Transfer Processes in Conductors with Regard to Nonlinear Effects

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(Submitted to JETP editor July 26, 1956)

J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 657-671 (October, 1956)

Relaxation processes are analyzed for a system of lattice electrons. The equations derived in a previous paper[1] to take account of the heating of an electron gas in strong electric fields are made more precise. The form of these equations is given for various conditions and the resulting transfer processes are investigated.

IN a previous work[1] we obtained kinetic equations allowing for the possible heating of an electron gas relative to the lattice. In contrast with the equations of Davydov[2], our equations took the quantum statistics of the electrons into consideration, and could therefore be applied to conductors whose electrons exist in a degenerate state. It is necessary to take the degeneracy of the electron gas into consideration at low temperatures for semimetals and for semiconductors with metallic-type conductivity (excepting the typical metals). The heating of an electron gas for good metals wtih attainable current densities first shows up in effects strongly depending on the temperature of the electrons, such as emission[3], for example, as apparently is confirmed by experiment[4].

For "good" metals at high temperatures, current densities for which it is possible to expect deviations from Ohm's law[5] and nonlinearities in other effects, on account of the heating of the electrons, are not yet attainable. For "poor" metals with a smaller number of charge carriers, however, current densities of the required magnitude are perfectly attainable, and corresponding departures have been observed, for example, in bismuth[6]. It is desirable therefore to investigate all combinations of transfer processes in conductors with regard to the heating of the electrons.

1. RELAXATION PROCESSES IN A METAL

Changes in the electron distribution function f as a result of electron-phonon, electron-electron and electron-impurity collisions, and changes in the phonon distribution function N_ω as a result of phonon-electron, phonon-phonon and phonon-impurity collisions are designated in the following manner:

$$[\partial f / \partial t]_{\text{coll}} = \Sigma^{ef}(f, \ N_\omega) \qquad (1.1)$$

$$+ \ \Sigma^{ee}(f, \ f) + \Sigma^{en}(f),$$

$$[\partial N_\omega / \partial t]_{\text{coll}} = \Sigma^{fe}(N_\omega, \ f) \qquad (1.2)$$

$$+ \ \Sigma^{ff}(N_\omega, \ N_\omega) + \Sigma^{fp}(N_\omega).$$

If we look for solutions of the form $f = f_0 + f'$, $N_\omega = N_\omega^0 + N_\omega'$, where $f_0 = f_0(p)$, $N_\omega^0 = N_\omega^0(k)$ are the symmetric parts of the distribution functions, depending only on the absolute values of the electron and phonon momenta and not on their directions, then the expressions (1.1) and (1.2) take the form

$$[\partial f / \partial t]_{\text{coll}} = \Sigma_{00}^{ef}(f_0, \ N_\omega^0) \qquad (1.3)$$

$$+ \ \Sigma_{10}^{ef}(f', \ N_\omega^0) + \Sigma_{01}^{ef}(f_0, \ N_\omega')$$

$$+ \ \Sigma_{00}^{ee}(f_0, \ f_0) + 2 \ \Sigma_{10}^{ee}(f', \ f_0) + \Sigma_1^{en}(f'),$$

$$[\partial N_\omega / \partial t]_{\text{coll}} = \Sigma_{00}^{fe}(N_\omega^0, \ f_0) \qquad (1.4)$$

$$+ \ \Sigma_{10}^{fe}(N_\omega', \ f_0) + \Sigma_{01}^{fe}(N_\omega^0, \ f')$$

$$+ \ \Sigma_{00}^{ff}(N_\omega^0, \ N_\omega^0) + 2 \ \Sigma_{01}^{ff}(N_\omega^0, \ N_\omega') + \Sigma_1^{fp}(N_\omega') \ .$$

Here we have neglected the terms $\Sigma_{11}^{ee}(f', f')$, $\Sigma_{11}^{ff}, \Sigma_{11}^{ef}, \Sigma_{11}^{fe}$, since it is presumed that $f' \ll f_0$, and also the terms $\Sigma_0^{ep}(f_0)$, $\Sigma_0^{fp}(N_\omega^0)$, since when electrons and phonons collide with impurities their energies are not changed.

If the usual assumption is made that the distribution functions f_0 and N_ω^0 are equilibrium distributions, with the same temperature, then operators of the form Σ_{00} reduce to zero.

A change in the directed motion of the electrons at the expense of a different kind of transition is expressed by the operators Σ_{10}^{ef}, Σ_{1}^{ep}, $2\Sigma_{10}^{ee}$; the influence of the latter on the temperature dependence of the electrical conductivity has been analyzed by Landau and Pomeranchuk[7] and also by Ginzburg and Silin[8]. The operator Σ_{01}^{ef} takes account of the "trapping" of electrons by phonons[9]. We will not consider it here.

As a result of the influence of external fields and gradients, the symmetric part of the distribution functions will depart from equilibrium. The form it will take is determined in part by the external field and gradients, and in part by the operators $\Sigma_{00}^{ef}(f_0, N_\omega^0)$, $\Sigma_{00}^{ee}(f_0, f_0)$. The first term plays the principal role in determining the change in the average energy or the effective temperature of the disordered motion of the electrons, since electron-electron interactions cannot change the average electron energy.

There are substantial changes in the average energy of the random motion of the electrons if the time required for the transfer of the energy obtained by the electron from the field is greater than the time required for the transfer of momentum. In order to ascertain the conditions under which it is possible to observe evidence of this effect in metals, we will estimate the order of magnitude of the different terms in (1.3) and (1.4), which are inversely proportional to the time or to the mean free path for the corresponding type of interaction. In making this estimate we will everywhere use the following values for the physical constants of a metal: number of free electrons $n \sim 3 \times 10^{22}$, limiting energy $\epsilon_0 \sim 5 \times 10^{-12}$ erg ~ 3 ev or a degeneracy temperature $T_0 = \epsilon_0/k \sim 3 \times 10^4$ °K (the corresponding velocity $v \sim 10^8$ cm/sec), a Debye temperature $\Theta_d \sim 10^2$ °K, velocity of sound v_s $\sim 5 \times 10^5$ cm/sec.

The second term in (1.3) is determined by the mean free path over which an electron, interacting with a phonon, loses its momentum. For $T \gg \Theta_d$

$$l_{ef} \sim l_{ef}^0 \Theta_d / T \sim v\Theta_d / \omega_m T \qquad (1.5)$$

$$\sim (hv / k\Theta_d)\,\Theta_d / T \sim 10^{-5}\Theta_d / T \sim 10^{-3} / T.$$

Near 300° K the electrical conductivity corresponding to the value chosen for the constant in l_{ef} is

$$\sigma = e^2 nl / mv \sim 5 \times 10^{16} \quad \text{cgs} \quad = 5 \times 10^1 \; 1/\Omega \; \text{cm}.$$

At low temperatures ($T \ll \Theta_d$)

$$l_{ef} \sim (v / a\omega_m)(\Theta_d / T)^5 \qquad (1.6)$$

$$\sim 10^{-7}(\Theta_d / T)^5 \sim 10^{-7}(10^2 / T)^5.$$

The numerical coefficient a can be determined from the ratio of the ideal electrical conductivities at high ($T' \gg \Theta_d$) and at low ($T \ll \Theta_d$) temperatures,

$$l_{ef} / l'_{ef} = \sigma(T) / \sigma(T') = a^{-1}(\Theta / T)^4 T' / T. \quad (1.7)$$

The value of a can amount to 500. For definiteness we assume $a \sim 100$.

