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Two Electron Charge Exchange of α-Particles in Helium

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The cross section for capture of two electrons by a fast α-particle colliding with a helium atom is calculated in the Born approximation.

THE effect of charge exchange, which occurs when positive ions pass through matter, has been studied thoroughly, both experimentally and theoretically. But in most cases, only collisions with single electron charge exchange have been considered. Recently, Fogel', Krupnik and Safranov[1], in the study of the processes of charge exchange of protons in gases, have observed the capture of two electrons in a single elementary process. They found a value of $\sigma = 2.3 \times 10^{-18}$ cm^2 for the cross section of a 21 kev proton transformed into a negative H^- ion in the collision with a hydrogen molecule.

The theory of multiple charge exchange can be treated by the same approximation methods as those used for the theory of atomic collisions;[2] these permitted the computation of single electron processes. For instance,[3] we used the method of perturbing the stationary states to determine the cross section for the simplest two-electron process $H^+ + He \to H^- + He^{++}$ in the case where the velocity of the proton is much smaller than the velocity of the electrons in the atom. In the present paper, the opposite limiting case of fast collisions is considered, and the Born approximation is applied.

Let us consider the case of the collision of a two-electron atom containing nucleus 1 (atomic number Z_1 mass number A_1) with a nucleus 2 (Z_2, A_2); as a result of the collision, both electrons are captured by 2. If we separate the center-of-mass motion, the Hamiltonian of the problem can be written in the form*

$$\hat{H} = -(1/2\mu_2)\,\Delta_\rho + \hat{H}_2 - Z_1(1/r + 1/r') + Z_1 Z_2/|\mathbf{r} - \mathbf{s}|; \tag{1}$$

where $\mathbf{r}$ and $\mathbf{r}'$ ($\mathbf{s}$, $\mathbf{s}'$) are the radius vectors of the electrons with respect to the nucleus 1 (2); ρ is the radius vector of the center of mass of the two-electron atom 1 with respect to the nucleus 2; $\mu_2 = MA_1(A_2 + 2\epsilon)(A_1 + A_2 + 2\epsilon)^{-1}$ is the reduced mass ($\epsilon = M^{-1}$ is the ratio of the electron mass and the nucleon mass); and $\hat{H}_2$ is the Hamiltonian of the two-electron atom 2

$$\hat{H}_2 = -\frac{1}{2}\left(1 + \frac{\varepsilon}{A_2}\right)(\Delta_{\mathbf{s}} + \Delta_{\mathbf{s}'}) - \frac{\varepsilon}{A_2}(\nabla_{\mathbf{s}}\nabla_{\mathbf{s}'}) - Z_2\left(\frac{1}{s} + \frac{1}{s'}\right) + \frac{1}{|\mathbf{s} - \mathbf{s}'|}. \tag{2}$$

We denote by $\chi_n^{(2)}(\mathbf{s}, \mathbf{s}')$ the orthonormal eigenfunctions of the operator $\hat{H}_2$

$$\hat{H}_2\chi_n^{(2)} = W_n^{(2)}\chi_n^{(2)}.$$

The function $\chi_0^{(2)}$ describes the ground state of the atom 2, and $W_0^{(2)}$ is the binding energy of the electrons in this state.

We seek a solution of the Schrödinger equation $\hat{H}\Psi = W\Psi$ in the form

$$\Psi = \sum_n F_n(\rho)\,\chi_n^{(2)}(\mathbf{s}, \mathbf{s}'). \tag{3}$$

We have the following equation for $F_0(\rho)$

$$(\Delta_\rho + k_2^2)F_0(\rho) = -2\mu_2\iint d\mathbf{s}\,d\mathbf{s}'\,\chi_0^{(2)*}(\mathbf{s}, \mathbf{s}')\,V(\rho, \mathbf{s}, \mathbf{s}')\,\Psi(\rho, \mathbf{s}, \mathbf{s}'),$$

$$V = Z_1(1/r + 1/r' - Z_2/|\mathbf{r} - \mathbf{s}|), \tag{5}$$

$$k_2^2 = 2\mu_2(W - W_0^{(2)}) = 2\mu_2 E_2; \tag{6}$$

where E_2 is the kinetic energy of the outgoing particles in the center-of-mass system. We are interested in the solutions of (4) which behave, at large ρ, as $F_0(\rho) \sim \rho^{-1}e^{ik_2\rho}f(\vartheta)$. Obviously,

$$f(\vartheta) = \frac{\mu_2}{2\pi}\iiint d\rho\,d\mathbf{s}\,d\mathbf{s}' \times e^{-i\mathbf{k}_2\rho}\chi_0^{(2)*}(\mathbf{s}, \mathbf{s}')\,V(\rho, \mathbf{s}, \mathbf{s}')\,\Psi(\rho, \mathbf{s}, \mathbf{s}') \tag{7}$$

and the cross section for the capture of the two electrons into the ground state of atom 2 is equal to

$$d\sigma = (k_2/k_1)|f(\vartheta)|^2\,d\Omega. \tag{8}$$

*All the magnitudes are expressed in the system where $\hbar = e = m = 1$.

