agree qualitatively with the data for dialogite. It follows from the data of Bizette and Tsai that an uncompensated moment is directed perpendicular to the trigonal axis; this supports our first proposal.

In closing, the authors convey their profound thanks to Prof. P. G. Strelkov for his constant interest in the work.

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Dispersion Relations for Scattering and Photoproduction

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A derivation is given of the dispersion relations for the following problems: scattering of pions by nucleons (excluding the case of scattering at small but nonzero angles), photoproduction of pions on nucleons, scattering of nucleons and antinucleons by nucleons. The method of consideration is based on the general requirement of the impossibility of the propagation of signals with velocities exceeding that of light, and does not make use of the concept of the *S* matrix for demonstration of the analytical properties of the scattering amplitude. The issue is decided as to whether requirement of microcausality is a necessary condition for the validity of the dispersion relations and it is pointed out that for certain types of violation of causality, the dispersion relations are preserved.

RECENTLY, Goldberger and others[1-4] obtained dispersion relations for the problem of the scattering of mesons from nucleons. In their derivation of the dispersion relations, these authors relied on the concept of microcausality; they made use of a series of general situations of quantum field theory in its present-day formulation. In view of the great generality of the dispersion relations, there is interest in giving another (and, as it appears to us, simpler) derivation of these relations, relying essentially only on the requirement of the impossibility of the propagation of signals with velocities greater than the velocity of light. The method set forth allows us to draw several conclusions on the problem of whether it is necessary, for the existence of dispersion relations, that the propagation velocity of the interaction be smaller than the velocity of light everywhere, even at microscopic distances (of the order of a nuclear distance), or whether it suffices to fulfill this condition only for macroscopic distances. In this case it appears that the dispersion relations are preserved if we assume that the interaction can propagate, not inside the light cone $t^2 - r^2 > 0$, but inside the hyperboloid $t^2 - r^2 > -l_0^2$ (l_0 is a distance of the order of a nuclear distance), i.e., when a condition which violates causality is imposed in the interval.

In the present paper, the dispersion relations are considered for the scattering and photoproduction of pions on nucleons, and for the problem of scattering of nucleons and antinucleons by nucleons.

1. SCATTERING OF PIONS BY NUCLEONS

Let us consider the scattering amplitude (without charge exchange) of $\pi^{\pm}$-mesons by protons, $f_{\pm}(\omega, \theta)$, in which we first limit ourselves to the case of forward scattering*,

* We shall neglect Coulomb scattering.

Let $f_{\pm}(\omega, 0) \equiv f_{\pm}(\omega)$, given as a function of the frequency ω on the real axis for $\omega > \mu$ (μ = mass of meson, $\hbar = c = 1$), analytically continued over the entire complex plane ω. We shall show that $f_{\pm}(\omega)$ has no poles in the upper half plane of ω and vanishes sufficiently rapidly on a semicircle of large radius $\Omega \to \infty$ in this half plane.

We shall carry out our analysis in the laboratory system of coordinates, where the nucleon is at rest before and after the collision. In this case, it is appropriate for us to represent the wave function of the nucleon in the form of a wave packet (naturally, of sufficiently large dimensions that the condition of finding the nucleon in a state of rest is satisfied with any degree of accuracy necessary for us). The wave function $\psi_{\pm}(\mathbf{r})$ of the $\pi^{\pm}$-meson that is scattered forward is chosen (at large distances from the scatterer) in the following fashion from the wave function of the incident meson $e^{ikz-i\omega t}$ (origin of the coordinates at the center of the wave packet):

$$\psi_{\pm}(\mathbf{r}) = f_{\pm}(\omega)\, e^{ikz-i\omega t}/r = \int K_{\pm}(t,\ t',\ \mathbf{r},\ \mathbf{r}')\, e^{ikz'-i\omega t'} ds\, dt'. \tag{1}$$

Integration on the right side of (1) is carried out over the region inside the light cone $(t - t')^2 > (\mathbf{r} - \mathbf{r}')^2$, where it suffices to carry out the spatial integration, in accordance with Huygen's principle, over any closed surface surrounding the point $\mathbf{r}$ that does not penetrate the region where the wave function of the pion differs from that function free of the presence of the nucleon. As such a surface we shall choose a plane which is perpendicular to the direction of the momentum of the incident meson (the z axis) and which passes beyond the wave packet [the infinite semicircle closing it does not give a contribution to the integral (1)]. The function K* depends on the coordinates of the wave packet, i.e., on the energy of the nucleon in addition to the coordinates $\mathbf{r}$, $\mathbf{r}'$ and the times t, t' of the incident and scattered particles.

For forward scattering in the laboratory system, the initial and final energy of the nucleon is equal to M, the rest mass, and does not depend on ω; therefore, the entire dependence of the right side on ω is determined by the factor $e^{-i\omega t'+ikz'}$. Multiplying (1) by $e^{-ikz+i\omega t}$ and introducing the new variables $\tau = t - t'$, $\boldsymbol{\rho} = \mathbf{r} - \mathbf{r}' = \{\xi, \eta, \zeta\}$, we get (1) in the form

$$\frac{1}{r} f_{\pm}(\omega) = \int_{\tau > \rho} d\tau\, d\xi\, d\eta K_{\pm}(\tau, \rho, \mathbf{r}, t)\, e^{i\omega\tau - ik\xi}. \tag{2}$$

It is evident from (2) that when ω is found in the upper half plane ($\mathrm{Im}\,\omega > 0$), an exponentially decaying (with increase in τ) factor appears under the integral of Eq. (2). This factor guarantees the convergence of the integral. It then follows that $f_{+}(\omega)$ cannot have poles in the upper half plane of ω. In order to analyze the behavior of $f_{\pm}(\omega)$ on a large semicircle in the upper half plane of ω, we note that for $\omega \to \infty$, $k \approx \omega - \mu^2/2\omega$ and, consequently, $\mathrm{Im}(\omega\tau - k\zeta) \to \mathrm{Im}\,\omega(\tau - \zeta) + O(\mu/\omega)\mu\zeta$. Inasmuch as $\tau > \zeta$, then $\mathrm{Im}(\omega\tau - k\zeta) > 0$, and tends toward infinity for $\mathrm{Im}\,\omega \to \infty$. Thus, $f_{\pm}(\omega)$ disappears, or at least does not increase, on the large semicircle in the upper half plane of ω.