The mean free path for the scattering of phonons by electrons[9] is, for $T \gg \Theta_d$,

$$l_{fe} \sim v_s \tau_{fe} \sim (v_s / \omega_m)\,\epsilon_0 / k\Theta_d \sim 10^{-5}, \quad (1.8)$$

and for $T \ll \Theta_d$,

$$l_{fe} \sim (v_s / \omega_m)\,\epsilon_0 / kT \sim 10^{-3} / T. \qquad (1.9)$$

At low temperatures this value is less than l_{ff} and essentially determines the temperature part of the thermal resistance of the lattice. In order of magnitude it agrees with the value of l_{fe} obtained from the expression for the thermal conductivity, the experimental value of which, for $T < 50$° K, is[10]

$$\varkappa_p \sim 2 \times 10^{-3} \, T^2 \; \text{watt/cm-deg}$$

$$\sim 2 \times 10^4 \, T^2 \; \text{erg/cm-sec-deg}.$$

Noting that the specific heat $c_p \sim n_a k (T/\Theta)^3$, where the number of atoms is $n_0 \sim n$, we obtain $l_{fe} \sim \varkappa_p / c_p v_s \sim 10^{-2}/T$.

At high temperatures, the phonon distribution function is established principally by way of phonon-phonon interactions. For $T \gg \Theta_d$ the mean free path for phonon collisions is

$$l_{ff} = (v_s / \omega_m) \varepsilon_0 / kT \tag{1.10}$$

$$\sim (h / mv_s^2)(\Theta_d / T) \sim l_{ff}^0 \Theta_d / T \sim 10^{-5}\Theta_d / T$$

(here noting that $\epsilon_0 \sim p^2 / 2m \sim h^2\omega_m^2 / 2mv_s^2$ $\sim (k\Theta_d)^2 / 2mv_s^2$).

For this value of the mean free path, the thermal conductivity at 300° K is

$$\varkappa_p \sim \tfrac{1}{3}c_p l_{ff} v_s \sim \tfrac{1}{3}n_a k l_{ff} v_s$$

$$\sim 10^{-6} \text{ erg/cm-sec-deg,}$$

which agrees with the experimental value for the thermal conductivity of insulators.

If one considers that equilibrium in the lattice can be established only by way of transfer processes, then at low temperatures the mean free path l_{ff} increases exponentially, in accordance with the fact that there are only a very small number of the larger phonons taking part in this process. If, however, one evaluates the operator Σ^{ff} without regard for this circumstance, which can only decrease the mean free path, then the estimate gives[9]

$$l_{ff} \sim l_{ff}^0 (\Theta / T)^5 \sim 10^{-5}(\Theta_d / T)^5. \tag{1.11}$$

The rate at which equilibrium is established with respect to the energies of the disordered motion among electrons and phonons, which is determined by the operator Σ_{00}^{ef}, is inversely proportional to the mean free path over which an energy kT is transferred. For high temperatures $T \gg \Theta_d$ this path length is

$$l_{ef}^\varepsilon \sim (v / \omega_m) T / \Theta_d \sim l_{ef}^0 T / \Theta_d \tag{1.12}$$

$$\sim l_{ef} (T / \Theta_d)^2 \sim 10^{-5}T / \Theta_d.$$

Here it is clear that for $T \gg \Theta_d$, l_{ef}^ϵ is greater than l_{ef} by a factor $(T/\Theta_d)^2 \sim (kT)^2 / mv^2\epsilon_0$.

For $T \ll \Theta_d$:

$$l_{ef}^\varepsilon \sim (v / \omega_m)(\Theta_d / T)^3 \tag{1.13}$$

$$\sim a (T / \Theta_d)^2 l_{ef} \sim 10^{-5}(\Theta_d / T)^3.$$

At low temperatures this quantity becomes dependent on the electronic part of the thermal conductivity. The coefficient a is taken as ~ 100, as

before, to agree with the experimental value of the thermal conductivity for these temperatures:

$$\varkappa_{ef} \sim c_e l_{ef}^\varepsilon v \sim nk \frac{kT}{\varepsilon_0} l_{ef}^\varepsilon v$$

$$\sim 10^{11} / T^2 \text{ erg/cm-sec-deg.}$$

Here, in particular, it is clear that it is possible to estimate a from measurements of $\varkappa_{ef}/\sigma_{ef}$, since

$$a \sim \varkappa_{ef} e^2 n \varepsilon_0 v_s / \sigma_{ef} c_e (kT)^2 v.$$

It follows from (1.13) that l_{ef}^ϵ actually remains greater than l_{ef} down to a rather low temperature, $T > \Theta_d a^{-\frac{1}{2}} \sim 10°$ K. At still lower temperatures l_{ef} increases much more rapidly than l_{ef}^ϵ.

The rate at which the equilibrium distribution of electrons is established, at some given temperature, as a result of electron-electron interactions is determined by the operator $\Sigma_{00}^{ee} \sim v/l_{ee}$, where l_{ee} is the mean free path over which the electrons redistribute their energy, as well as their momentum. Taking account of the statistics we obtain, in order of magnitude[3],

$$l_{ee} \sim 1 / q_{ee}n (kT / \varepsilon_0)^2 \sim 3 \times 10^{-3} (10^2 / T)^2, \tag{1.14}$$

where a value of 10^{-15} cm^2 is taken for the collision cross section q_{ee}.

For completely free electrons, the term $\Sigma_{10}^{ee}(f_1, f_0)$ has to reduce to zero, since interelectronic interactions cannot change the average momentum of the electrons. However, because of the electronic transfer processes which are possible in lattices, there exists a limiting mean free path for such an interaction. As it turns out, its order of magnitude is the same as that of (1.14)[8].

Finally, the mean free path for scattering by impurities and lattice defects is

$$l_{ep} \sim 1 / \pi^2 q n_p \sim 10^{-4} \tag{1.15}$$

for $q \sim 10^{-15}$ and an impurity concentration $n_p \sim 10^{18}$, that is, $n_p/n_a \sim 3 \times 10^{-5}$.

For convenience in comparing the role of the processes by which equilibrium is established at various temperatures, we write the approximate values of the mean free paths given by Eqs. (1.5) – (1.15) in tabular form. (The relaxation times corresponding to these are $\tau_e \sim l_e v$, $\tau_f \sim l_f v_s$.)

T	l_{ef}	l_{fe}	l_{ff}	l^{ε}_{ef}	l_{ee}	l_{en}
10^4	10^{-7}	10^{-5}	10^{-7}	10^{-3}	3×10^{-7}	10^{-4}
10^3	10^{-6}	10^{-5}	10^{-6}	10^{-4}	3×10^{-5}	10^{-4}
300	3×10^{-6}	10^{-5}	3×10^{-6}	3×10^{-5}	3×10^{-4}	10^{-4}
100					3×10^{-3}	10^{-4}
30	3×10^{-5}	3×10^{-5}	$\times10^{-3}$	3×10^{-4}	3×10^{-2}	10^{-4}
10	10^{-2}	10^{-4}	1	10^{-2}	3×10^{-1}	10^{-4}
1	10^3	10^{-3}	10^5	10	3	10^{-4}
0.1	$\gg10^3$	10^{-2}				10^{-4}

From the table it is clear that at $T \sim 10^3$ °K and above, energy equilibrium is achieved at the expense of interelectronic interactions, since in this temperature region $l_{ee} \ll l^{\varepsilon}_{ef}$. For lower temperatures ($T < 300°$) the electron energy distribution is determined principally by electron-phonon interactions ($l^{\varepsilon}_{ef} \ll l_{ee}$) with the exception of only very low temperatures of the order of a few degrees, where l^{ε}_{ef} again becomes larger than l_{ee}.