The above equations are exact; later we make the following approximations:

1. Born approximation: the wave function in (7) is replaced by the expression

$$\Psi \approx e^{i\mathbf{k}_1\sigma}\chi_0^{(1)}(\mathbf{r}, \mathbf{r}'), \qquad (9)$$

where σ is the radius vector of the incoming nucleus 1 with respect to the center of mass of the two-electron atom 2.

2. The following approximate expressions are used for the ground state wave functions of the atoms 1 and 2

$$\chi_0^{(1)} = (\alpha_1^3/\pi)\, e^{-\alpha_1(r+r')}, \qquad \chi_0^{(2)} = (\alpha_2^3/\pi)\, e^{-\alpha_2(s+s')}. \qquad (10)$$

The criterion for the validity of the Born approximation is usually given by the inequality $Z_1Z_2/v \ll 1$ but the study of the single electron process $H^+ + H \to H + H^+$ has shown[4] that the Born approximation gives results in good agreement with the experimental data for smaller velocities, down to $v \sim 1$. Apparently this is related to the fact that in the initial state as well as in the final one, one of the colliding particles is neutral, and the Coulomb interaction practically vanishes for distances somewhat larger than the atomic radius. Therefore a plane wave is a sufficiently good description of the motion of the atom and of the ion, even for not very high energies.

The same situation takes place in the case of a two-electron capture by the process

$$He^{++} + He \to He + He^{++}. \qquad (11)$$

In this case, the criterion for the validity of the Born approximation is not obvious. If one requires $4v \leq 1$, then $E_{lab} \gtrsim 1.5$ mev; but if one relaxes the condition to $v_0 \lesssim v$ (where $v_0 = \alpha$ is the mean velocity of the electrons in the helium atom), the condition on the energy becomes weaker: $E_{lab} \gtrsim 0.3$ mev. (Note that in the case of proton charge exchange in hydrogen, both criteria coincide: $v \sim 1$.) Notwithstanding the fact that the nuclei 1 and 2 are identical in the process (11), the symmetrization of the wave function is not necessary because, for high energies, the nuclei are practically distinguishable.

For the sake of simplicity, all the following formulas refer to the process (11); their generalization to the case of arbitrary Z and A does not present any difficulty.

After an obvious change of integration variables, the amplitude $f(\vartheta)$ for a double charge exchange of α-particles in helium is equal to

$$f(\vartheta) \approx 4(J_1 - J_2)/\pi^3\varepsilon\alpha^2; \qquad (12)$$

$$J_1 = \int d\mathbf{R}\, e^{i\mathbf{gR}} \int \frac{d\mathbf{r}}{r} e^{-i\varkappa\mathbf{r} - r - |\mathbf{r}-\mathbf{R}|} \times \int d\mathbf{r}'\, e^{-i\varkappa\mathbf{r}' - r' - |\mathbf{r}'-\mathbf{R}|}, \qquad (13)$$

$$J_2 = \int \frac{d\mathbf{R}}{R} e^{i\mathbf{gR}} \left(\int d\mathbf{r}\, e^{-i\varkappa\mathbf{r} - r - |\mathbf{r}-\mathbf{R}|}\right)^2; \qquad (14)$$

$$\mathbf{g} = \frac{1}{\alpha}\left(\mathbf{k}_1 - \frac{\mathbf{k}_2}{1+\varepsilon/2}\right), \quad \varkappa = \frac{\varepsilon(\mathbf{k}_1 + \mathbf{k}_2)}{4\alpha(1+\varepsilon/2)}, \qquad (15)$$

$$k_1^2 = k_2^2 = k^2 \approx (2/\varepsilon)\, E_{lab}, \quad \times \cos\vartheta = (\mathbf{k}_1\mathbf{k}_2)/k^2, \quad \alpha = 1.69. \qquad (16)$$

Let us express the integrals J_1 and J_2 in a form suitable for numerical integration. We represent

$$Q(\mathbf{R}) = \int d\mathbf{r} \exp(-i\varkappa\mathbf{r} - r - |\mathbf{r} - \mathbf{R}|)$$

in a Fourier integral

$$Q(\mathbf{R}) = \frac{8}{\pi}\int d\boldsymbol{\xi}\, e^{i\boldsymbol{\xi}\mathbf{R}}(1+\xi^2)^{-2}[1 + (\boldsymbol{\xi}+\varkappa)^2]^{-2},$$

using the identity[5]

$$\frac{1}{a^2b^2} = \int_0^1 \frac{6x(1-x)\,dx}{[ax + b(1-x)]^4},$$

letting $a = 1 + (\xi + \varkappa)^2$, $b = 1 + \xi^2$; we denote

$$p^2 = \varkappa^2 x(1-x) + 1, \quad q^2 = \varkappa^2 y(1-y) + 1. \qquad (17)$$

