zero the matrix element $(\psi_f| S | \psi_i)$ vanishes. In fact, by assumption, the wave function ψ_i of the initial state transforms according to the representation of the group P_n to which corresponds Young's diagram in the form of two horizontal lines of equal length. Since a charged π-meson appears in the final state, the isotopic spin changes and with it the charge symmetry of the nucleus. The operator S is symmetric under arbitrary permutation of the charge variables. In agreement with the above, the matrix element $(\psi_f | S | \psi_i)$ vanishes.*

From the selection rule (2) and the usual rules for combining angular momenta, it follows that the isotopic spin of the nucleus in the final state may be either 1 or 2. Making use of the well-known expressions for the matrix elements of a vector (see for instance Landau and Lifshitz[2]), it can be shown that the photoproduction cross section for charged mesons of opposite signs for both possible values of the isotropic spin of the nucleus in the final state are given by

$$d\sigma_1(\pi^+) = d\sigma_1(\pi^-), \qquad d\sigma_2(\pi^+) = d\sigma_2(\pi^-). \quad (3)$$

Summing these cross sections over the possible final states, we obtain

$$d\sigma(\pi^+) = d\sigma(\pi^-).$$

The experimental values of $r = d\sigma(\pi^-)/ d\sigma(\pi^+)$ for light nuclei (D, C^{12}, N^{14}, O^{16}) are, within the limits of experimental error, equal to unity.[3]

The decrease of r as the charge increases is evidently explained by the capture of negative π-mesons by nuclei.

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*The matrix element $(\psi_f|S|\psi_i)$ vanishes identically if we limit ourselves to dipole interactions. Here, however, no such limitation is made.

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[2] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, Part I, GTTI (1948) p. 111.

[3] R. M. Littauer add D. Walker, Phys. Rev. 82. 746 (1951); 86, 838 (1952).

Translated by E. J. Saletan
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The Magnetic Field in the Two-dimensional Motion of a Conducting Turbulent Liquid

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THE problem of magnetic fields arising spontaneously in the motion of a liquid has been considered by Batchelor.[1] He came to the conclusion that the magnetic field increases without limit for sufficient conductivity in the given field. His conclusion was based on nonrigorous considerations of the analogy between the magnetic field and a velocity vortex.

In the present work, the particular case of two-dimensional motion is considered: $v_z = 0$, v_x and v_y depend only on x and y; the liquid is incompressible, div $\mathbf{v} = 0$. In this case, we have succeeded in treating the problem rigorously. The results differ essentially from the conclusions of Batchelor: In two-dimensional motion in the absence of external fields, the initial magnetic field can increase no more than a definite number of times, and thereafter certainly dies out. In the presence of external fields on the boundaries of the region of motion, the fields in the moving liquid in the stationary state are proportional to the external fields. In the absence of mean regular flow the turbulently moving, conducting liquid behaves as a diamagnet with permeability μ inversely proportional to the intensity of the turbulent mixing.

Following Batchelor, we set up the equation in the quasi-stationary approximation, neglecting the displacement current and the density of free charges. We employ $c = 1$ and the Heaviside system (without 4π), φ = scalar potential, $\mathbf{A}$ = vector potential, div $\mathbf{A} = 0$, $\mathbf{J}$ = current, div $\mathbf{J} = 0$, the specific resistance of the liquid is r.

The equations have the form:

$$r\mathbf{j} = \mathbf{E} + \mathbf{v}\times\mathbf{H};\ \mathbf{H} = \mathrm{curl}\ \mathbf{A};\ \mathbf{E} = (\partial \mathbf{A}/\partial t) - \nabla\varphi; \quad (1)$$

$$\mathbf{J} = \mathrm{curl}\,\nabla^2 \mathbf{A}.$$

It follows from this equation that

$$(\partial \mathbf{A}/\partial t) + \mathbf{v}\times \mathrm{curl}\ \mathbf{A} = r\nabla^2 \mathbf{A} + \nabla\varphi. \quad (2)$$

Taking the curl of (2), we obtain an equation which reduces to Batchelor's:

$$(\partial \mathbf{H}/\partial t) + \mathrm{curl}\,(\mathbf{v} \times \mathbf{H}) = r\nabla^2 \mathbf{H}. \qquad (3)$$

We now go to the case of two-dimensional motion of an incompressible fluid. The equation for H_z can be separated; employing div $\mathbf{v} = 0$, we get

$$\frac{\partial H_z}{\partial t} + v_x \frac{\partial H_z}{\partial x} + v_y \frac{\partial H}{\partial y} = \frac{dH_z}{dt} = r\nabla^2 H_z. \quad (4)$$

Equation (4) is entirely analogous to the heat conduction equation in a moving liquid.

