is thus possible that the spectrometer can be used in work on thermonuclear reactions, since hydrogen, deuterium and tritium atoms possess magnetic moments. With a neutron spectrometer it is possible to measure the cross sections of neutron interactions with nuclei and polarization effects, and to obtain the spectra of neutron sources. The measurable energy range goes from thermal ranges to the order of 10 mev. For large energies the size of the apparatus and the power of the generator are increased, but pulsed operation is possible.

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Transformation Properties of the Electron-Positron Field Amplitudes

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One of the conditions for the relativistic invariance of a theory is that the state vectors Φ of the field transform according to some representation of the Lorentz group. Since any vector Φ can be obtained by operating on the vacuum state Φ_0 with creation and annihilation operators, the transformation of Φ reduces to the transformations of Φ_0 and of the creation operators, that is, of the field amplitudes. It will be shown that the field amplitudes do not transform according to the spinor representations of the Lorentz group, and that the amplitudes corresponding to electron and positron states transform independently of each other according to the same representations. We shall consider the inhomogeneous Lorentz group $\mathfrak{L}$, including space reflections but not time reflections. The question of time reflections is more complicated and requires special consideration. It is clear that the vacuum state Φ_0 is invariant under the group $\mathfrak{L}$. In order to derive the transformation properties of the field amplitudes we shall write the field operator $\psi(x)$ in the interaction representation in the following relativistically invariant form (in the following we make use of Feynman's[1] notation and set $\hbar = c = 1$)

$$\psi(x) = (2\pi)^{-3/2} \times \int \{u(p)\, a(p)\, e^{-ipx} + v(p)\, b^{+}(p)\, e^{ipx}\}\, d\Gamma. \quad (1)$$

The integration in (1) is taken over the hypersurface given by $p^2 = m^2$, $p_4 \equiv \epsilon > 0$; $d\Gamma = (m/\epsilon)\, d^3 p$ is the invariant element of the hypersurface; $u(p) \equiv \| u_\mu^\alpha(p) \|$ is a matrix of four rows and two columns given by the two solutions of the Dirac equation for positive energy, normalized according to the condition $\bar{u}^\alpha u^\beta = \delta_{\alpha\beta}$; $v(p)$ is the charge conjugate matrix; $a(p) \equiv \binom{a_1(p)}{a_2(p)}$ is the column operator consisting of electron annihilation operators, and $b^+(p)$ is the analogous column operator of positron creation operators. The operators a, b are normalized by the invariant δ-function on Γ, that is, they satisfy the anticommutation relations

$$[a(p), a^+(q)]_+ = \Delta(p-q), \text{ и т. д. } (\Delta(p) = (\varepsilon/m)\,\delta(\mathbf{p})).$$

Let us first consider the homogeneous Lorentz transformation. (Similar, though somewhat simpler, considerations hold for translations.) Under a transformation L the spinor field $\psi(x)$ transforms according to

$$\psi(x) \rightarrow \psi'(x) = S_L \psi(L^{-1}x), \quad (2)$$

where S_L is the spinor representation of the Lorentz group, which satisfies the condition $S_{L_1} S_{L_2} = S_{L_1 L_2}$. It is easy to see that the transformation (2) is equivalent to the following system of transformations in p-space:

$$u(p) \rightarrow S_L u(L^{-1}p); \qquad a(p) \rightarrow a(L^{-1}p) \quad (3)$$

with similar systems of transformations for v and b^+. From the relativistic invariance of the Dirac equation, however, it follows that

$$S_L u(L^{-1}p) = u(p)\, Z_L(p), \quad (4)$$

where the second degree matrix $Z_L(p)$ is determined by the relation $Z_L(p) = \bar{u}(p) S_L u(L^{-1}p)$.

The following properties of the matrix $Z_L(p)$ are easily established:

1) $Z_L(p)$ generates a representation of the Lorentz group, that is,

$$Z_{L_1}(p)\, Z_{L_2}(L_1^{-1}p) = Z_{L_1 L_2}(p). \quad (5)$$

2) The representation (5) is unitary

$$Z_L^+(p) = Z_L^{-1}(p) = Z_{L^{-1}}(L^{-1}p).$$

Shirokov[2] has found a similar representation of the Lorentz group.