However, because the corresponding relaxation times are so large, the process of establishing electronic energy equilibrium will be quite slow if the electrons are not able to exchange energy with the boundaries of the sample or of the individual crystals.

Regarding the establishment of the phonon distribution, it is clear from a comparison of the tabulated values of l_{ff} and l_{fe} that at 300° K and above, phonon equilibrium is established at the expense of phonon-phonon interactions, and for lower temperatures is determined principally by the interaction with electrons.

The heating of an electron gas relative to the lattice is sufficient to bring about changes in the kinetic coefficients before the velocity of the directed motion has become equal to that of the disordered motion, if the frequency of collisions of electrons with phonons with the exchange of energy $\sim kT$ ($\nu^{\varepsilon} = 1/\tau^{\varepsilon}_{ef} = v/l^{\varepsilon}_{ef}$) is less than the frequency of all possible collisions in which momentum is transferred*:

$$\nu = v/L = v(1/l_{ef} + 1/l_{ep} + 1/l_{ee}), \quad (1.16)$$

* As is clear from the Table, l_{ee} begins to compete with l_{ef}, from Eq. (1.16), and can influence the temperature part of the resistance of metals, only for temperatures $T \lesssim 10°$; that is, in the region where impurity scattering predominates.

that is, if the inequality $L \ll l^{\varepsilon}_{ef}$ is satisfied. For high temperatures $l^{\varepsilon}_{ef} \gg l_{ef}$, for the Debye temperatures $l^{\varepsilon}_{ef} \sim l_{ef}$, for temperatures still lower than the Debye temperature, and down to 10-20° K, l^{ε}_{ef} can again be considered larger than l_{ef} [on account of the coefficient a in Eq. (1.13)]. For the very lowest temperatures, again $l^{\varepsilon}_{ef} \ll l_{ef}$. However, at low temperatures $l_{ef} \gg L \sim l_{ep}$ for the impurity concentrations which we selected for $T \lesssim 30°$; consequently, from this point of view the heating of an electron gas is possible both at high and at low temperatures, excluding the region of the Debye temperature.

2. ELEMENTARY CALCULATION OF THE TEMPERATURE OF THE ELECTRONS IN AN ELECTRIC FIELD

It is possible to obtain an estimate of the temperature difference between the electrons and the lattice in the precence of a single external electric field from qualitative considerations. Per unit volume and per unit time, the electrons acquire an energy

$$\sigma E^2 = (e^2 nL/mv)E^2 \sim (eEL)^2 n/\varepsilon_0\tau \quad (2.1)$$

(where ϵ_0 is the limiting energy of the electrons, σ is the conductivity and E is the electric field).

The emission of phonons by electrons is proportional to $N_\omega + 1$, the absorption to N_ω. Therefore, on the whole the electron gives energy to the lattice in $1/N_\omega$ of all the collisions. In unit time it gives up an energy $\hbar\omega/N_\omega\tau_1$, where τ_1 is the time between separate discrete collisions, in which a phonon of energy $\hbar\omega$ is emitted or absorbed. For high temperatures, $N_\omega \sim kT/\hbar\omega \gg 1$, and the induced phonon emission predominates

over spontaneous emission. Therefore, the transfer of energy from electrons to phonons and the mean free path of the electrons regardless of their temperature are completely determined by the distribution of phonons whose equilibrium is established at the expense of phonon-phonon interactions, as we have seen. The momentum of a phonon at high temperatures is comparable with that of the electrons, $\hbar\omega/v_s \sim p$. Therefore, the time τ_1 between separate collisions coincides with τ_{ef}. To summarize, if we designate the average energy lost by an electron in the time t by $\Delta\bar{\epsilon}$, we obtain for the energy transferred by an electron to the lattice per unit time

$$\overline{\Delta\epsilon}\,/\,t \sim \hbar\omega\,/\,N_\omega\tau_1 \tag{2.2}$$

$$\sim (\hbar\omega)^2\,/\,kT\tau_{ef} \sim \epsilon_0 mv_s^2\,/\,kT\tau_{ef}\,.$$

Moreover, if once considers that, for heating which still does not remove the degeneracy ($k\Theta/\epsilon_0 < 1$), according to the Pauli principle only $nk(\Theta-T)/\epsilon_0$ of the total number n of electrons can transfer this energy, where Θ is the temperature of the electrons and T is that of the lattice, then comparison with (2.1) gives

$$\alpha = (\Theta - T)\,/\,T \sim (eEL)^2\,l_{ef}\,/\,mv_s^2\epsilon_0 L. \tag{2.3}$$

For low temperatures ($kT \ll k\Theta_d = \hbar\omega_m$) the picture changes. Whenever electrons and phonons exist in equilibrium, practically the only excitations are phonons with energies $\hbar\omega \lesssim kT$, as can be seen from the expression for the number of phonons

$$N_\omega = [\exp(-\hbar\omega/kT) - 1]^{-1}.$$

Therefore, electrons cannot absorb phonons of large frequency. But electrons at the temperature T also cannot emit phonons of large frequency, since transitions of electrons with the emission of a phonon take place in the zone of diffusion $\sim kT$. This fact also makes it possible to calculate the mean free path l_{ef} for the momentum changes at low temperatures. However, in order for electrons to be heated, it is necessary that phonons with energy $\hbar\omega > kT$ be emitted. The absence of such phonons in the spectrum of the equilibrium distribution at the temperature T and the long relaxation time for the phonon-phonon interactions will lead to a disturbance of the phonon equilibrium distribution. Nevertheless, in problems connected with the rapid transfer of energy from the electrons

to the lattice, the energy of the lattice oscillations can be characterized by a temperature T, with suitable reservations. On the other hand, the possibility of investigating marked heating of the electrons relative to the lattice at low temperatures is limited by the fact that the electron mean free path is still not only determined by the lattice temperature T, but also depends on the temperature of the electrons, and, consequently, also on the field. Therefore, it is possible to investigate transition processes by introducing a mean free path for the electrons, but the results are correct only with the assumption that $\Theta/T \lesssim 1$. In this case the electron loses energy $k\Theta \sim kT$ in the time

$$\tau_{ef}^\epsilon \sim a\,(T/\Theta_d)^2\,\tau_{ef} \sim a\,(kT)^2\,\tau_{ef}\,/\,mv_s^2\epsilon_0.$$

Per unit time, a fraction $nk(\Theta - T)/\epsilon_0$ of the electrons give up an energy

$$n\,\frac{k(\Theta - T)}{\epsilon_0}\,\frac{kT}{\tau_{ef}^\epsilon} \sim \frac{\Theta - T}{T}\,\frac{nmv_s^2}{a\tau_{ef}}\,.$$

Comparison with (2.1) leads to an expression differing from (2.2) only in the numerical factor a.

