where a_0 is of the order of magnitude of the difference in a on the different boundaries of the medium. Consequently, the mean (not the mean square!) value of the field in the volume $\bar{h} \approx a_0 / L$ and the value of the tangential field at the boundary of the region, where the liquid is at rest, are related by the following:

$$\bar{h} = (r/\varkappa)\, h_t|_s. \qquad (13)$$

By analogy with macroscopic electrodynamics, we shall consider the flow in the turbulent liquid as molecular flow, the neutralized true field $\bar{h}$ we denote by $\mathfrak{B}$ and introduce $\vec{\mathfrak{H}}$; here curl $\vec{\mathfrak{H}} = 0$ in the region where there are no irregularities of flow dependent on the turbulence. From Eq. (13) we get $\mathfrak{B} = (r/\varkappa)\, \vec{\mathfrak{H}} = \vec{\mathfrak{H}}/\,\mathrm{Rem}$. Thus, macroscopically, the turbulent conducting liquid behaves as a diamagnet * with very small permeability $\mu \sim /\mathrm{Rem}$.

Apropos of the analogy noted by Batchelor between the vortex velocity and the magnetic field, it should be noted that for the realization of an actually stationary turbulence, a supply of mechanical energy is necessary. The supply of energy comes about either at the expense of nonpotential volume forces or at the expense of the motion of the surfaces bounding the liquid. With consideration of these factors, the set of equations and boundary conditions for the vortex are not identical to the equation and boundary conditions for a magnetic field in the absence of external magnetic fields and attendant electromotive forces.

Once again, we note that direct step-by-step consideration of the three-dimensional case has, up to the present time, not been possible, and the question of the growth of field in the three-dimensional case remains unknown.

---

*Czada[3] has come to the conclusion that a conducting turbulent liquid is a paramagnet with large permeability. His analysis , based on a consideration of the damping of the field (p. 140) and of the energy of the field (p. 143) is not convincing, since the energy of the pulsating components of the field can be regarded macroscopically as a part of the turbulent energy, but the damping of the field can be connected not only with the conductivity, but also with a transition of the energy into mechanical form.

[1] G. K. Batchelor, Proc. Roy Soc. (London) 201A, 405 (1950); Problems in Contemporary Physics 2, 134 (1954). (Russian translation).

[2] W. M. Elsasser, Rev. Mod. Phys. 22, 1 (1950).

[3] I. K. Czada, Problems in Contemporary Physics 2, 136 (1956); I. K. Czada, Acta Phys. Hung. 1, 235 (1952).

Translated by R. T. Beyer
27

On the Composition of Primary Cosmic Rays

V. L. GINZBURG AND M. I. FRADKIN
P. N. Lebedev Physical Institute
(Submitted to JETP editor May 23, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 523-525
(September, 1956)

We consider the question of the chemical constitution of primary cosmic rays in the framework of the theory of the origin of cosmic rays developed in Refs. 1-3 and in many previous works discussed there. The concentration of cosmic particles of type i, which are designated by $N_i(\mathbf{r}, t)$, can be found from the system of equations

$$\partial N_i/\partial t = \nabla(D_i \nabla N_i) - N_i/T_i + \sum_{j>i} p_{ij} N_j/T_j + q_i, \quad (1)$$

where $q_i(\mathbf{r}, t)$ is the number of particles of type i per unit volume per unit of time, injected into interstellar space by the source of cosmic rays (the envelopes of supernovae and novae), $D_i(\mathbf{r})$ is the diffusion coefficient of cosmic rays in interstellar space, $T_i(\mathbf{r})$ is the lifetime of particles of type i, until their break-up in collisions with atomic nuclei of the interstellar medium (i.e., principally protons) and p_{ij} is the number of particles of type i, formed by the splitting of particles of type j. In case of nuclei, one can almost always assume that the collisions lead to the formation of nuclei of another sort, while the energy per nucleon of the primary and secondary nucleons are the same. Similarly, it is possible to understand N_i in (1) to be the concentration of particles in any energy interval, lying above the energy $E_0 \sim 10^9$ ev/nucleon observed in cosmic rays on the earth. For protons, the collisions are no longer catastrophic, but by observing the well-known precautions it is possible in this case to use Eqs. (1) for the concentration of protons N_p with energy $E > E_0$. On the other hand, for electrons which experience continuous magnetic retardation losses, use of Eq.