Making use of two properties of $f_{\pm}(\omega)$ in the upper half plane of ω given above (the absence of poles and the vanishing on a semicircle of large radius), it is easy to obtain a correction (with the help of the Cauchy theorem) between the real and imaginary parts of $f_{\pm}(\omega)$:

$$\mathrm{Re} f_{\pm}(\omega) = \frac{1}{\pi} \mathrm{P} \int_{-\infty}^{\infty} \frac{\mathrm{Im} f_{\pm}(\omega')}{\omega' - \omega} d\omega', \tag{3}$$

where the integration on the right side of (3) in the neighborhood of the pole $\omega' = \omega$ is taken in the sense of the principal value, and all the singularities of the amplitude of $f_{\pm}(\omega)$ which lie on the real axis, are detoured above. Equation (3) is correct if $f_{\pm}(\omega)$ vanishes with sufficient rapidity as $\omega \to \pm\infty$. If this is not so, then the correct result can be obtained by considering the difference $f_{\pm}(\omega) - f_{\pm}(\omega_0)$ in place of $f_{\pm}(\omega)$. Here ω_0 is some fixed frequency. This is equivalent to a subtraction from both sides of (3) of its value for $\omega = \omega_0$.

We now turn our attention to a consideration of the scattering at an angle not equal to zero. In this case, as was shown by Salam[4], it is appropriate to transform to a system of coordinates in which the sum of the initial p and the final p′ of the

* The function K is directly connected with the nucleus by a two-particle equation for the system meson-nucleon. Within the framework of the present formalism, Eq. (1) can be obtained from the properties of the two-particle equation upon consideration of the fact that the nucleon is centered in a small region near the origin of the coordinates.

momenta of the nucleon is equal to zero: $\mathbf{p} + \mathbf{p}' = 0$. It is easy to see that in this system of coordinates, $\omega = \omega'$, $k = k'$, and in place of (1), we will have

$$f_{\pm}(\omega, \theta)/r \tag{4}$$
$$= \int K_{\pm}(t, t', \mathbf{r}, \mathbf{r}') e^{i\omega(t-t')} e^{-i(kr-\mathbf{kr}')} ds\, dt'.$$

The integration in (4) is carried out over a plane which passes through $\mathbf{p}$. Then the function K will depend on ω only through the absolute value of the momentum of the nucleon, $\mathbf{p}$. We denote $q = 2|\mathbf{p}| = 2k \sin(\theta/2)$ and shall consider the scattering amplitude as a function of ω for fixed q. By the same arguments as in the case of forward scattering, we can show that $f_{\pm}(\omega, q)$ has no poles in the upper half plane of ω. For the investigation of its behavior on the large circle, we denote $t - t' = \tau$, $\mathbf{r} - \mathbf{r}' = \boldsymbol{\rho}$, and make use of the fact that for large $r\,(r \sim \rho \gg r')$ we have $r = \rho + \mathbf{nr}'$. Here $\mathbf{n}$ is a unit vector in the direction of the scattered meson. Then (4) can be written in the form

$$\frac{1}{r} f_{\pm}(\omega, q) = \int\limits_{\tau > \rho} d\tau\, d\xi\, d\eta K(\tau, \boldsymbol{\rho}, \mathbf{r}, t, q) \tag{5}$$
$$\times \exp\{i[\omega\tau - k\rho] - i(k\mathbf{nr}' - \mathbf{kr}')\}.$$

If ω tends toward infinity and q is fixed, then $\theta \to 0$. In this case we can consider

$$(k\mathbf{nr}' - \mathbf{kr}') \approx k\theta \sqrt{\xi^2 + \eta^2} \approx q \sqrt{\xi^2 + \eta^2},$$

so that the second term in the exponent in (5) is dependent only on q. Inasmuch as K also depends only on q, then the entire dependence on ω for large ω is determined by the factor $e^{i(\omega\tau - k\rho)}$. But $\mathrm{Im}(\omega\tau - k\rho) > 0$ and tends toward infinity for $\mathrm{Im}\,\omega \to \infty$. This permits us to draw the conclusion that $f_{\pm}(\omega, q)$ vanishes on the large semicircle in the upper half plane of ω. Thus $f_{\pm}(\omega, q)$, considered as a function of ω for fixed q, must satisfy the relation (3).

Below we shall be interested in the small momenta q transferable to the nucleon, i.e., in small angle scattering. In this case we can consider the nucleon nonrelativistically and describe the scattering amplitude in the form

$$f_{\pm}(\omega, q) = f_{\pm}^{(1)}(\omega, q^2) + i\sigma[\mathbf{kk}'] f_{\pm}^{(2)}(\omega, q^2), \tag{6}$$

where σ is the spin vector of the nucleon. It is clear that the dispersion relations (3) will exist independently for the functions $f^{(1)}$ and $f^{(2)}$. Upon substitution of (6) in (3), the integration over the region $\omega < 0$ reduces to integration over $\omega > 0$ with the help of the relations

$$f_{+}^{(1)}(-\omega, q^2) = f_{-}^{(1)*}(\omega, q^2), \tag{7}$$
$$f_{+}^{(2)}(-\omega, q^2) = -f_{-}^{(2)*}(\omega, q^2).$$

In order to prove (7) we can consider, as proportional to the scattering amplitude, the invariant matrix element $M_{\pm\alpha\beta}(p, p'; k, k')$ which is characterized by a certain arbitrary Feynman diagram (α, β = spin indices). Consideration of the arbitrary diagram (on which we shall not linger) shows that*

$$M_{\pm\alpha\beta}(p', p; -k, -k') = M^{*}_{\pm\beta\alpha}(p, p'; k, k').$$

Actually $M_{\pm}$ depends (except for spin factors) only on the three invariants: pp', $(p + p')k$, $(p - p')k$ which, in the system of coordinates for which $\mathbf{p} = -\mathbf{p}'$, are equal, respectively, to

$$pp' = M^2 + {}^1\!/_2 q^2, \quad (p + p')k = \sqrt{M^2 + {}^1\!/_4 q^2}\, \omega,$$
$$(p - p')k = -{}^1\!/_2 q^2.$$

It is seen that in the given coordinate system the substitution $k \to -k$, $p \to p'$ is equivalent to the substitution $\omega \to -\omega$ for fixed q^2. Then, considering that $[\mathbf{kk}']$, upon the substitution $k \to -k$, $p \to p'$, transforms into itself, we obtain (7). It is evident that for $q^2 = 0$, the first of the relations in (7) transforms into a relation for the forward scattering amplitude in the laboratory system:

$$f_{+}(-\omega) = f_{-}^{*}(\omega). \tag{7'}$$

Substituting (6) and (7) into (3), we get

$$\frac{1}{2} \mathrm{Re}[f_{+}^{(1)}(\omega, q^2) \pm f_{-}^{(1)}(\omega, q^2)] \tag{8}$$
$$= \frac{2}{\pi} \int\limits_0^\infty \left\langle \begin{matrix}\omega' \\ \omega\end{matrix} \right\rangle d\omega' \frac{{}^1\!/_2\, \mathrm{Im}\,[f_{+}^{(1)}(\omega', q^2) \pm f_{-}^{(1)}(\omega', q^2)]}{\omega'^2 - \omega^2},$$