3. THE KINETIC EQUATION FOR DETERMINING THE HEATING OF AN ELECTRON GAS

The heating of electrons in transfer processes can be calculated with the aid of the kinetic equations obtained earlier[1] by transforming the integral operator $\Sigma_{00}^{ef}(f_0, N_\omega^0)$ in (1.3) to a differential one. The system of equations for the symmetric part of the distribution function f_0, with respect to the velocity, and for the nonsymmetric part $f' = (1/v)\,\mathbf{v}\cdot\mathbf{f}_1$ has the form

$$\frac{\partial f_0}{\partial t} + \frac{v}{3}\nabla f_1 + \frac{1}{N}\frac{\partial}{\partial\epsilon}(NS_1) \tag{3.1}$$

$$+ \Sigma_{ee}(f_0, f_0) = 0,$$

$$\frac{\partial \mathbf{f}_1}{\partial t} + v\nabla f_0 + eEv\frac{\partial f_0}{\partial\epsilon} \tag{3.2}$$

$$+ \frac{e}{mc}[\mathbf{H}\times\mathbf{f}_1] + \frac{v}{L}\mathbf{f}_1 = 0,$$

where in energy space the energy flow density is

$$S_1 = \tfrac{1}{3}e\mathbf{E}\cdot\mathbf{f}_1 v - S = \tfrac{1}{3}e\mathbf{E}\cdot\mathbf{f}_1 v \tag{3.3}$$

$$- (2mv_s^2\epsilon v\,/\,al)[f_0(1 - f_0)\,/\,kT + \partial f_0/\partial\epsilon].$$

Here ϵ is the energy, $v = \partial\epsilon/\partial p$ is the velocity, m is the effective mass, e is the charge of the electrons,

v_s is the velocity of sound, T is the temperature of the lattice, $l = l_{ef}$ is the mean free path over which the electron loses momentum through interaction with a phonon, and L is the mean free path with respect to all interactions [see Eq. (1.16)]; E and H are the external electric and magnetic fields, C is the velocity of light, N is twice the density of the electron levels (doubled because of the spin degeneracy; for a free electron gas, $Nd\epsilon = (2\pi\hbar)^{-3} 8\pi p^2 dp$), and a is the numerical coefficient discussed above (for $T \gg \Theta_d$, $a \sim 1$). Together with the normalization requirement

$$n = \int f_0 N d\epsilon \qquad (3.4)$$

the system of equations totally determines the electronic distribution function $f = f_0 + (1/v)\, \mathbf{v} \cdot \mathbf{f}_1$.

Equation (3.1) with S_1 taken from (3.3) differs from the corresponding expression obtained previously[1] by a factor $1/a$ in front of the square brackets, which was not considered in the system of equations used in that paper, and also in the last term, which determines the influence of the electron-electron interaction on the rate of change of the energy flow density in energy space. Furthermore, the third term of (3.1) is written in a form which does not assume any concrete form for the dependence of the density of electronic energy levels N on the energy ϵ, or of the energy ϵ on the momentum p; we can easily convince ourselves of this by modifying the equations in the earlier paper by expanding $\Sigma_{00}^{ef}(f_0, N_\omega^0)$ in $\Delta\epsilon/\epsilon^2$ instead of in $\Delta p/p$ *.

* Instead of (6, 8, 9) derived earlier[1] we obtain in this case.

$$(a - b) f_0 = \Sigma_{00}^{ef} (f_0, N_\omega^0) = A_1 f_0 \frac{\partial f_0}{\partial\varepsilon} - A_2 f_0 \frac{\partial^2 f_0}{\partial\varepsilon^2}$$
$$- (1-f_0) \frac{1}{N} \frac{\partial}{\partial\varepsilon} (N A_1 f_0)$$
$$- (1-f_0) \frac{1}{N} \frac{\partial^2}{\partial\varepsilon^2} (N A_2 f_0) = \frac{1}{N} \frac{\partial}{\partial\varepsilon} (NS),$$

where

$$S = - (1 - f_0) \left[A_1 f_0 + \frac{1}{N} \frac{\partial}{\partial\varepsilon} (N A_2 f_0) \right] - A_2 f_0 \frac{\partial f_0}{\partial\varepsilon}.$$

The equality $S = 0$ for $f_0 = f_{00} = [\exp (\epsilon - \mu) / kT + 1]^{-1}$ gives

$$A_1 = \frac{1}{kT} \left[A_2 - \frac{1}{N} \frac{\partial}{\partial\varepsilon} (N A_2) \right].$$

Substituting A_1 into the expression for S, we obtain

$$S = - A_2 \{ f_0 (1 - f_0) / kT + \partial f_0 / \partial\varepsilon \}.$$

As in our earlier Eqs. (14) and (17)[1], A_2 is

$$A_2 = \overline{\frac{(\Delta\varepsilon)^2}{2\tau_1}} = \int \frac{(\Delta\varepsilon^2)}{2} W (\varepsilon, \Delta\varepsilon, \vartheta)\, d\Delta\varepsilon\, d\Omega = \frac{2m v_s^2 v\varepsilon}{al},$$

The essential assumptions made in obtaining this system of equations were the following: 1) the conductor is isotropic; 2) $f_1 \ll f_0$. This is equivalent[5] to the condition $eEL/\bar\epsilon \ll 1$. The formal solution shows that this condition is realized for any amount of heating (or for any field) if only the quantity $\delta = 2m v_s^2 L / kTal \ll 1$ [see (2.36) and the first footnote appearing on the bottom of p. 504 of this article], which is satisfied for high, as well as for sufficiently low temperatures; 3) the phonon distribution is assumed to be an equilibrium distribution, and the electron mean free path does not depend on Θ (or on the field). As we have seen, this assumption is justified for any Θ at high temperatures; at low temperatures it is approximately correct but only in the region $\alpha = (\Theta - T)/T \lesssim 1$.

Integrating (3.1) over phase space, we obtain the equation of conservation of charge:

$$(\partial n / \partial t) + \text{div}\, (\mathbf{j} / e) = 0, \qquad (3.5)$$

and integrating over $\epsilon N d\epsilon$, the equation of conservation of energy

$$(\partial\varepsilon_e / \partial t) + \text{div}\, \mathbf{W}_e - \mathbf{E} \cdot \mathbf{j} + Q = 0. \qquad (3.6)$$

Here the electric current

$$\mathbf{j} = \frac{e}{3} \int v \mathbf{f}_1 N d\varepsilon, \qquad (3.7)$$

the electronic heat flow

$$\mathbf{W}_e = \frac{1}{3} \int \varepsilon v \mathbf{f}_1 N d\varepsilon, \qquad (3.8)$$

the energy transferred per unit volume per unit time from the electrons to the lattice

$$Q = \int S N d\varepsilon, \qquad (3.9)$$

and the average energy of the n electrons

$$\varepsilon_e = n\bar\varepsilon = \int \varepsilon f_0 N d\varepsilon; \qquad (3.10)$$

S has to be substituted from (1.22),

Equations (3.5) and (3.6) have to be complemented by the equation of conservation of energy for the lattice, which can be obtained by integrating the equation for phonons over phase space,

$$\partial\varepsilon_e / \partial t + \text{div}\, \mathbf{W}_p - Q = 0 \qquad (3.11)$$

and from the boundary conditions. This makes it possible to determine the temperature of the lattice uniquely for different processes.