(1) is already impossible, but we shall not consider this case (see Ref. 4 where all the questions raised in the present note are considered in great detail).

We denote the concentration of protons, α-particles, nuclei Li, Be and B, nuclei C, N, O and F, and nuclei with $Z \geq 10$ by N_p, N_α, N_L, N_M and N_H, respectively. From experiment, $N_M / N_H \approx 3.2$; $N_\alpha / N_p \approx 0.1$; $N_H / N_p \approx 1.6 \times 10^{-3}$. With respect to the concentration of nuclei of group L, we have contradictory data, but we use the lowest value[5,6] for $N_L / N_M \leq 0.1$ (i.e., $N_L / N_H \leq 0.32$) since such a value for N_L is most difficult to explain theoretically. For the lifetime of the nuclei we take the value $T_p = 4 \times 10^8$ years, $T_H : T_M : T_L : T_\alpha : T_p = 1:2:3:5:20$, which differ but little from the values in Ref. 1. It is possible to suppose that in a time $T_i \leq 4 \times 10^8$ years the structure of the galaxy and the intensity of cosmic rays has changed but little, and by virtue of this, we set $\partial N_i / \partial T = 0$ in Eqs. (1). Further, if we consider the sources of cosmic rays to be distributed uniformly, then the diffusion is unimportant and we obtain

$$N_i = \sum p_{ij} N_j T_i / T_j + T_i q_i; \qquad (2)$$

$$N_M / N_H = (T_M / T_H)(q_M / q_H + p_{MH}) = 3.2; \quad q_M / q_H = 1.33;$$

$$N_L / N_H = (T_L / T_H)[p_{LH} + p_{LM}(q_M / q_H + p_{MH})] = 1.8; \quad N_L / N_M = 0.56,$$

where the values $p_{LM} = p_{LH} = 0.23$, $p_{MH} = 0.27$ are assumed, and where all are minimum values; furthermore, we assume that $q_L = 0$, so that on the average in nature the elements of group L are 105 times less abundant than elements of group M.

In the equilibrium condition, even if we completely neglect the role of secondary protons (i.e., set $p_{ij} = 0$), from Eqs. (2) we get $q_p / q_H = N_p T_H / N_H T_p \approx 30$; if we consider that $p_{ij} \neq 0$, then it is possible to to show that all the protons are secondaries. At the same time, in nature, the elements of group H are, on the average, 3000 times less abundant than the protons, Thus, if we proceed from Eqs. (2), it is necessary to suppose that the sources either are nearly completely without hydrogen, or the acceleration of protons is extremely ineffective in comparison with the acceleration of nuclei. Both of these assumptions seem to be inadmissible. Further, if $N_L / N_M \leq 0.1$, then the result (2) directly contradicts the experimental data (in this case a lowering of the value $N_L / N_M \approx 0.56$ via the choice of other permissible values of the constants is not possible)*.

Furthermore, the assumption of a uniform distribution of sources, in fact contradicts the picture adopted in Refs. 1-3, according to which cosmic rays are generated in the envelopes of supernovae and novae, located in the galactic plane and, possibly, concentrated at the galactic center. Upon emergence from the layer of thickness $2h \sim 2 \times 10^{21}$ cm which is occupied by sources, cosmic rays diffuse through the rare interstellar gas, occupying a volume of radius $R_0 \sim 5 \times 10^{22}$ cm. The corresponding diffusion coefficient $D \sim lv/3 \lesssim 3 \times 10^{37}$ cm^2/year, which yields an effective mean free path (regions with quasi-uniform magnetic fields), equal to 100 parsecs (this number corresponds to a maximum as shown in Ref. 7).