* Here it is not necessary to pay any attention to the sign of the infinitely small imaginary contribution $i\epsilon$, inasmuch as the factor which arises from it upon integration over the virtual states is compensated by a corresponding factor which appears in front of the matrix element, and the rule of detouring poles in the integration over ω is determined not by the imaginary contribution but by our requirement on detouring the poles on the upper side.

$$\frac{1}{2}\operatorname{Re}\left[f_{+}^{(2)}(\omega, q^2) \pm f_{-}^{(2)}(\omega, q^2)\right] = \frac{2}{\pi}\int_0^\infty \left\langle {\omega \atop \omega'} \right\rangle d\omega' \frac{{}^1\!/_2 \operatorname{Im}\left[f_{+}^{(2)}(\omega', q^2) \pm f_{-}^{(2)}(\omega', q^2)\right]}{\omega'^2 - \omega^2}. \quad (9)$$

The symbol $<{\omega' \atop \omega}>$ means that for the upper sign (plus) in the square brackets, we must use ω' and for the lower sign (minus), ω. On the right side of (8), (9), there appears an integration over the nonphysical region $0 < \omega' < \omega_{min} = (\mu^2 + q^2/4)^{1/2}$. In the case of forward scattering ($q^2 = 0$, $\omega_{min} = \mu$), only the bound states of the meson-nucleon system make a contribution in this region. For computation of this contribution we make use of an expression for the forward scattering amplitude[1]

$$f_{\pm}(\omega) = i\int d^4x e^{ikx} \langle p \mid T(j_{\pm}^*(x), j_{\pm}(0)) \mid p\rangle. \quad (10)$$

The imaginary part of the amplitude will be

$$\operatorname{Im} f_{\pm}(\omega) = \pi \int d\mathbf{x} e^{-i\mathbf{k}\mathbf{x}} \sum_n \{\langle p \mid j_{\pm}^*(\mathbf{x}) \mid n\rangle \langle n \mid j_{\pm}(0) \mid p\rangle \times \delta(E_p - E_n + \omega) - \langle p \mid j_{\pm}(0) \mid n\rangle \langle n \mid j_{\pm}^*(\mathbf{x}) \mid p\rangle \delta(E_p - E_n - \omega)\}, \quad (11)$$

where the summation in (11) is carried out over all the states of the meson-nucleon system. The choice of signs in Eq. (11) is determined by the rule for detouring the poles. We make this choice, keeping in mind the substitution of (11) in (8), where the poles in the integration in ω are detoured on the upper side.

The only bound state in the scattering of pions by protons is the neutron, and the matrix element is (operating in terms of meson theory) the exact vertex part

$$\langle n \mid j_{-}(0) \mid p\rangle = ig\sqrt{2}\,\bar{u}(p_n)\,\Gamma_5(p_n, p; k)\,u(p), \quad (12)$$

where g is the renormalized charge. In the given case, $\Gamma_5 = \gamma_5$, inasmuch as all the external momenta correspond to free particles. Further calculations repeat those of Ref. 2 and lead [after subtraction from both parts of (8) of the real part of $f(\mu)$] to the same expressions for the dispersion relations.

In the case of the scattering amplitude at angles not equal to zero, the minimum value of ω, corresponding (for fixed q) to the scattering of a real meson, is $\omega_{min} = (\mu^2 + q^2/4)^{1/2}$. On the other hand, determining the energy ω_1, beginning with which meson-nucleon states can be realized, from the equality $(M + \mu^2) \leq (k + p)^2$, we find

$$\omega_1 = (M\mu - q^2/4)/(M^2 + q^2/4)^{1/2}.$$

Thus in the interval $\omega_1 < \omega < \omega_{min}$ on the right side of (8) and (9) there enters the contribution of the nonphysical states of the meson-nucleon system, and this contribution cannot be calculated on the basis of contemporary theory. Therefore, the consideration of the dispersion relations makes sense only for small q^2, where the contribution of the nonphysical states is small.

Let us first consider the spin-flip scattering amplitude $f_{\pm}^{(2)}(\omega, q^2)$, wherein we limit ourselves to the case $q^2 = 0$: $f_{\pm}^{(2)}(\omega, 0) \equiv f_{\pm}^{(2)}(\omega)$. In the nonphysical region $0 < \omega < \mu$, the contribution gives only a bound state of one neutron. Inasmuch as we are only interested in terms proportional to q, we can calculate this contribution, starting with its value for the forward scattering amplitude. Actually, the position of the pole, with accuracy up to terms linear in q, remains the same as in the case of the amplitude of forward scattering (since all the invariants depend only on q^2). The matrix element corresponding to the scattering amplitude can be written in the general case as

$$M(p, p'; k, k') = \bar{u}(p')\{F_1 + k_\mu\gamma_\mu F_2\}\,u(p), \quad (13)$$

where F_1 and F_2 are functions of the invariants pp', $(p + p')k$, $(p - p')k$, which we can take at the value $q^2 = 0$ in our approximation. Comparing (13) with the value of the matrix element for the case of scattering in the forward direction, we get

$$F_1|_{q^2=0} = 0, \quad F_2|_{q^2=0} = -g^2/M.$$

Computing (13) for these values of F_1 and F_2, we find for the contribution from the bound state, with accuracy up to linear terms in q,

$$\operatorname{Im} f_{+}(\omega, q) = (\pi g^2/M)\{\omega - (1/2M)\, i\sigma[\mathbf{k}\mathbf{k}']\}\,\delta(\omega - E_p + E_n). \quad (14)$$

The dispersion relations for the scattering amplitude $f_{\pm}^{2}(\omega)$ have the form* [$f = (\mu/2M)g$]

* Equations (15) coincide with the dispersion relations for the spin-flip scattering amplitude obtained in Ref. 3.

$$\frac{1}{2}\operatorname{Re}[f_+^{(2)}(\omega)\pm f_-^{(2)}(\omega)] \qquad (15)$$

$$=\frac{f^2}{\omega^2-(\mu^2/2M)^2}\left\langle{2\omega/\mu^2 \atop 1/M}\right\rangle$$

$$+\frac{2}{\pi}\int_\mu^\infty\left\langle{\omega \atop \omega'}\right\rangle d\omega'\,\frac{{}^1\!/_2\operatorname{Im}[f_+^{(2)}(\omega')\pm f_-^{(2)}(\omega')]}{\omega'^2-\omega^2}.$$

We now go on to the consideration of the nonspin-flip scattering amplitude $f^{(1)}(\omega, q^2)$. As in the earlier case, we shall consider q^2 small, and in the expansion in q^2 we limit ourselves to the zeroth and first terms. The zeroth term evidently gives the dispersion relations for the forward scattering amplitude. For computation of the first term, it is appropriate to expand the region of integration on the right side of (8) to three intervals between the points $\omega = \omega_1$ and $\omega = \omega_{\min}$. For small q^2, the first of these integrals corresponds to the bound state of the system meson + proton , i.e., the neutron, the second to the nonphysical states of the system meson – nucleon and the third to the real states of the system meson – nucleon.