4. SOLUTION OF THE KINETIC EQUATIONS. CONSERVATION AND TRANSFER EQUATIONS

Disregarding the term $\partial f_1 / \partial t$, it is possible to solve Eq. (3.2) with respect to f_1

$$f_1 = -eL \frac{G + (eL / cmv) G \times H + (eL / cmv)^2 H (G \cdot H)}{1 + (eL / cmv)^2 H^2}, \qquad (4.1)$$

where $G^{-1} = e^{-1} \nabla - E \partial/\partial\epsilon$ is an operator which acts on the function f_0. The equation obtained by substitution of Eq. (4.1) for f_1 into (3.1) can be solved for f_0 if the first two terms in (3.1) are ignored in comparison with the last in the first approximation. This means that the processes being considered are sufficiently slow and the gradients sufficiently small, that is, the field changes insignificantly along either the electron-electron mean free path, or along the mean free path for the transfer of energy from the electron to the lattice. Concerning the relation between the last two possibilities we can consider two extreme cases: the third term is noticeably smaller than the fourth (strong interelectronic interactions), or the reverse.

In the first case the equation $\Sigma_{00}^{ee}(f_0, f_0) = 0$ gives the Fermi distribution

$$f_0 = (e(\epsilon - \mu)/k\Theta + 1)^{-1} \qquad (4.2)$$

with a chemical potential μ and an undetermined parameter Θ. The connection between these two quantities and the number n of electrons and the temperature T of the lattice is established with the aid of the normalization condition (3.4), but their dependence on the coordinates and on the fields is found in the next approximation by substituting Eq. (4.2) for f_0 into Eq. (3.6).

For the reverse case of interelectronic interactions which are small in comparison with those between electrons and phonons, we substitute Eq. (4.1) for f_1 into Eq. (3.3) for S_1 and obtain

$$S_1 = \tfrac{1}{3} e (E \cdot f_1) v - S = -(2mv_s^2 \epsilon v / al) \qquad (4.3)$$

$$\times \{f_0(1 - f_0)/kT + (\partial f_0/\partial \epsilon)(1 + \alpha_H)\},$$

where

$$\alpha_H = \alpha \frac{1 + \beta^2\gamma^2}{1 + \beta^2} = \frac{(eEL)^2}{6mv_s^2\epsilon} \frac{al}{L} \frac{1 + \beta^2\gamma^2}{1 + \beta^2}, \qquad (4.4)$$

$$\beta = \frac{eL}{cmv} H, \quad \gamma = \cos(E \cdot H).$$

The solution of the equation $S_1 = 0$ has the form

$$f_0 = \left(e^{(z-\zeta)/kT} + 1\right)^{-1}, \qquad z = \int^\epsilon d\epsilon / (1 + \alpha_H), (4.5)$$

where ζ is a constant of integration, whose connection with the number n of electrons and the temperature T of the lattice is established with the aid of the normalization condition (as is easily seen, the possibility of normalization is guaranteed by the requirement that α goes to infinity no faster than ϵ). The dependence of ζ and n on the coordinates and on the fields can be established in the next approximation with the aid of the conservation equation (3.6). Knowledge of the function f_0 makes it possible to evaluate the transfer equations (3.3) and (3.8), which together with the conservation equations (3.6) for electrons and (3.11) for phonons completely determine all galvanomagnetic, thermomagnetic and thermoelectric phenomena.

1. We assume that the distribution function has the Fermi form (4.2), and that it is possible to speak of a temperature for the electrons. It is a particularly simple matter to establish the second assumption in general form for the following situations:

a) There is a single stationary field (div W_e = 0, $\partial \epsilon_e/\partial t = 0$). Substituting (4.2) into (4.3), we obtain

$$Q - Ej = -\int NS_1 d\epsilon = \frac{\Theta - T}{BT} \qquad (4.6)$$

$$-Ej = \frac{\Theta - T}{BT} - \frac{1}{B(\alpha_H)},$$

$$\frac{1}{B} = -\frac{2mv_s^2}{a} \int \frac{\epsilon v N}{l} \frac{\partial f_0}{\partial \epsilon} d\epsilon, \qquad (4.7)$$

$$\frac{1}{B(\alpha_H)} = -\frac{2mv_s^2}{a} \int \frac{\epsilon v N}{l} \alpha_H \frac{\partial f_0}{d\epsilon} d\epsilon.$$

Here for the temperature of the electrons we obtain

$$(\Theta - T) / T = B / B(\alpha_H). \qquad (4.8)$$

In the first approximation, for the case $k\Theta/\mu \ll 1$,

$$\frac{\Theta - T}{T} = \alpha_H^0 = \frac{B_0 j^2}{\sigma_0} \qquad (4.9)$$

$$= \frac{e^2 j^2}{6 (\sigma_0 / L)^2 m v_s^2 \epsilon_0} \frac{al}{L} \frac{1 + \beta_0^2 \gamma^2}{1 + \beta_0^2},$$

where the index 0 denotes that the corresponding expression is taken at the limit of the Fermi distribution.

For small magnetic fields or for **E** perpendicular to **H**, $j = \sigma E$ and the expression for the temperature can be written in the form

$$\frac{\Theta - T}{T} = \alpha_H^0 = \frac{(eEL)^2}{6 m v_s^2 \epsilon_0} \frac{al}{L} \frac{1 + \beta_0^2 \gamma^2}{1 + \beta_0^2} . \qquad (4.10)$$

For $|k\Theta/\epsilon_0| \gg 1$ (classical statistics)

$$(\Theta - T)/T = \overline{\epsilon v \alpha_H / l} \mid \overline{\epsilon v / l} \qquad (4.11)$$

(where the line denotes average value).

For small magnetic fields and for $l = $ const, $L = $ const, and also for $\epsilon = p^2/2m$, $N \sim \epsilon^{1/2}$, we obtain from the considerations $\alpha_H \sim 1/\epsilon$ and $\epsilon = 3/2\, k\Theta$

$$\frac{\Theta - T}{T} = \frac{2\overline{v}}{m\overline{v^3}} = \frac{3}{4} \alpha_H \qquad (4.12)$$

$$= \frac{(eEl)^2}{12 m v_s^2 k\Theta} \frac{al}{L} \frac{1 + \gamma^2 \beta^2}{1 + \beta^2} .$$

When $l \sim \epsilon^{1/2}$ ($\alpha_H = $ const) the temperature $(\Theta - T)/T = 3\,\alpha_H/2$.

Solving (4.12) for $k\Theta$, we obtain for the temperature of the electrons an equation which, when written in the form*

$$k\Theta = \frac{1}{2} (kT + \sqrt{(kT)^2 + 2 (eEL)^2 / 3\delta}) \qquad (4.13)$$

agrees with the old results for classical statistics, without accounting for additional scattering by impurities, however. (In the above equation $\delta = 2 m v_s^2 L / kTal$ is the ratio of the energy given up by the electron to the total energy during the time required for the total loss of momentum; for a plasma $\delta = 2m/M$, where M is the mass of the atom.)