For a point source, located at a distance r from the observation point (Earth), a solution of the system (1) for N_H, N_M and N_L is [assuming $q_i = Q_i \delta(r)$; $q_L = 0$; $D_i = D = \text{const}$; $T_i(\mathbf{r}) = \text{const}$; $p_{Hj} = 0$]:

$$N_H = (Q_H / 4\pi Dr) \exp(-r / \sqrt{DT_H}); \qquad (3)$$

$$N_M = \frac{Q_H}{4\pi Dr}\left\{\frac{Q_M}{Q_H}\exp\left(-\frac{r}{\sqrt{DT_M}}\right) + \frac{p_{MH}T_M}{T_M - T_H}\left[\exp\left(-\frac{r}{\sqrt{DT_M}}\right) - \exp\left(-\frac{r}{\sqrt{DT_H}}\right)\right]\right\}$$

$$\times \; N_L = \frac{Q_H}{4\pi Dr}\left\{\frac{T_L}{T_L - T_H}\left[p_{LH} - \frac{p_{LM}p_{MH}T_H}{T_M - T_H}\right] \times \left[\exp\left(-\frac{r}{\sqrt{DT_L}}\right) - \exp\left(-\frac{r}{\sqrt{DT_H}}\right)\right] + \frac{p_{LM}T_L}{T_L - T_M}\left[\frac{Q_M}{Q_H} + \frac{p_{MH}T_M}{T_M - T_H}\right] \times \left[\exp\left(-\frac{r}{\sqrt{DT_L}}\right) - \exp\left(-\frac{r}{\sqrt{DT_M}}\right)\right]\right\}.$$

The values of $N_M/N_H = 3.2$ and $N_L/N_M \leq = 0.1$ are derived from this, if $r \leq 0.7\sqrt{DT_H} \approx 1.8 \times 10^{22}$. Simultaneously, we obtain $3.2 \geq Q_M/Q_H \geq 2.5$, and an analogous estimate for protons leads to the conclusion that $Q_p/Q_H \sim N_p/N_H \sim 10^3$. The value $r = 1.8 \times 10^{22}$ is only 1.4 times less than the distance of the sun from the galactic center (R_0

$= 2.5 \times 10^{22}$). If we consider the possibility of certain changes in the parameters, and also assume that the sources are distributed in a certain region, it is possible to get even better agreement between the calculated and observed values of N_i/N_j (for details, see Ref. 4). Thus, by including diffusion and also the nature of the source distribution for the cosmic ray and a rare interstellar medium, the question of the composition of cosmic rays is satisfactorily resolved within the framework of a theory of the origin of cosmic rays**. Because of insufficient reliable knowledge, the set of parameters used here must be investigated further and be made more precise. In particular, it is necessary to obtain a reliable value of N_L/N_M at the limits of the atmosphere, since on the basis of the theory it would be impossible to obtain the value of $N_L/N_M \ll 0.1$ without any essential changes.

---

* This statement was not made clear in the development in Refs. 1 and 2, since it was assumed that from experiment $N_L/N_M = 0.4 - 0.5$ (see Ref. 6); furthermore, only in Ref. 3 is this difficulty examined, with certain connections to the assumptions about preferential acceleration of nuclei in comparison with protons.

** For example, there is contained in Ref. 8 the conclusion that the smallness of the ratio N_L/N_M indicates that the lifetime of all nuclei $T_i \leq 4 \times 10^6$ years; therefore, this is entirely without basis. This conclusion was due first of all to the use of a radically different picture of the distribution of gas and magnetic fields in the galaxy (for criticism of the picture, see Ref. 8 and certain other works, see Refs. 2, 4 and 9).

[1] V. L. Ginzburg, Usp. Phys. Nauk **51**, 343 (1953); Fortschr. d. Phys. **1**, 659 (1954).

[2] V. L. Ginzburg, Izv. Akad. Nauk SSSR, Fiz. Ser. **20**, 5 (1956); Rep. of Fifth Conf. on the Problem of Cosmology, Acad. Sci. Press, 1956, p. 438.

[3] Ginzburg, Pikel'ner and Shklovskii, Astrono. Zh. **32**, 503 (1955); **33**, 447 (1956).

[4] V. L. Ginzburg and M. I. Fradkin, Astron. Zh. **33**, 577 (1956).

[5] B. Peters, *Progress in Cosmic Ray Physics*, pp. 193-242, Ch. IV, 1952.

[6] Kaplon, Noon and Racette, Phys. Rev. **96**, 1408 (1954).

[7] S. B. Pikel'ner, Rep. Third Conf. on the Problems of Cosmology, Acad. Sci. Press, 1954, p. 123.

[8] B. Rossi, Nuovo Cimento, Suppl. **2**, 275 (1955).

[9] V. L. Ginzburg, Dokl. Akad. Nauk SSSR **99**, 703 (1954).

Translated by B. Hamermesh
103