It is easy to see that for small q^2 the integral over the region $\omega_1 < \omega < \omega_{\min}$ is proportional to $q^2\operatorname{Im}f(\mu, 0)$ and contributes nothing in our approximation [inasmuch as $\operatorname{Im}f(\mu, 0) = 0$]. For the computation of the contribution from the bound state we make use of the general expression for the scattering amplitude[1] and describe it first in the form which it has before the exclusion of the δ-function which describes the law of conservation of momentum:

$$f_\pm \sim i\int d\mathbf{x}\,d\mathbf{y}\,dt e^{i\omega t-i\mathbf{kx}+i\mathbf{k'y}} \qquad (16)$$

$$\times\langle p'\,|\,T[j_\pm^*(\mathbf{x},\ t), j_\pm(\mathbf{y},\ 0)]\,|\,p\rangle$$

$$=\int d\mathbf{x}\,d\mathbf{y}\,e^{-i\mathbf{kx}+i\mathbf{k'y}}\sum_n\left\{\frac{\langle p'\mid j_\pm^*(\mathbf{x})\mid n\rangle\langle n\mid j_\pm(\mathbf{y})\, p\rangle}{E_p-E_n+\omega+i\varepsilon}\right.$$

$$\left.+\frac{\langle p'\mid j_\pm(\mathbf{y})|n\rangle\langle n|j_\pm^*(\mathbf{x})\mid p\rangle}{E_{p'}-E_n-\omega-i\varepsilon}\right\}$$

(The imaginary contribution was chosen from the condition for the detouring of the poles on the upper side.) Taking the complex conjugate of (16), it is not difficult to see that for these terms in the sum over n where the denominator does not vanish,

$$f_{\pm\,\alpha\beta}^{*(n)}(p,\ p';\ k,\ k') = f_{\pm\,\beta\alpha}^{(n)}(p',\ p;\ k',\ k),$$

and for the pole terms,

$$f_{\pm\,\alpha\beta}^{*(n)}(p,\ p';\ k,\ k') = -f_{\pm\,\beta\alpha}(p',\ p;\ k',\ k).$$

Considering the nonspin-flip scattering amplitude ($f_{\pm\alpha\,\beta}^{(1)} \sim \delta_{\alpha\,\beta}$),and taking it into account that in the coordinate system we have chosen, $f_\pm$ depends only on the invariants pp', $(p + p')k$, $(p - p')k$, which do not change upon the substitution $p \to p'$, $k \to k'$, we come to the conclusion that the imaginary part of the amplitude $f^{(1)}$ corresponds to the contribution from those terms in (16) where the denominator vanishes. This permits us to write it in the form

$$\operatorname{Im}f_\pm^{(1)}(\omega,\ q^2) = \pi\int d\mathbf{x}e^{-i\mathbf{kx}} \qquad (17)$$

$$\times\sum_n[\langle p'\,(j_\pm^*(\mathbf{x})\,|\,n\rangle\,\langle n\,|\,j_\pm(0)\,|\,p\rangle$$

$$\times\,\delta(E_p-E_n+\omega)$$

$$-\langle p'\,|\,j_\pm(0)\,|\,n\rangle\,\langle n\,|\,j_\pm^*(\mathbf{x})\,|\,p\rangle\,\delta(E_{p'}-E_n-\omega)]\,|_1.$$

(The subscript 1 denotes that it is necessary to separate out the part which does not contain σ.) With the help of (17), the contribution from the neutron state can be computed in the same way as was done in the case of the forward scattering amplitude. The computations lead to the following expression for the imaginary part of the amplitude [with accuracy up to $(\mu/M)^2$]:

$$\operatorname{Im}f_+^{(1)}(\omega,\ q^2) = -(2\pi g^2/4M^2) \qquad (18)$$

$$\times\ [k^2-{}^1\!/_2\,q^2]\,\delta(\omega-\mu^2/2M-q^2/4M).$$

The expansion of the integral in the region $\omega_{\min} < \omega' < \infty$ in powers of q^2 is elementary. In this case we must replace μ in the lower limit for the same reasons for which the integral over the region $\omega_1 < \omega' < \omega_{\min}$ was omitted. Collecting the results, we obtain, after subtraction from both sides of the equation of its value for $\omega = \mu$, the dispersion relations for $f_+^{(1)} \pm f_-^{(1)}$:

$$\frac{1}{2}\operatorname{Re}\frac{\partial}{\partial q^2}[f_+^{(1)}(\omega,\ q^2)\pm f_-^{(1)}(\omega,\ q^2)]|_{q^2=0} \qquad (19)$$

$$-\frac{1}{2}\left\langle{\omega \atop \mu}\right\rangle\operatorname{Re}\frac{\partial}{\partial q^2}[f_+^{(1)}(\mu,\ q^2)\pm f_-^{(1)}(\mu,\ q^2)]|_{q^2=0},$$

$$=f^2\left\langle{1/M\mu^2 \atop \omega/\mu^4}\right\rangle\frac{k^2}{\omega^2-(\mu^2/2M)^2}+\frac{2k^2}{\pi}\int_\mu^\infty\left\langle{\omega' \atop \omega}\right\rangle d\omega'$$

$$\times\frac{{}^1\!/_2\operatorname{Im}(\partial/\partial q^2)[f_+^{(1)}(\omega',\ q^2)\pm f_-^{(1)}(\omega',q^2)]\,|_{q^2=0}}{(\omega'^2-\omega^2)(\omega'^2-\mu^2)}.$$

The equations (19) can also be obtained from the corresponding equations of Salam[4].

2. PHOTOPRODUCTION OF PIONS ON NUCLEONS

The dispersion relations for the photoproduction of pions on nucleons can be obtained by the method completely analogous to this, with the help of which the relations were found for the scattering of pions on nucleons. We shall consider the amplitudes of photoproduction of π^+-mesons on protons $f_+(\omega)$ and of π^--mesons on neutrons $f_-(\omega)$ at an angle of 0°, as functions of the frequency of the quantum ω. We shall carry out the analysis in a system of coordinates in which the sum of the momenta of the nucleon in the initial and final states is equal to zero: $\mathbf{p} + \mathbf{p}' = 0$. Here, of course, a proper system of coordinates corresponds to each value of the frequency of the quantum ω. Such a choice of a coordinate system brings it about that the amplitude of photoproduction possesses simple properties upon the replacement of ω by $-\omega$. In this system of coordinates the energy of the quantum is equal to the energy of the meson and the initial momentum of the nucleon $\mathbf{p}$ is directed against the momentum of the quantum $\mathbf{k}$ and in absolute magnitude is equal to

$$p = \tfrac{1}{2}\left(\omega - \sqrt{\omega^2 - \mu^2}\right). \qquad (20)$$

The threshold of photoproduction lies (in this system) at $\omega = \mu$. Near the threshold, p reaches its maximum value $p_{max} = \mu/2$, and for large ω ($\omega \gg \mu$), $p \approx \mu^2/2\omega$, i.e., it tends toward zero for $\omega \to \infty$. Inasmuch as, even for $p = p_{max}$, the kinetic energy of the nucleon $E^{kin}_{max} = \mu^2/8M \approx 3$mev is very small, we can regard the nucleons nonrelativistically with very high accuracy.

