presence of the cathode, which gives an appreciable current in cold emission for an electric field strength $\sim 10^6$ v/cm and a temperature of $\sim 20°$ K. As for the strength of the magnetic field, we refer to what recently has been published in Ref. 5, whose authors write of producing an intensity $H = 6.5 \times 10^5$ G with a pulse lasting from 1 to 10 milliseconds, still far from the value of the threshold needed with existing technology. In spite of the fact that the time interval between such pulses far exceeds the normal interval between operating pulses of contemporary accelerators, the gain in the average current of polarized electrons of high-energy and the value of the polarization may prove to be much greater in comparison with other methods.

Depolarization of the electrons in the acceleration process may be examined for each concrete case with the aid of the formula, derived in Ref. 6.

The difference in the penetrability of the potential barrier, represented in the Figure, for electrons with different spin orientations may also be used for measuring the polarization of the electron beam*‐ incident on the surface of the metal. In this case, the small work function and low temperature are not necessary; the potential of the measuring electrode must be chosen so that the usual classical "turning point" for electrons is somewhat lower than the peak of the barrier.

To summarize, it is possible to say that a practical realization of the source proposed here is not easy to carry out. The practicality is determined mainly by the magnitude of the circular polarization of bremsstrahlung from polarized electrons. This question is being studied in detail at the present time.

The author sincerely thanks K. D. Sinel'nikov and I. M. Lifshitz for valuable discussions.

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* This possibility has been shown by K. D. Sinel'-nikov.

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Spectral Representation of Green's Function in the Nonrelativistic Many-Body Problem

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IT was previously shown[1,2] that the quantum Green's function gives very extensive information about the properties of systems of interacting particles. In this connection there is interest in investigating the general properties of Green's functions which are independent of any method of approximation. In work in the quantum theory of fields[3,4] the fruitfulness of the spectral approach to this problem was shown. In nonrelativistic theory there are certain complications, connected with the absence of Lorentz invariance; however, as will be shown below, there also exist analogous spectral theorems. For concreteness we will speak about a system of many electrons; the transition to a system of Bose particles will not present any difficulties.

We denote by $\Psi^*(x)$ and $\Psi(x)$ creation and destruction operators for electrons (in the Heisenberg representation); the symbol $<..>_0$ will mean the average over the ground state of the system of interacting particles, Φ_0 is the wave function of the ground state, $\Phi_{\nu,E}$ are the wave functions for the excited states, with characteristic energy E and (possibly) some other quantum number ν (the energy of the ground state is assumed to be zero). By definition

$$G_+(x, y) = i \langle \Psi(x) \Psi^*(y) \rangle_0; \quad x = \{x, x_0\}, x_0 = t, \quad (1)$$

$$G_c(x, y) = i \langle T\{\Psi(x)\Psi^*(y)\} \rangle_0 \quad (2)$$
$$= \theta(x_0 - y_0) G_+(x, y) + \theta(-x_0 + y_0) G_+(y, x).$$

If we assume

$$G_+(x, y) = i(\Phi_0, \Psi(x)\Phi_0)(\Phi_0, \Psi^*(y)\Phi_0) \quad (3)$$
$$+ i \sum_\nu \int dE (\Phi_0, \Psi(x)\Phi_{\nu,E})(\Phi_{\nu,E}\Psi^*(y)\Phi_0)$$

and use the equation of motion

$$dL/dx_0 = i(HL - LH) \quad (4)$$

(L is any operator not explicitly time-dependent, H is the total Hamiltonian of the system, $\hbar = 1$), we arrive at

$$(\Phi_0, \Psi(x)\Phi_{\nu,E}) = e^{-iEx_0}\varphi_{\nu,E}(x),$$

from which

$$G_+(x, y) = i\sum_\nu \int dE e^{-iEx_0} F_{\nu,E}(x, y), \quad (5)$$

where $F_{\nu,E}(x, y) = \varphi(x)\varphi^*(y)$ [obviously, $\Phi_0, \Psi(x)\Phi_0 = 0$]. We note that, generally speaking, $F_{\nu,E}(x, y)$ is some generalized function. In particular, it may include a δ-function singularity, so that actually the integral over E also contains a sum over possible discrete states. The spectral representation is obtained from (5) by multiplying by $(1/2\pi)\exp\{ip_0(x_0 - y_0)\}$ and integrating over $x_0 - y_0$ (to make a Fourier transformation over the spatial variables is not always convenient; because of the presence of external fields we may not have spatial uniformity). Setting

$$G_c(p_0; x, y) \quad (6)$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ip_0(x_0 - y_0)} G_c(x, y) d(x_0 - y_0),$$

we derive by virtue of (2) and (5)

$$G_c(p_0; x, y) = i \quad (7)$$

$$\sum_\nu \int dE \{\delta_+(p_0 - E)F_{\nu,E}(x, y) - \delta_-(p_0 - E)F^*_{\nu,E}(y, x)\}.$$

We emphasize that the limits of integration are not fixed beforehand, but are determined by the nature of the system (the structure of the function $F_{\nu,E}$). In particular, for an ideal Fermi gas in a completely degenerate state an elementary calculation gives

$$G_c(p_0, x, y) = \frac{i}{2\pi^2}\frac{m}{|x - y|} \quad (8)$$

$$\times \left\{\int_0^\infty dE \sin|x - y|\sqrt{2m(E + W_F)}\,\delta_+(p_0 - E) - \int_{-W_F}^0 dE \sin|x - y|\sqrt{2m(E + W_F)}\,\delta_-(p_0 - E)\right\}$$