It is easy to convince oneself that in the absence of external fields, H_z only diminishes. If at any point H_z is maximal, then in the neighborhood, $\nabla^2 H_z < 0$, so that $dHz/dt < 0$, i.e., the maximum is eliminated. In order to be convinced of this cancellation, we set $H_z = 0$ at infinity or on the boundaries of the region and find, integrating by parts,

$$\frac{d}{dt}\int H_z^2\,dV = \int H_z \frac{\partial H_z}{\partial t}\,dV \qquad (5)$$
$$= -r\int (\nabla H_z)^2\,dV.$$

We consider the field which is obtained after the damping out of H_z. In this field, $j = j_z$, $\varphi = 0$, $E = E_z$. A has only one component, A_z, which we denote as a in what follows. The two-dimensional vector of the magnetic field with components H_x and H_y we denote by h. Expanding Eq. (3), we get the equation for $\mathbf{h}$:

$$(\partial \mathbf{h}/\partial t) + (\mathbf{v}\cdot\nabla)\mathbf{h}$$
$$= (d\mathbf{h}/dt) = (\mathbf{h}\cdot\nabla)\mathbf{v} + r\nabla^2\mathbf{h}. \qquad (6)$$

Along with the dissipation term $r\nabla^2\mathbf{h}$ this equation contains the terms $(\mathbf{h}\cdot\nabla)\mathbf{v}$ which describe the growth of the magnetic field for increase in length of the magnetic force lines, noted by Batchelor. Thus it is impossible to confirm that $\mathbf{h}$ or $\mathbf{h}^2$ is canceled and that $\int h^2\,dV$ decreases monotonically.

It appears basic for us that the equation for a has the form of the heat conduction equation and a cannot increase (see above, the behavior of H_z).

In the two-dimensional case, for $H_z = 0$, we easily obtain from (2):

$$(\partial a/\partial t) + (\mathbf{v}\cdot\nabla)\,a = da/dt = r\nabla^2 a. \qquad (7)$$

Hence, in the absence of external fields,

$$\frac{d}{dt}\int a^2 dV = -r\int (\nabla a)^2\,dV = -r\int h^2 dV. \quad (8)$$

We represent the distribution of a, which is characterized by amplitude a_0 and length dimension L, which exceeds the maximum scale of the turbulent pulsation l. We denote the pulsating velocity by u; the turbulent coefficient of diffusion, the coefficient of thermal conductivity and the effective turbulent kinematic viscosity are expressed by the formula $\varkappa = ul$. For initial uniform distribution of a, evidently,

$$(\sqrt{\overline{h^2}})_0 \approx a_0 / L. \qquad (9)$$

The macroscopic leveling of a in the process of turbulent exchange is characterized by a time of the order $\tau = L^2/n = L^2/ul$, so that, with account of Eq. (8),

$$r\overline{h^2} = d\overline{a^2}/dt = -\overline{a^2}/\tau = -(ul/L^2)\,\overline{a^2}. \qquad (10)$$

Thus, in a time of the order τ, the mean field increases to the amoont

$$\sqrt{\overline{h^2}} = (a_0/L)\sqrt{ul/r} = \sqrt{ul/r}\,(\sqrt{\overline{h^2}})_0, \qquad (11)$$

but then both $\overline{a^2}$ and $\overline{h^2}$ fall exponentially with a period of the order of τ. The quantity ul/r is similar to the Reynolds number ul/ν (ν = molecular kinematic viscosity). We denote it by Rem $= ul/r^2$. As is evident from (11), the increase in the field is limited to $\sqrt{\mathrm{Rem}}\; h_0$; in the two-dimensional case the possibility is excluded of the increase of the field from the arbitrarily small (for example, depending on fluctuations) to a finite value for a finite r (even for small ν).