We note, moreover, one property of the function $p(\omega)$ which follows from its analytic continuation in the region of negative ω:

$$p(-\omega) = -p(\omega) \qquad (21)$$

(the points of the loop are detoured above).

The analytical properties of the amplitude of photoproduction can be made clear by writing the expression analogous to (1):

$$f_\pm(\omega)/r = \int\limits_{(t-t')^2 > (\mathbf{r}-\mathbf{r}')^2} K_\pm(t, t', \mathbf{r}, \mathbf{r}', p) \times e^{i\omega(t-t') - i(qz - kz')}\, dt' ds \qquad (22)$$

(q = momentum of the meson), where the integration over ds is carried out in a plane perpendicular to the momentum of the quantum. It follows from (22) that $f_\pm(\omega)$ does not have poles in the upper half plane. Actually, for $\operatorname{Im}\omega > 0$ there appears in the integrand an exponentially decaying factor (for $t' \to -\infty$), which guarantees the convergence of the integral. The function K, by virtue of invariance, can depend on ω only through the combinations $E_{kin}(t - t') = (p^2/2M)(t - t')$ or pz'. Inasmuch as the integration is not carried out over z', the second of these combinations does not affect the convergence of the integral. So far as the first is concerned, it could change the convergence of the integral only if it were multiplied by a large factor ($\sim M/\mu$) which is completely improbable, becuase E_{kin} must enter on a par with M.

For the investigation of the behavior of $f_\pm(\omega)$ on a large circle in the upper half plane of ω, we must make use of the fact that as $\omega \to \infty$, p tends to zero and $q \to k$. Thus, for sufficiently large ω, the factor $\omega[t - t' - (z - z')]$ appears in the exponent. Its imaginary part is positve for $\operatorname{Im}\omega > 0$ and tends toward infinity for $\operatorname{Im}\omega \to \infty$. Since the dependence of K on ω vanishes for sufficiently large $\omega\,(p \to 0)$, the conclusion as to the vanishing of the function $f_\pm(\omega)$ on the semicircle of large radius in the upper half plane follows directly.

For the photoproduction of mesons at the angle of 0°, the amplitude of photoproduction must have the form

$$f_\pm(\omega) = \boldsymbol{\sigma}\mathbf{e}F_\pm(\omega), \qquad (23)$$

where $\mathbf{e}$ is the vector of polarization of the quantum (since the amplitude must be pseudoscalar and must contain $\mathbf{e}$ linearly). The function $F_\pm(\omega)$ has the same analytical properties as the function $f_\pm(\omega)$ and consequently satisfies the relation (3).

The connection of $F(\omega)$ with $F(-\omega)$ can be obtained, as in the case of the scattering of pions, with the help of a consideration of an arbitrary matrix element $M_{\pm\alpha\beta}(p, p'; k, q)$, which corresponds to a certain Feynman diagram. Such a consideration yields the fact that $M^*_{\pm\alpha\beta}(p', p; -k, -q) = M_{\mp\beta\alpha}(p, p'; k, q)$. M depends only on the invariants pp', $(p + p')k$, $(p - p')k$, which, in our system of coordinates are, respectively, $pp' = E_p^2 + p^2$, $(p + p')k = 2E_p\omega$, $(p - p')k = 2p\omega$. It is evident from these equations that the substitution $p \to p'$, $k \to -k$ is equivalent to the substitution $\omega \to -\omega$. Taking into account Eq. (23) and the

Hermitian character of the matrices σ, we find

$$F^*_+(\omega) = F_-(-\omega). \tag{24}$$

Thus $\frac{1}{2}(F_+ + F_-)$ and $\frac{1}{2}(F_+ - F_-)$ satisfy Eq. (8).

In order to remove from consideration in the integral of the imaginary part of the amplitude the region below the threshold of photoproduction ($\omega < \mu$ in the chosen coordinate system), we can make use of the phase equality pointed out by Fermi[5] for scattering and photoproduction of pions. Inasmuch as for scattering of pions, the phases are purely imaginary for $\omega < \mu$, then for photoproduction in the corresponding states of the nascent pions, the phases will also be purely imaginary for $\omega<\mu$. Consequently, the imaginary parts of the functions $F_\pm$, which are obtained after exclusion (from the expression for the amplitude) of the factor $\boldsymbol{\sigma}\mathbf{e}$, must vanish for $0 < \omega < \mu$.

The contribution of the bound state requires separate consideration. In the case of photoproduction of pions, this could be a proton (or a neutron). The energy E_n of this bound state ought to be determined (in a way similar to what takes place in the scattering of pions by nucleons) by one of the equalities $E_p \pm \omega - E_n = 0$, where the momentum is $\mathbf{p}_n = \mathbf{p}' + \mathbf{k}$ or $\mathbf{p}_n = \mathbf{p} - \mathbf{k}$. With the help of these equalities, it is easy to find that the bound state corresponds to $\omega = 0$, i.e., to the electromagnetic field with constant potential. Such a field, by virtue of gauge invariance, can contribute nothing to the amplitude of photoproduction, so that the contribution from the bound state also vanishes. To sum up, the dispersion relations for the amplitude of photoproduction take the form

$$\frac{1}{2}\,\mathrm{Re}\,[F_+(\omega) \pm F_-(\omega)] \tag{25}$$
$$= \frac{2}{\pi}\int_\mu^\infty \left\langle \frac{\omega'}{\omega} \right\rangle \frac{\frac{1}{2}\mathrm{Im}\,[F_+(\omega') \pm F_-(\omega')]}{\omega'^2 - \omega^2}\, d\omega',$$

where the integration on the right begins at the threshold of photoproduction ($\omega = \mu$).

The amplitude of photoproduction of a single pion ought to tend to zero for high energies, as also each amplitude of a definite inelastic process[6,7]. Therefore, the integrals in (25) ought to converge. The dispersion relation for the amplitude of photoproduction of π^0-mesons has a form identical with the relation for $\frac{1}{2}(F_+ + F_-)$.

3. SCATTERING OF NUCLEONS AND ANTINUCLEONS BY NUCLEONS

The dispersion relations for the scattering of nucleons by nucleons were considered in a paper by Fainberg and Fradkin[8]. Their method was analogous to the method of Goldberger[1]. We shall show how these relations can be obtained by means of our method.

We limit ourselves to the case of forward scattering. We shall consider the scattering amplitude of protons by protons f_+ and antiprotons by protons f_-, in the laboratory system of coordinates, as functions of the energy of the incident particle, neglecting the Coulomb interaction. We shall consider both the incident particle and the particle at rest to be polarized in (or against) the direction of the momentum of the incident particle. We introduce the notation: r, s for the polarization of the incident and the second particle before the collision, and r', s' after collision ($r, s, r', s' = \pm 1$).

