prof. P. A. Cerenkov for valuable discussions.

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On the Derivation of a Formula for the Energy Spectrum of Liquid He^4

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As is well known, Feynman [1], using a wave function of a special form, has obtained an expression for the spectrum of the elementary excitations in liquid He^4. A hydrodynamical derivation of this formula is presented below.

We shall begin with the Hamiltonian for a quantum liquid in the form[2]

$$\hat{H} = \frac{1}{2}\int m\hat{\mathbf{v}}n\hat{\mathbf{v}}d\tau + \hat{H}^1[n], \tag{1}$$

where n is the number of atoms per unit volume and $\hat{H}^1[n]$ is the velocity-independent part of the Hamiltonian. We shall assume it to be a function of n. We set $n = \bar{n} + \delta n$ and expand H in terms of the second order in δn. The first-order term drops out and we obtain

$$\hat{H} = \hat{H}^1[\bar{n}] + \frac{m\bar{n}}{2}\int \hat{\mathbf{v}}^2 d\tau + \int \varphi(\mathbf{r}, \mathbf{r}')\, \delta n \delta n' d\tau d\tau', \tag{2}$$

where φ is the second functional derivative of $\hat{H}^1$ with respect to n. Transforming now to Fourier components

$$\delta n = \sum_{\mathbf{k}} n_{\mathbf{k}} e^{i\mathbf{kr}}$$

and taking into account the equation of continuity in the form

$$\delta\dot{n} + \bar{n}\,\mathrm{div}\,\hat{\mathbf{v}} = 0, \tag{3}$$

as well as the fact that φ depends only upon $|r - r'|$, we obtain

$$\hat{H} = \hat{H}^1[\bar{n}] + \sum_{\mathbf{k}} \left(\frac{m\,|\dot{n}_{\mathbf{k}}|^2}{2\bar{n}k^2} + \frac{1}{2}\varphi_{\mathbf{k}}|n_{\mathbf{k}}|^2 V \right). \tag{4}$$

This expression has the form of a sum of the Hamiltonians of oscillators having frequencies:

$$\omega^2(\mathbf{k}) = (k^2\varphi_{\mathbf{k}}\bar{n}/m)\,V. \tag{5}$$

For the determination of φ_k we note that the average value of the potential energy of an oscillator in the ground state is equal to $\hbar\omega/4$, whence

$$\tfrac{1}{2}\varphi_{\mathbf{k}}\overline{|n_{\mathbf{k}}|^2}V = \tfrac{1}{4}\hbar\omega(\mathbf{k}). \tag{6}$$

As is well known, however, $S(k) = \overline{|n_k|^2}/\bar{n}$ is the Fourier component of the correlation function for the atoms of a liquid, which can be determined from diffraction experiments. Substituting φ_k from (6) into (5), we find for the energy of excitation

$$E(\mathbf{k}) = \hbar\omega(\mathbf{k}) = \hbar^2k^2/2mS(\mathbf{k}),$$

which agrees with Feynman's result.

In conclusion, I would like to express my thanks to L. D. Landau for his advice.

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