Let us consider the turbulent motion in a closed region of size L, on the boundaries of which $v_n = 0$ (the index n = normal to the surface S bounding the region) in the presence of external fields. It follows from Eq. (7) that the quantity $h_t = (\mathrm{grad}\,a)_n$ should be continuous at the boundary. From the analogy between (7) and turbulent heat conduction, noting that r plays the role of molecular thermal conductivity and considering the flow a as a heat flow, we find:

$$r\,(\mathrm{grad}\,a)_n|_S \approx a_0\varkappa / L, \qquad (12)$$

where a_0 is of the order of magnitude of the difference in a on the different boundaries of the medium. Consequently, the mean (not the mean square!) value of the field in the volume $\bar{h} \approx a_0 / L$ and the value of the tangential field at the boundary of the region, where the liquid is at rest, are related by the following:

$$\bar{h} = (r / \varkappa) \, h_t |_s. \qquad (13)$$

By analogy with macroscopic electrodynamics, we shall consider the flow in the turbulent liquid as molecular flow, the neutralized true field $\bar{h}$ we denote by B and introduce $\vec{\mathcal{H}}$; here curl $\vec{\mathcal{H}} = 0$ in the region where there are no irregularities of flow dependent on the turbulence. From Eq. (13) we get $B = (r/\varkappa)\, \vec{\mathcal{H}} = \vec{\mathcal{H}}/\text{Rem}$. Thus, macroscopically, the turbulent conducting liquid behaves as a diamagnet * with very small permeability $\mu \sim /\text{Rem}$.

Apropos of the analogy noted by Batchelor between the vortex velocity and the magnetic field, it should be noted that for the realization of an actually stationary turbulence, a supply of mechanical energy is necessary. The supply of energy comes about either at the expense of nonpotential volume forces or at the expense of the motion of the surfaces bounding the liquid. With consideration of these factors, the set of equations and boundary conditions for the vortex are not identical to the equation and boundary conditions for a magnetic field in the absence of external magnetic fields and attendant electromotive forces.

Once again, we note that direct step-by-step consideration of the three-dimensional case has, up to the present time, not been possible, and the question of the growth of field in the three-dimensional case remains unknown.

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*Czada[3] has come to the conclusion that a conducting turbulent liquid is a paramagnet with large permeability. His analysis, based on a consideration of the damping of the field (p. 140) and of the energy of the field (p. 143) is not convincing, since the energy of the pulsating components of the field can be regarded macroscopically as a part of the turbulent energy, but the damping of the field can be connected not only with the conductivity, but also with a transition of the energy into mechanical form.

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Translated by R. T. Beyer
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On the Composition of Primary Cosmic Rays

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WE consider the question of the chemical constitution of primary cosmic rays in the framework of the theory of the origin of cosmic rays developed in Refs. 1-3 and in many previous works discussed there. The concentration of cosmic particles of type i, which are designated by $N_i(\mathbf{r}, t)$, can be found from the system of equations

$$\partial N_i/\partial t = \nabla(D_i \nabla N_i) - N_i/T_i + \sum_{j>i} p_{ij} N_j/T_j + q_i, \quad (1)$$

where $q_i(\mathbf{r}, t)$ is the number of particles of type i per unit volume per unit of time, injected into interstellar space by the source of cosmic rays (the envelopes of supernovae and novae), $D_i(\mathbf{r})$ is the diffusion coefficient of cosmic rays in interstellar space, $T_i(\mathbf{r})$ is the lifetime of particles of type i, until their break-up in collisions with atomic nuclei of the interstellar medium (i.e., principally protons) and p_{ij} is the number of particles of type i, formed by the splitting of particles of type j. In case of nuclei, one can almost always assume that the collisions lead to the formation of nuclei of another sort, while the energy per nucleon of the primary and secondary nucleons are the same. Similarly, it is possible to understand N_i in (1) to be the concentration of particles in any energy interval, lying above the energy $E_0 \sim 10^9$ ev/nucleon observed in cosmic rays on the earth. For protons, the collisions are no longer catastrophic, but by observing the well-known precautions it is possible in this case to use Eqs. (1) for the concentration of protons N_p with energy $E > E_0$. On the other hand, for electrons which experience continuous magnetic retardation losses, use of Eq.