Proof of the fact that the scattering amplitude has no poles in the upper half plane of ω, and vanishes (or at least does not increase) on a semicircle of large radius, runs identically with that for the case of scattering of pions on nucleons and leads, on the basis of Cauchy's theorem, to a relation similar to (3). In order to investigate the relation of the scattering amplitude of proton-proton for positive ω with the scattering amplitude of antiproton-proton for negative ω, we consider an arbitrary matrix element $M^{\text{nonex}}_{++}(p_1, p_1'; p_2, p_2'; r, r'; s, s')$ (p_1, p_1' are the 4-momenta of the incident particle, p_2, p_2' of the second particle). The nonexchange matrix element can be written in the form

$$M^{\text{nonex}}_{++}(p_1,\ p_1';\ p_2,\ p_2';\ r,\ r';\ s,\ s')$$
$$= \bar{u}^{r'}(p_1')\, M_1(p_1',\ p_1)\, u^r(p_1)$$
$$\times\, \bar{u}^{s'}(p_2')\, M_2(p_2',\ p_2)\, u^s(p_2),$$

where, for example, $u^r(p_1)$ is a spinor corresponding to the 4-momentum p_1 and polarization r. We make the substitutions $p_1 \to -p_1$, $p_1' \to -p_1'$, $p_2 \to p_2'$, $s \to s'$ in Eq. (26), and take its complex conjugate. We get

$$M^{*\text{nonex}}_{+}(-p_1,\ -p_1';\ p_2',\ p_2;\ r,\ r';\ s';\ s) \tag{27}$$
$$= \bar{u}^r(-p_1)\, M_1(-p_1,\ -p_1')\, u^{r'}(-p_1')$$
$$\times\, \bar{u}^{s'}(p_2')\, M_2(p_2',\ p_2)\, u^s(p_2).$$

The right-hand side of Eq. (27) is the scattering matrix element for the antiproton (with momentum p_1) by the proton (with momentum p_2 and polarization s). Here $u^r(-p_1)$ is the spinor corresponding to the negative energy $-E_1$ and polarization r. The spinor $v(p)$, which represents the wave function of the antiproton, is connected with $u(-p)$ by means of the matrix of charge conjugation $v(p) = u^*(-p)C$. Making use of the explicit form of the matrix C, it is not difficult to prove that in the case of nucleons polarized along (or against) the direction of the momentum, the polarization r of the spinor with negative energy $u(-p)$ corresponds to the polarization $-r$ of the spinor with positive energy $v(p)$. Thus

$$M_{++}^{*\,\mathrm{nonex}}(-p_1, -p_1'; p_2', p_2; r, r'; s', s) = M_{-}^{\mathrm{nonex}}(p_1, p_1'; p_2, p_2'; -r_1 - r'; s, s'). \quad (28)$$

For the exchange matrix element there is an analogous relation with just this difference, that the right side corresponds to a diagram in which the nucleon and antinucleon lines exchange their roles.

In the laboratory system of coordinates, $E_2 = E_2' = M$, $\mathbf{p}_2 = \mathbf{p}_2' = 0$ and the vector $\mathbf{p}_1 = \mathbf{p}_1'$ cannot enter linearly since the polarizations are pseudoscalars. Therefore, the substitution $p_1 \to -p_1, p_1' \to -p_1'$, $p_2 \to p_2'$ is equivalent to the substitution $\omega \to -\omega$, which permits us to write for the scattering amplitude [on the basis of Eq. (28)]:

$$f_+(-\omega; r, r'; s, s') = f_-^*(\omega; -r; -r'; s', s). \quad (29)$$

Below we shall be interested only in coherent scattering without change in the spin of each of the particles: $r = r'$, $s = s'$. We denote the amplitude of such scattering by $f_\pm^{r,s}(\omega)$. Then, from (3), with the help of Eq. (29), we get the two relations

$$\frac{1}{2}\operatorname{Re}[f_+^{r,s}(\omega) \pm f_-^{-r,s}(\omega)] - \frac{1}{2}\operatorname{Re}[f_+^{r,s}(M) \pm f_-^{-r,s}(M)] = \frac{2}{\pi}(\omega^2 - M^2) \times \int_{1}^{\infty} \left\langle \frac{\omega'}{\omega} \right\rangle d\omega' \frac{{}^1\!/_2 \operatorname{Im}[f_+^{r,s}(\omega') \pm f_-^{-r,s}(\omega')]}{(\omega'^2 - \omega^2)(\omega'^2 - M^2)_J}. \quad (30)$$

(To assure convergence of the integrals, we subtract from each part of the equation its value for $\omega = M$.)

In order to make clear the properties of the imaginary part of the forward scattering amplitude in the interval $0 < \omega < M$, we return to the general expressions for the amplitudes, which have the form:

$$f_+^{r,s}(\omega) = i\frac{\omega}{2\pi}\int d^4x e^{-ip_1x}\bar{u}_\alpha^r(p_1)\langle p_2, s\,|\,T[\chi_\alpha(x), \bar{\chi}_\beta(0)]\,|\,p_2, s\rangle u_\beta^r(p_1); \quad (31)$$

$$f_-^{r,s}(\omega) = i\frac{\omega}{2\pi}\int d^4x e^{-ip_1x}\bar{u}_\alpha^r(-p_1) \times \langle p_2, s\,|\,T[\chi_\alpha(0), \bar{\chi}_\beta(x)]\,|\,p_2, s\rangle u_\beta^r(-p_1). \quad (31')$$

In (31), (31′), $\chi(x)$ denotes the interaction operator standing on the right-hand side of the equation for the nucleon ψ-function in the Heisenberg representation:

$$\gamma_\mu \partial\psi(x)/\partial x_\mu - m\psi(x) = \chi(x).$$

In the case of a pseudoscalar symmetric theory, $\chi(x) = ig\tau_i\gamma_5\psi(x)\varphi_i(x)$. Considering that the interaction is carried out by pseudoscalar mesons, we omit, on the right side of (31), (31′), terms proportional to $\bar{u}^r(p_1)\gamma u^r(p_1)$, which vanish for forward scattering. The imaginary part of the amplitude of scattering can be written in the form of a sum over the entire system $|n\rangle$ of intermediate states:

$$\operatorname{Im} f_+^{r,s}(\omega) = \frac{\omega}{2}\sum_n \int d\mathbf{x} e^{-i\mathbf{p}_1\mathbf{x}}\bar{u}_\alpha^r(p_1) \times [\langle p_2, s\,|\,\chi_\alpha(\mathbf{x})\,|\,n\rangle\langle n\,|\,\bar{\chi}_\beta(0)\,|\,p_2, s\rangle \times \delta(M - E_n + \omega) + \langle p_2, s\,|\,\bar{\chi}_\beta(0)\,|\,n\rangle\langle n\,|\,\chi_\alpha(\mathbf{x})\,|\,p_2, s\rangle\,\delta(M - E_n - \omega)]\,u_\beta^r(p_1); \quad (32)$$