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* Here it is clear that even for strong fields which totally remove the degeneracy and make $\Theta \gg T$, the inequality $eEL/\overline{\epsilon} \sim eEL/k\Theta \sim \sqrt{\delta} \ll 1$ is satisfied if only $\delta \ll 1$.

b) The magnetic field is absent. With the aid of Eq. (3.7) for **j**, we eliminate the field **E** from the Eq. (3.8) for the electronic heat flow in coordinate space W_e, and obtain the equation of conservation of energy in the form

$$\frac{\partial \epsilon_e}{\partial t} + \frac{\Theta - T}{BT} = \frac{j^2}{\sigma} - \frac{j}{e} \Theta \nabla \chi + \nabla (\varkappa_e \nabla \Theta), \quad (4.14)$$

where $Q = \int S N d\epsilon = (\Theta - T)/BT$ is the energy given to the phonons by the electrons, σ is the electrical conductivity, χ is the thermal electromotive force, and $\varkappa_e$ is the electronic thermal conductivity *. The coefficients $\sigma, \chi, \varkappa_e$ are expressed in terms of f_0 in the usual way, except with T replaced by Θ. For $\Theta = T$, Eq. (4.14) coincides with the standard equation in the theory of metals. The energy lost from the conductor because of the current passing through it is $\Omega = \int (W_e + W_p)dS$, where the integral is taken over the surface of the conductor.

With the help of Eqs. (4.14) and (3.11), and the transfer equations (3.7) and (3.8), we can derive all the thermoelectric effects related to the heating of the electron gas.

The expression for the temperature of the electrons in the stationary case in the absence of spatial gradients, which is obtained from Eq. (4.14), agrees with Eq. (4.8), in which it is necessary to set the magnetic field equal to zero ($\alpha_H = \alpha$).

2. If the distribution function has the form (4.5), then from the normalization condition (3.4) we can find ζ, which, as simple transformations will show, is equal to the following expression in the second approximation for the case $kT(1 + \alpha)/\epsilon \ll 1$:

$$(4.15)$$

$$\zeta = z_0 + \Delta z = z_0 - \frac{(\pi kT)^2}{6} \left\{ \frac{1}{N} \frac{d}{d\epsilon} [N (1 + \alpha)] \right\}_0 ,$$

$$z_0 = \int^{\epsilon_0} \frac{d\epsilon}{1 + \alpha_H} .$$

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* The electronic thermal conductivity is determined here, as well as at low temperatures, by the mean free path for momentum transfer (and not by the mean free path for energy transfer, as is usual), since in transforming the term $\Sigma_0^{ef} (f', N_\omega^0)$ in our earlier paper[1] we did not take energy transfer into account, and consequently ignored the temperature part of the thermal resistance at low temperatures, a procedure which is justifiable only in the vicinity of the residual resistance.

We are now in a position to calculate the kinetic coefficients in the transfer equations. The calculations show that, in the absence of spatial gradients, the results obtained in the first approximation in $kT(1 + \alpha)/\epsilon$ with the aid of the nonequilibrium distribution function (4.5) agree exactly with the results of the first approximation in $k\Theta/\mu$, obtained with the aid of a distribution function similar to the equilibrium function (4.2) with a temperature Θ, determined from (4.9). Consequently, in this approximation for the galvanomagnetic phenomena, one is unable to say anything about the dependence of α_H [in Eq. (4.5)] on energy. The form of the electron distribution, which depends on the relation between electron-electron and electron-phonon interactions, plays a secondary role in comparison with the primary effect of the heating of the electrons, which is determined by the field and by the electron-phonon interaction. Therefore, even for large heating, say for $kT(1 + \alpha)/\epsilon \gg 1$ (obviously, this case cannot be realized for metals), the results obtained with the help of the functions (4.2) and (4.5) differ little.

In the presence of spatial gradients, the difference in the forms of the distribution functions (4.2) and (4.5) can be told in advance from the magnitude of the thermoelectric coefficients. This becomes apparent if one considers that thermoelectric phenomena are rather minute effects in the theory of metals, which appear only in the absence of total degeneracy. Here also the difference did not amount to enough that it was necessary to compute it for a qualitative study of the primary effect of the influence of the heating on the magnitude of the thermoelectric coefficients. For example, the equation for the thermal electromotive force, determined as the coefficient of ∇T [in Eq. (4.14) the coefficient was derived with reference to the electronic temperature Θ] takes the form, for the case (4.2),

$$\chi_T = \chi(1 + \alpha^0) \tag{4.16}$$

$$= \frac{\pi^2 k^2 T (1 + \alpha^0)^2}{\epsilon} \left\{ \frac{\epsilon}{L v N} \frac{d}{d\epsilon} (L v N) \right\}_0$$

[$\Theta = T(1 + \alpha^0)$ is the temperature of the electrons], and for the case (4.5)

$$\tag{4.17}$$

$$\chi_T' = \frac{\pi^2 k^2 T (1 + \alpha^0)^2}{\epsilon_0} \left\{ \frac{\epsilon}{L v N (1 + \alpha)} \frac{d}{d\epsilon} [L v N (1 + \alpha)] \right\}_0$$

From the above it is clear that for understanding the principal characteristics of most of the effects, it is quite possible to use a distribution function of the form (4.2), even though the equilibrium distribution may not in fact be realized. At the same time, calculations carried out with the function (4.2) are made considerably easier.

5. GALVANOMAGNETIC AND THERMOELECTRIC PHENOMENA

Before proceeding with an investigation of these phenomena, let us say a few words about the character of the hypotheses which have been made and the experimental conditions which they presuppose.

For the determination of the temperature of the electrons in the case of a single stationary field [Eq. (4.8)] we implicitly assumed that the temperature of the lattice, T, was given, and did not consider Eq. (3.11) at that time, since it follows from this equation that

$$(\Theta - T)/BT = \text{div } \mathbf{W}_p = -\nabla(\varkappa_p \nabla T),$$

that is, the total flow of energy given up by the electrons to the lattice has to be able to be removed by thermal conduction in the lattice. On the other hand, in throwing away the term div $\mathbf{W}_e$ in Eq. (3.6) we actually assumed that the inequality $\nabla(\varkappa_p \nabla T) \gg \nabla(\varkappa_e \nabla \theta)$ was satisfied. With the weakening of this inequality, the relative heating of the electrons decreases, and in the case of the reverse inequality there is no noticeable heating. A similar relation can be satisfied either in the case $\nabla\Theta \to 0$, that is, when the transfer of energy by electrons to an external medium at the boundaries of the conductor is somehow hampered, or in the case $\varkappa_p \gg \varkappa_e$. For good conductors, it is known that the latter inequality is not satisfied, not only for high, but also for low temperatures in the region of residual resistance, because in this region

$$\varkappa_e \sim c_e l_{ep} v \sim nk (kT/\epsilon_0) l_{ep} v$$

$$\gg \varkappa_p \sim c_p l_{fp} v_s \sim n_a (T/\Theta_d)^3 l_{fp} v_s.$$

Furthermore, the current density for which the relative heating of electrons $(\Theta - T)/T$ remains appreciable is so large, in the case of metals, that the heat evolved simply cannot be removed from the surface of the conductor. The latter difficulty can to some extent be ameliorated by placing the conductor in liquid helium, which possesses a very

large thermal conductivity. However, this does not save the situation, in view of the inequality $\varkappa_p \ll \varkappa_e$ in the region of residual resistance. Consequently, it is impossible to obtain noticeable related heating of electrons in the stationary case for metals. For semiconductors with classical statistics the inequality $\varkappa_e \ll \varkappa_p$ leads to the requirement $n \ll n_a l_{ff} v_s / l_{ef} \sim v_s n_a / v$ and is realized for many semiconductors.