In view of (4), the transformation (3) can be written in the form

$$u(p) \to u(p); \qquad a(p) \to Z_L(p)\, a(L^{-1}p).$$

Carrying through similar considerations for the amplitudes b (making use of the charge conjugation of the u and v), we arrive at the following result: the transformation (2) reduces to the following system of transformations for the field amplitudes:

$$a(p) \to Z_L(p)\, a(L^{-1}p); \qquad b(p) \to Z_L(p)\, d(L^{-1}p). \quad (6)$$

Under this transformation the quantities $u(p)$ and $v(p)$ are not transformed at all. The transformation for the conjugate amplitudes a^+ and b^+ follow uniquely from (6).

The transformaion (6) can be represented in operator form. Let A be an operator in the space of the amplitudes $a_\alpha(p)$, $b_\alpha(p)$. The symbol $\langle A\rangle$ shall denote the expression

$$\langle A\rangle = \sum_{\alpha,\beta=1}^{4}{}' \int a_\alpha^+(p)\,(p\alpha\,|\,A\,|\,q\beta)\, a_\beta(q)\, d\Gamma_p d\Gamma_q$$

(in order to condense the notation we have put $a_3 = b_4$, $a_4 = b_3$).

We arrive at the following relations:

$$e^{-\langle A\rangle} a_\alpha(p)\, e^{\langle A\rangle} = \sum_{\beta=1}^{4} \int (p\alpha\,|\,e^{A}|\,q\beta)\, a_\beta(q)\, d\Gamma_q; \quad (7)$$

$$e^{-\langle A\rangle} a_\alpha^+(p)\, e^{\langle A\rangle} = \sum_{\beta=1}^{4} \int a_\beta^+(q)\,(q\beta\,|\,e^{-A}\,|\,p\alpha)\, d\Gamma_q.$$

If, for A we take the operator $(\frac{1}{2})\, i\epsilon_{\mu\nu} M_{\mu\nu}$, where $M_{\mu\nu}$ is the four-dimensional angular momentum for the representation (5), then (7) will describe the proper Lorentz transformations (6). Space reflections can be handled in a similar way.

The transformation corresponding to a translation can be put in the form of Eq. (7), setting

$$\langle A\rangle = i\alpha_\mu P_\mu = i\alpha_\mu \int p_\mu \{a^+(p)\, a(p) + b^+(p)\, b(p)\}\, d\Gamma.$$

In addition to the Lorentz transformation many other transformations can be represented in the form of Eq. (7); examples are charge conjugation, gauge transformations, etc. For instance, for a gauge transformation (with a constant phase α) we must take

$$\langle A\rangle = i\alpha Q = i\alpha \int \{a^+(p)\, a(p) - b^+(p)\, b(p)\}\, d\Gamma.$$

[1] R. P. Feynman, Phys. Rev., **76**, 749 (1949).

[2] Iu. M. Shirokov, Dokl. Akad. Nauk SSSR **94**, 857 (1954); 99, 137 (1954).

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Determination of the Spins of K-Particles and Hyperons

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At present very little is known about the spin values of heavy mesons and hyperons. Some information can be obtained from a detailed study of the decay products[1-3]. But even for the τ-particles, which occupy the most privileged position among all the "strange" particles, there are inconsistent data[4,5].

As "strange" particles cannot be transformed into "common" particles as the result of a strong interaction, they are not produced with large probability in reactions analogous to the reaction $p + p \rightleftarrows d + \pi$, by means of which it would be possible to determine the spin of one unknown particle. The reaction

$$K^- + d \rightleftarrows \Sigma^- + p, \quad (1)$$

which is analogous to the reaction $\pi^- + d \leftrightarrows n + n$, is not forbidden for slow scalar K-particles, as is was for π-mesons, inasmuch as Σ^- and p do not appear to be identical particles. Lee[6] has shown that the reaction (1) can be used for determining the spins of the new particles.

It is not difficult to see that for the same purposes the analogous reactions with various other nuclei can be used together with (1), for example,

$$K^- + H^3 \rightleftarrows \Sigma^- + d, \quad K^- + He_2^4 \rightleftarrows \Sigma^- + He_2^3, \quad (2)$$

$$K^- + He_2^4 \rightleftarrows \Lambda^0 + H^3 \text{ , etc.} \quad (3)$$