$$\operatorname{Im} f_-^{r,s}(\omega) = \frac{\omega}{2}\sum_n \int d\mathbf{x} e^{-i\mathbf{p}_1\mathbf{x}}\bar{u}_\alpha^r(-p_1) \times [\langle p_2, s\,|\,\bar{\chi}_\beta(\mathbf{x})\,|\,n\rangle\langle n\,|\,\chi_\alpha(0)\,|\,p_2, s\rangle \times \delta(M - E_n + \omega) + \langle p_2, s\,|\,\chi_\alpha(0)\,|\,n\rangle\langle n\,|\,\bar{\chi}_\beta(\mathbf{x})\,|\,p_2, s\rangle\,\delta(M - E_n - \omega)]\,u_\beta^r(-p_1). \quad (32')$$

[The sign before the second terms in parentheses in Eqs. (32) and (32′) are chosen from the condition of detouring the poles above.] By virtue of the law of conservation of momentum, the total momentum in the intermediate state is $\mathbf{p}_n = \pm \mathbf{p}$. It is not difficult to see that the second term of (32′) is in fact equal to zero. Actually, in this term, only states with two or more nucleons can give a contribution in the sum over n. Since the momentum of such a state must be equal to p, then $E_n > \sqrt{(2M)^2 + p^2}$ and $E_n + \omega$ is always larger than M (even for $\omega < M$). In the first term of Eq. (32), only states with two or more nucleons also can enter into the sum over n, i.e., $E_n > \sqrt{(2M)^2 + p^2}$. It then follows that the quantity $M - E_n + \omega < M - \sqrt{(2M)^2 + p^2} + \omega$ in the δ-function cannot vanish for $\omega < M$. Thus, for $\omega < M$, only the second term of (32) and the first of (32′) can be different from zero.

In the second term of (32), there can enter as intermediate states those of one, two and more mesons. The contribution of the state with a single meson can be calculated precisely, since the matrix element is an exact vertex part*. We have

$$\operatorname{Im} f_{+1}^{r,s} = \frac{\omega}{2} \, |\langle p_1, s | g \overline{\psi}(0) \gamma_5 \tau_3 u^r(p_1) \varphi_3(0) | n \rangle|^2 \delta(M - \sqrt{\mu^2 + p^2} - \omega) = -\pi g^2 (\mu/2M)^2 (\omega/M) \delta_{r,s} \delta\left(\omega - M + \frac{\mu^2}{2M}\right). \quad (33)$$

It is of interest to note that the pole lies only $\mu^2/2M$ (~ 5 mev in the center-of-mass system) lower than the zero of the kinetic energy. The contribution of intermediate states with two and more mesons cannot be computed on the basis of contemporary theory. It can only be pointed out that these states have especially high energy, so that, for example, the term in the imaginary part, corresponding to a two-meson intermediate state, will differ from zero only for $-(\omega - M) < 4\mu^2/2M$ and, naturally, will not have the character of a δ-function.

The first term in (32′) corresponds to processes of annihilation of the antinucleon with the nucleon. It is not difficult to verify that in the intermediate state here there ought to be at least two pions, so that for $\omega < M$, a term analogous to (33) does not arise.

* There can be no intermediate state with a single K-meson because of the conservation of strangeness.

Besides the pole in the scattering amplitude which corresponds to a single-meson intermediate state, one needs to take into consideration another pole arising from the virtual level (with energy ϵ) of a system of two protons in the ^1S state.

Finally, we get the following dispersion relations for the scattering of protons by protons and antiprotons by protons*:

$$\begin{aligned} \operatorname{Re}\Big[f_{\pm}^{r,s}(\omega) &- \frac{1}{2}\left(1 \pm \frac{\omega}{M}\right) f_{\pm}^{r,s}(M) - \frac{1}{2}\left(1 \mp \frac{\omega}{M}\right) f_{\mp}^{-r,s}(M) \Big] \\ &= \mp f^2 \frac{1}{\mu^2} \frac{\omega^2 - M^2}{\omega - M + \mu^2/2M} \delta_{r,s} - \frac{2}{\sqrt{M\varepsilon_1}} \frac{\omega - M}{\omega - M + \varepsilon_1} \left\langle {\delta_{-r,s} \atop 0} \right\rangle \\ &+ \frac{1}{\pi}(\omega^2 - M^2) \left\{ \int_0^{M - 4\frac{\mu^2}{2M}} \frac{d\omega'}{\omega'^2 - M^2} \frac{\operatorname{Im} f_{+}^{r,s}(\omega')}{\omega' \mp \omega} + \int_0^M \frac{d\omega'}{\omega'^2 - M^2} \frac{\operatorname{Im} f_{-}^{-r,s}(\omega')}{\omega' \pm \omega} \right\} \\ &+ \frac{\omega^2 - M^2}{2\pi^2} \int_M^\infty \frac{d\omega'}{\sqrt{\omega'^2 - M^2}} \frac{1}{2} \left\{ \frac{\sigma_{+}^{r,s}(\omega')}{\omega' \mp \omega} + \frac{\sigma_{-}^{-r,s}(\omega')}{\omega' \pm \omega} \right\}. \end{aligned} \quad (34)$$

As a consequence of the fact that in the first part of the dispersion relations there enter integrals from the imaginary parts in the energy region $\omega < M$, it is not clear whether they can be proved to be any sort of poles. Some interest attaches to the estimate of the first "pole" term in the right-hand side of (34) which arises from the single meson intermediate state. Since this term contains $\delta_{r,s}$ and is proportional to p^2 for small momenta, it must be related to the ^3P state. Estimating the remaining terms in the first part of (34), we can see that for small energies they all (except the third) are considerably smaller than the first (at least in the ratio μ/M). The third term does not contain a small parameter in comparison with the first, but inasmuch as integration over $\omega' - M$ is begun in it with an energy four times larger than the "kinetic" energy of the point of the pole in the first term, and extends to M, a basis is provided for thinking that

* For the scattering of protons and antiprotons by neutrons, the same relations hold, only with this difference: the pole term from the single meson intermediate state will be twice as large and a contribution is added from the bound state—the deuteron.

it also cannot in any appreciable degree compensate the "pole" term at low energies ($\omega_{kin} \sim \mu^2/2M$). If we consider that the "pole" term cannot compensate the remaining terms at low energies, then with its help we can determine the P-phase in the proton-proton scattering. (It amounts to about 1° at an energy of 5 mev in the laboratory system.) Unfortunately, the experimental data presently available on this problem are sufficiently indeterminate, although they lend support to the idea that the P-phase at these energies is somewhat smaller[9]. If making the experimental data more precise shows that the P-phase is actually significantly less than that value which is required by the "pole" terms, then this will mean that the role of the third and fourth terms in (34) (which could not be determined from experiment) is significant even for small energies (except for the trivial case in which all the scattering is determined by a virtual level). This possibly reduces to zero the value of the dispersion relations in the nucleon-nucleon scattering.