In the nonstationary case the equation for the temperature Θ (4.8) is somewhat changed. The temperatures Θ and T are determined from Eqs. (3.6) or (4.14) and (3.11), which, considering that $\epsilon_e = c_e \Theta$, $\epsilon_p = c_p T$, take the following form in the absence of heat exchange with the surroundings:

$$c_e \partial \Theta / \partial t + (\Theta - T) / BT = j^2 / \sigma, \qquad (5.1)$$

$$c_p \partial T / \partial t - (\Theta - T) / BT = 0.$$

For squared current impulses lasting for a length of time $t \gg \tau_T = c_p c_e TB / (c_p + c_e)$ (for metals $\tau_T \sim 10^{-13}$ sec), along with the assumption that c_e, c_p, j^2 / σ and BT do not depend on the temperature, the solution of the system of Eq. (5.1) gives

$$T - T_0 = R j^2 t / (c_p + c_e), \qquad (5.2)$$

$$\Theta - T_0 = R j^2 / (c^p + c_e), \qquad (5.3)$$

$$\Theta - T = TBR j^2 c_p / (c_p + c_e), \qquad (5.4)$$

where T_0 is the initial temperature, and $R = 1/\sigma$. Equation (5.4) remains valid also if c_p, c_e, etc., do depend on temperature, provided only that $c_p \gg c_e$. This formula differs from Eq. (4.8) only in the factor $c_p / (c_p + c_e)$, which for metals can be markedly less than unity only at low temperatures. Thus for the momentum process the relation $c_p \gg c_e$ is the most advantageous one for realizing electron heating.

We will state some final results for the effects mentioned in the heading of this Sec:

1. *Electrical conductivity.* If we calculate the temperature correction for $k\Theta/\mu \ll 1$ in the usual way, and recall that the results obtained for the galvanomagnetic effects, obtained using Eqs. (4.2) and (4.5), are identical here, we obtain

$$(\sigma - \sigma_0) / \sigma_0 = (k\Theta / \varepsilon_0)^2 \Lambda_{10} \qquad (5.5)$$

$$= (kT)^2 (1 + \alpha^0)^2 \Lambda_{10} / \varepsilon_0,$$

where σ_0 is the electrical conductivity for weak currents, α^0 is determined from (4.9) with $H = 0$, and

$$\Lambda_p = \frac{\pi^2}{6} \frac{\varepsilon^2}{Lv N\varepsilon^{n-1}} \left\{ \frac{d^2}{d\varepsilon^2} (Lv N\varepsilon^{n-1}) \right. \qquad (5.6)$$

$$\left. - \frac{1}{N} \frac{dN}{d\varepsilon} \frac{d}{d\varepsilon} (Lv N\varepsilon^{n-1}) \right\}$$

is a coefficient of order of magnitude unity under ordinary conditions.

As already discussed in previous papers[2,5], deviations from Ohm's law, according to Eq. (5.5), can be approached for experimentally attainable current densities only for poor metals like bismuth.

2. *Resistance changes in a magnetic field.* We can obtain a relation similar to (5.5):

$$\frac{\sigma - \sigma_0}{\sigma} = \left[\frac{(kT)^2 (1 + \alpha_H)^2}{\varepsilon^2} \frac{\Lambda_1^1 + \beta^2 (2\Lambda_1^2 - \Lambda_1^1)}{1 + \beta^2} \right]_0 \qquad (5.7)$$

where the coefficients Λ_n^m differ from Eq. (5.6) for Λ_n by a factor $1/\beta^{m-1}$ outside the curly brackets and a factor $\beta^{n-1}/(1 + \beta^2)$ in the parentheses ($\beta = eLH/cmv$).

For small currents, $\alpha_H = 0$, the result agrees with the well-known result obtained for the isotropic model of a metal in the usual theory, which predicts a change in the electrical conductivity in a magnetic field for $kT\beta/\epsilon \sim 1$. An accurate estimate, which answers to experimental fact, is obtainable only from an anisotropic model (nonspherical Fermi distribution in momentum space) and indicates a magnetc field dependence for $\beta \sim 1$. Our original equations were not applicable to the anisotropic model, and consequently we cannot say whether or not Eq. (5.7) is changed in this case. However, the fact that α_H and β in Eq. (5.7) correlate suggests that in an anisotropic model in a magnetic field, deviations from Ohm's law might set in sooner than the isotropic model would predict; namely, for $\alpha_H \sim 1$ instead of for $\alpha_H kT/\epsilon \sim 1$. Furthermore, for those metals for which the change of electrical conductivity in a magnetic field does not undergo saturation, the factor in (5.7) depending on the magnetic field can become large enough to reduce the magnitude of the current, so that we might expect a dependence of the electrical conductivity on the current.

For strong currents, changes in the Hall constant are determined by a formula similar to (5.7).

For thermoelectric phenomena the temperature of the electrons for weak currents is determined from the equation [see (4.14) and (4.16)]

$$\alpha = (\Theta - T)/T \qquad (5.8)$$

$$= Bj^2/\sigma - (Bj\Theta/e)\, db\Theta/dx = \alpha_{tr} + \alpha_{th}\ ,$$

where the thermal electromotive force $\chi = b\Theta$ and

$$b = \frac{\pi^2}{3}\left[\frac{k^2}{L\upsilon N}\right]\frac{d}{d\varepsilon}(L\upsilon N)\Big]_0$$

for the case $k\Theta/\mu \ll 1$.

Since the thermoelectric phenomena appear as second order effects in the theory of metals, vanishing for complete degeneracy, their deviations from linearity have already set in for $\alpha \sim 1$, instead of for $\alpha kT/\epsilon_0 \sim 1$, as in the case of the electrical conductivity.

3. *The Thomson coefficient* [b = const, $\Theta = \Theta(x)$].

$$\zeta = -Q_T/j\,\frac{d\Theta}{dx} = b\Theta/e \qquad (5.9)$$

will depend on the current for $\alpha \sim 1$. However, for such a value of α the required currents are already so large that in Eq. (5.8) $\alpha_{tr} \gg \alpha_T$. This circumstance makes it impossible for us to detect the influence of changes in the direction of the current on the temperature.

4. *The Peltier coefficient* [Θ = const, $b = b(x)$].

$$\Pi_{12} = -\frac{1}{j}\int_{(1)}^{(2)} Q_\pi\, dx = \frac{\Theta^2}{e}\,\Delta b, \quad (5.10)$$

where $\Delta b = b_2 - b_1$ is the change in the quantity b over the region Δx in going from one metal to the other. The temperature of the electrons in this region is determined from Eq. (5.8), in which it is necessary to set $/db\Theta/dx \sim \Theta\Delta b/\Delta x$. In contrast with the Thomson effect, in the Peltier effect the coefficient α_π can approach unity for currents for which α_{tr} is still small compared with 1. This occurs because of the smallness of the transition region Δx. However, the impossibility of controlling the temperature in the momentum system over such a small region as Δx apparently deprives the Peltier effect of this advantage in practice.