(W_F is the limiting Fermi energy). In this example there is clearly seen the well-known technical disadvantage of using the single particle Green's function for studying energy spectra: it gives the spectrum of "covered" electrons" without including the conservation of the number of particles in the intermediate states. In principle, of course, the excitation spectrum is easy to get by finding the appropriate energy differences; however, certain essential peculiarities of the spectra may thereby easily be lost if we calculate G_c by use of any approximate method. (This remark pertains to the method of mass operators developed in Ref. 1.) In this connection it is expedient to introduce another Green's function, in which the "pair" character of the phenomenon of excitation would be shown explicitly; such as, for example, the function

$$K(x, y) = \theta(x_0 - y_0)\langle\rho(x)\rho(y)\rangle_0; \quad (9)$$

$$\rho(x) = \Psi^*(x)\Psi(x).$$

If we assume, in agreement with Eq. (4),

$$(\Phi_0, \rho(x)\Phi_{\nu, E}) \quad (10)$$

$$= e^{-iEx_0}r_{\nu, E}(x); \quad r(x)r(y) = R(x, y),$$

we obtain

$$K(x, y) = R_0(x, y) \quad (11)$$

$$+ \sum_\nu \int dE e^{-iE(x_0 - y_0)}R_{\nu, E}(x, y)\theta(x_0 - y_0),$$

and, in spectral form,

$$K(p_0; x, y) = \delta_+(p_0)R_0(x, y) \quad (12)$$

$$+ \sum_\nu \int dE \delta_+(p_0 - E)R_{\nu, E}(x, y).$$

The frequencies that appear in (11) and (12) give the spectrum of the elementary excitations; in particular, by definition, $E \geq 0$ (at the lower limit we have the equal sign or the inequality, depending on whether there is an energy gap in the system). It is possible to show that in calculating $K_4(x, y)$ certain difficulties arise, connected with the necessity of defining the value of the Green's function, $G_c(x, y)$, for equal arguments. In a problem of the present type, however, this difficulty is illusory, and in fact the quantity $G_c(x, x)$ is easy to relate to the number density of particles n. So, for example, for a degenerate Fermi gas, we derive (for $x_0 > y_0$):

$$K(x, y) = n^2 + G_c(y, x) G_c(x, y) \qquad (13)$$

$$= n^2 + \frac{1}{(2\pi)^6} \int dk dk' \exp\left\{i\,(k - k',\, x - y)\right.$$

$$\left. - i\left(\frac{k^2}{2m} - \frac{k'^2}{2m}\right)(x_0 - y_0)\right\}$$

$$\times \theta\left(\frac{k^2}{2m} - W_F\right)\theta\left(-\frac{k'^2}{2m} + W_F\right).$$

A spectral representation analogous to (7) and (12) is easily derived for all other Green's functions.

We wish to express our great thanks to N. N. Bogoliubov for discussion of this work and for valuable advice.

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Concerning the Existence of a Transition Layer on a Liquid Surface

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IN connection with the recent appearance of some work of Sivukhin,[1] we present several results of our investigations.

The black dots in Figs. 1 and 2 show our values for the ellipticity ρ of reflected light for four wavelengths and for a few homogeneous nonabsorbing liquids, for light incident at Brewster's angle. Figure 3 shows the dependence of the phase difference between the components of the reflected light on the angle of incidence φ for o-xylol at $\lambda = 5460$ A.

The general method is described in an earlier work,[2] and the experimental details, elsewhere.[3] Various methods of calculation, based on the assump-
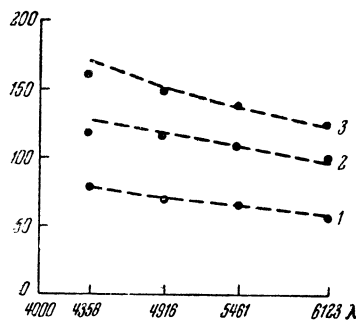

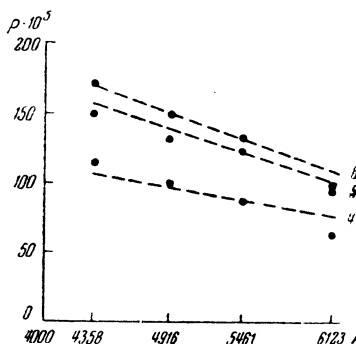FIG. 1. The dependence of ellipticity on wavelength: 1—ethylene glycol, 2—m—xylol, 3—CCl$_4$.

FIG. 2. The dependence of the ellipticity on wavelength for nitro benzene: 5—cyclohexanol, 6—chlorobenzene.
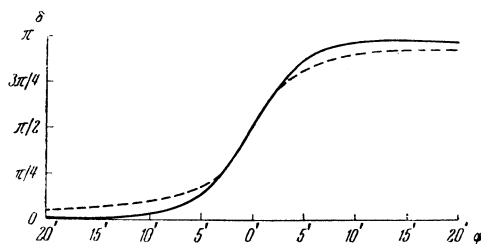

FIG. 3. The dependence of the phase difference between the components of the reflected light on the angle of incidence (Brewster's angle is taken as zero).

tion that there exists a transition layer on the surface, with no other hypotheses and without making any special assumptions as to the molecular structure of the layer, lead to the expressions[1,2]

$$\rho = (\pi/\lambda)\, V \sqrt{n^2 + 1}\,(\gamma_z - \gamma_x), \qquad (1)$$

$$\mathrm{tg}\,\delta = 4\,(\pi/\lambda)\,(\gamma_z - \gamma_x) \qquad (2)$$

$$\cos\varphi\,\sin^2\varphi/(\sin^2\varphi - \cos^2\psi),$$