We then have

$$Q(\mathbf{R}) = (48/\pi)\int_0^1 dx\,(1-x)\,x e^{-ix\varkappa\mathbf{R}} \times \int (\eta^2 + p^2)^{-4}\, d\eta\, e^{i\eta\mathbf{R}}$$

$$= 2\sqrt{2\pi}R^{5/2}\int_0^1 dx\,(1-x)\,x p^{-5/2} e^{-ix\varkappa\mathbf{R}} K_{5/2}(pR);$$

the integral J_2 is equal to

$$J_2 = 48\pi^3 \int_0^1\!\!\int_0^1 dx\, dy\, xy(1-x) \times (1-y)\, p^{-5} q^{-5} \lambda F_2(x, y), \qquad (18)$$

$$F_2 = 3 + 2(2p^2 + 3pq + 2q^2)\lambda \qquad (19)$$
$$+ 8(p^4 + 2p^3q + 3p^2q^2 + 2pq^3 + q^4)\lambda^2$$
$$+ 48pq(p^2 + q^2)(p+q)^2\lambda^3 + 128p^2q^2(p+q)^4\lambda^4,$$

$$1/\lambda = (p+q)^2 + [g - (x+y)\varkappa]^2. \qquad (20)$$

Similar manipulations yield

$$J_1 = 384\,\pi^3$$
$$\times \int_0^1\!\!\int_0^1 dx\,dy\,xy(1-x)\,p^{-5}q^{-3}\lambda^3 F_1(x, y), \qquad (21)$$

$$F_1 = q^3 + 2p(p+q)^2(p^2 - 2pq + 3q^2)\lambda \qquad (22)$$
$$+ 16p^2q(p+q)^4\lambda^2.$$

It is easy to see that for energies $E_{lab} \sim 1$ mev, when the Born approximation begins to be valid, each of the magnitudes p, q and $\varkappa^2$ does not exceed a few units, and $k^2 \sim 10^8$; therefore, for $\vartheta \gg 1/k \sim 10^{-4}$:

$$\lambda^{-1} = 2(1 + pq) + (1 - x - y + 2xy)\varkappa^2$$
$$+ (4k^2/\alpha^2)\sin^2(\vartheta/2)$$
$$\approx (2k/\alpha)^2 \sin^2(\vartheta/2) \gg 1,$$

and $F_2 \approx 3$. Formula (18) becomes

$$J_2 \approx \frac{36\pi^3\alpha^2}{\sin^2(\vartheta/2)}\left(\int_0^1 \frac{dx\,(1-x)\,x}{[\varkappa^2 x(1-x) + 1]^{5/2}}\right)^2,$$

and since, for $\lambda \ll 1$ $J_1 \ll J_2$, the amplitude $f(\vartheta)$ is equal to

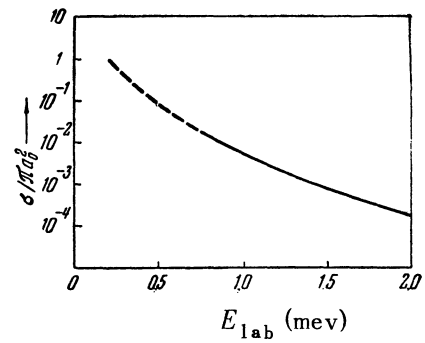
$$f(\vartheta) \approx -\frac{4}{\varepsilon k^2 \sin^2(\vartheta/2)}\left[1 + \left(\frac{\varepsilon k}{4\alpha}\right)^2 \cos^2\frac{\vartheta}{2}\right]^{-4}. \qquad (23)$$

The contribution to the charge exchange cross section σ from angles $\vartheta > \vartheta_0 \sim 10^{-3}$ to 10^{-2} amounts to

$$\frac{64\pi}{\varepsilon^2 k^4}\int_0^{1-(\vartheta_0/2)^2} \frac{dx}{(1-x)^2\left[1 + \left(\frac{\varepsilon k}{4\alpha}\right)^2 x\right]^8} \qquad (24)$$
$$\approx \frac{256\pi}{\varepsilon^2 k^4 \vartheta_0^2}\left[1 + \left(\frac{\varepsilon k}{4\alpha}\right)^2\right]^{-8}.$$

For angles $\vartheta < \vartheta_0$ the integrals J_1 and J_2 were obtained by numerical integration.

The results of our calculations of the cross section for the process (11), as a function of the energy of the incoming α-particle in the Laboratory system, are plotted in the graph. When comparing with experiment, one should remember that the capture into excited states of the helium atom has not been taken into account.



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