5. *The resistance of a polycrystal.* It appears that there is a medium, nevertheless, in which electron heating at the expense of thermoelectric heat can still be observed. This medium is a semiconductor, between whose individual grains a contact potential difference can exist. By a simple analog, this structure can serve as a closed circuit, consisting of fragments of two dissimilar metals. If the contacts exist at the same temperature, then the contact resistance of such a circuit will be zero, in accordance with the fact that the thermoelectric effects at different junctions have opposite sign. However, precisely because of this difference, the temperature of the electrons in the region of contact will, depending on the sign of α_{th}, either be greater or less than the average temperature of the electrons in the conductor. Determining the potential difference over the region h, inside of which there are two contacts,

$$V_h = \int_h E\,dx = \int_h (j/\sigma + (\chi/e)d\Theta/dx)\,dx$$

[for a closed circuit $\int \Delta\mu\, dx = 0$ and, consequently, $\int(\Delta b)\, dx = 0$] we obtain, for $\alpha_\pi \ll 1 + \alpha_{tr}$

$$V_h = jhR_0\left[1 + \frac{2B}{h\Delta xR_0}(\Pi_{12}^0)^2(1 + \alpha_{tr})^3\right], \quad (5.11)$$

$$\Pi_{12}^0 = \Pi_{12}(T).$$

Applied to a polycrystal, (5.11) gives

$$\frac{\sigma_0 - \sigma}{\sigma} = \frac{R - R_0}{R} = \frac{\Delta R}{R} \qquad (5.12)$$

$$= \frac{2B(\Pi_{12}^0)^2\sigma_0}{\Delta x\,\Delta y}(1 + Bj^2R_0)^3,$$

where Δy is the linear dimension of an individual grain, Δx is the width of the transition region between them. Actually, on account of the thermal conductivity, which we did not take into consideration, this region will expand, although it will not exceed the dimensions of Δy.

For $\alpha_{tr} \ll 1$ the additional resistance will not depend on the current density. If we take $\Pi_{12}^0 \sim (kT)^2/e\epsilon_0$, then for low temperatures (for $\Theta_d < T < T_0$ the effect is insignificant) we obtain in order of magnitude

$$(5.13)$$

$$\frac{\Delta R}{R} \sim \frac{kT}{mv_s^2}\left(\frac{T}{T_0}\right)^3\frac{al\,l_{ep}}{\Delta x\Delta y} \sim \frac{l_{ep}\upsilon h}{\Delta x\Delta y mv_s^2}\left(\frac{\Theta_d}{T_0}\right)^2\frac{\Theta_d}{T}$$

[noting that $l = l_{ef} \sim (vh/ak\,\Theta_d/T)$]. The quantity $l_{ep}vh/\Delta x\,\Delta y\dot{m}v_s^2 \sim 10^{-1}$ [for $l_{ep} \sim 10^{-4}$, $v \sim 10^8$, $v_s^2 \sim 10^{11}$, $\Delta x \sim \Delta y \sim 10^{-3}$), so that $\Delta R/R \sim 1$ for $T/\Theta_d \sim (\Theta_d/T_0)^3$.

This region of temperature is attainable only for conductors with low degeneracy temperatures (i.e., with small numbers of free electrons). Thus, for poor conductors, in the momentum regime, one can attempt to observe a reversal of the temperature dependence of the resistance in the low temperature region.

The theory which has been developed is entirely applicable to semiconductors, which in a series of cases have a region of temperatures in which the conduction electrons are degenerate, and for which it is impossible to use classical statistics. Recently, more and more evidence has appeared for the existence of these regions at low temperatures, even for pure semiconductors[11]. For such semiconductors an increase in resistance with decrease in temperature can also arise in the stationary case at low temperatures, in accordance with Eqs. (2.12) and (2.13) ($\varkappa_p \ll \varkappa_e$ at low temperatures because of the small number of electrons), in spite of the fact that the conductor ought to possess metallic-type conductivity in the region of degeneracy.

An investigation of the electrical properties of semiconductors in strong fields in the region of degeneracy has an advantage over an investigation in the region where classical statistics apply, in that in the first case it is much simpler to solve the problem of the number of free carriers of charge, whose increase with field apparently sets in long after the field has removed the degeneracy. This situation contrasts with that for the high temperature region, where classical statistics are obeyed, where it is impossible to be absolutely certain that the

increase in the number of free electrons with field shows up in the electrical conductivity and in other effects later than in the relative increase of the temperature of the electrons. On the contrary, there is reason to believe that the hypothesis, on which other work has been based[2,12], concerning the invariability of the number of charge carriers with field down to the critical fields where Ohm's law sets in, is in general wrong[5].

In conclusion, I express my profound gratitude to V. L. Ginzburg for his interest in the work and for his valuable comments.

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Translated by W. M. Whitney
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Other Errata

Page	Column	Line	Reads	Should Read
Volume 4				
38	1	Eq. (3)	$\ldots \dfrac{\pi r^2 \rho^o \rho^2 n}{\rho_s^2},$	$\ldots \dfrac{\pi r^2 \rho^o \rho_n}{\rho_s^2},$
196		Date of submittal	May 7, 1956	May 7, 1955
377	1	Caption for Fig. 1	$\delta_{35} = \eta - 21 \cdot \eta^5$	$\delta_{35} = -21^\circ \eta^5 \cdot$
377	2	Caption for Fig. 2	$\alpha_3 = 6.3^\circ \, \eta$	$\alpha_3 = -6.3^\circ \eta$
516	1	Eq. (29)	$s^2/c^2 \ldots$	s/c
516	2	Eqs. (31) and (32)	Replace $A_1 s^2/c^2$ by A_1	
497		Date of submittal	July 26, 1956	July 26, 1955
900	1	Eq. (7)	$\ldots \dfrac{i}{4\pi} \displaystyle\sum_{c,\,\alpha} \dfrac{\partial w_a\,(t,\,P)}{\partial P^\alpha} \ldots$	$\ldots 2\pi^2 i \displaystyle\sum_{c,\,\alpha} \ldots$ (This causes a corresponding change in the numerical coefficients in the expressions that result from the calculation of the effects of the plasma particles on each other).
804	2	Eq. (1)	$\ldots \exp\{-(\overline{T} - V')\}$	$\ldots \exp\{-(\overline{T} - V')\tau^{-1}\}$
Volume 5				
59	1	Eq. (6)	$v_l\,(1\partial F_0/\partial \mathbf{x}) + \ldots$ where E_l is the projection of the electric field E on the direction l	$\overline{(\mathbf{v}\partial F_0/\partial \mathbf{x})} + \ldots$ where the bar indicates averaging over the angle θ and E_l is the projection of the electric field E along the direction l
91	2	Eq. (26)	$\Lambda = 0.84\,(1+22/A)$	$\Lambda = 0.84/(1+22/A)$
253		First line of summary	$Tl^{204,\,206}$	$Tl^{203,\,205}$
318	1	Figure caption	$\ldots e^2 mc^2 = 2.8 \cdot 10^{-23}$ cm,	$\ldots e^2/mc^2 = 2.8 \cdot 10^{-13}$ cm,
398		Figure caption	$\ldots$ to a cubic relation. A series of points etc.	$\ldots$ to a cubic relation, and in the region 10 -20°K to a quadratic relation. A series of points ●, coinciding with points O, have been omitted in the region above 10°K.