4. CONCLUSION

The method carried out above of obtaining the dispersion relations was actually based on only one assumption: the impossibility of the propagation of signals with velocities exceeding that of light, inasmuch as all the other considerations (the replacement of ω by $-\omega$, and the calculation of the contribution from the poles) possess only an auxiliary character and could be replaced by other considerations which do not make use of the concept of the S-matrix in its present-day form. Therefore, the question is of great interest as to whether it is necessary to require the validity of these conditions for microscopic distances or whether it is sufficient to limit oneself to the macroscopic where such an assumption raises no doubt. The conclusion set forth above permits us to bring forth some arguments in support of the following possibility*. Actually, we assume that the region of propagation of the interaction is not limited by the light cone but extends over some small region beyond it, i.e., for example, that the signals can reach a point $\mathbf{r}$, t from all points $\mathbf{r}'$, t' which satisfy the condition $(t-t')^2-(\mathbf{r}-\mathbf{r}')^2 > -l_0^2$; l_0 is some constant of the order of a nuclear distance. We assume, and this is essential, that the condition which violates causality is superposed in the interval, because in the opposite case it would be difficult to represent that the violation of causality in a small region in one system of coordinates would not become a violation of causality in a large region in another system. Then, for example, in the case of scattering of pions by nucleons, we would have for the amplitude of the forward scattering [in place of Eq. (2)]

$$f_{\pm}(\omega)/r = \int\limits_{\rho^2-\tau^2+l_0^2} d\tau\, d\xi\, d\eta\, K_{\pm}(\tau,\ \rho,\ \mathbf{r},\ t)\, e^{i\omega\tau - ik\xi}. \quad (35)$$

The demonstration of the absence of poles in the upper half plane evidently remains without change. We consider the behavior of $f_{\pm}(\omega)$ on a large semicircle. Let $\xi = \eta = 0$ (this is the worst case). Then it is evident that for large ζ (ζ is a macroscopic distance), the lower limit of the integration over τ is equal to $\tau_{min} = \zeta - l_0^2/2\zeta$ and consequently, for sufficiently large ζ the correction due to l_0 different from zero can be made arbitrarily small. At first glance, it would seem that, from the presence of a finite mass, it is impossible in Eq. (35) to extend ζ to infinity, since in this case the exponent would be equal to $\exp\{-\frac{1}{2}i[\omega l_0^2/\zeta - \mu^2\zeta/\omega]\}$ (if $\tau = \tau_{min}$ and $\omega \gg \mu$) and, therefore, for $\omega = i\Omega$ gives an exponentially increasing function. This difficulty could be avoided in the following fashion. We choose a sufficiently large (macroscopic) ζ and let $\omega = \Omega e^{i\varphi}$, $\Omega \sim \mu\zeta/l_0$. Then the exponent will be of the order $\omega l_0 \lesssim 1$. Along with this, multiplying $f_{\pm}(\omega)$ by a sufficiently rapidly decreasing function, we can become convinced that the integral of $f_{\pm}(\omega)$ over the large semicircle of radius Ω will be very small (in the ratio of some power of l_0/ζ) and the dispersion relations are preserved. In practice, the form of this function, by which it is necessary to multiply $f_{\pm}(\omega)$ is determined by the behavior of $f_{\pm}(\omega)$ on the real axis, i.e., by the behavior of the cross section for large ω. Consequently, in the model considered, the violation of causality in the small region does not change in the course of the proof*.

* Our attention was called to this point by I. Ia. Pomeranchuk.

* For the case of the scattering of gamma quanta, the conclusion that the dispersion relations do not change if the condition of causality is violated in the small, can be obtained also from the results of Ref. 10 if, in obtaining Eq. (3.19) of this paper from (3.18), we introduce a condition imposed in the interval that violates causality.

However, we can choose the condition that violates causality in some fashion which does not impose it in the interval. Here, generally speaking, one chould expect that the dispersion relations will no longer be maintained.

We have therefore come to the following conclusion: if the experimental data are in contradiction with the dispersion relations, then this will mean that at small distances, the propagation of signals with velocities exceeding that of light can go on; at the same time, in accord with experimental data with dispersion relations, we cannot exclude the violation of causality at small distances, in particular the propagation of the reaction between two points lying not inside the light cone but inside the hyperboloid appears to be possible.

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On a Singularity of the Field of an Electromagnetic Wave Propagated in an Inhomogeneous Plasma

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The effect of field growth is investigated in a region where the plasma dielectric constant becomes zero. The problem of the absorption influence is fully explained. The relationship between this effect and plasma resonance is established.

In solving the problem of propagation of electromagnetic waves in an inhomogeneous plano-stratified medium the simplest case is that of normal incidence. Under the conditions of complete reflection it is most convenient to use the linear approximation of the dielectric constant $\epsilon(z)$ in the neighborhood of its zero (point of reflection). In fact, the consideration of this simplest case enables one to explain completely the field of a standing wave in the reflection region (see Ref. 1, Sec. 66). An analogous situation occurs for oblique incidence.

Zhekulin[2] carried out a detailed investigation of solutions describing the oblique incidence of radio waves on a plano-stratified isotropic ionosphere. In such a medium waves with different polarizations of the electric vector E (perpendicular and parallel to the incidence plane) are propagated independently of each other. It turns out that the reflection problem of the wave, with an electric vector perpendicular to the incidence plane, does not differ in principle from the well known case of normal incidence. They differ only in the displacement of the incident wave reflection level. However, the equation describing the wave with a different polarization of the electric vector is of a more specific type; in this case, the point at which the dielectric constant of the medium ϵ, $\epsilon(z)$ becomes zero is a singularity. Zhekulin

ERRATA TO VOLUME 4

	reads	should read
P. 218, column 2, Eq. (10)	$\ldots \xi(\sqrt{3}+2)(2-\sqrt{3})$	$\ldots \xi(\sqrt{2}+2)/(2-\sqrt{3})\ldots$
P. 219, column 1, Eq. (11)	$\ldots (t\xi)^{\sqrt{3/2}}\ldots$	$\ldots (t\xi)]^{\sqrt{3/2}}\ldots$
P. 219, column 1, Eq. (12)	$y^2 = \rho^{2/3}$	$y^2 - \rho^{2/3} \gg 1$
P. 223, column 1, Eq. (45)	$\ldots (E_0\mu^{3/4})^{\sqrt{3/4}}$	$\ldots (E_0\mu^{3/4})^{\sqrt{3}/4}$
P. 223, column 2, Eq. (46)	$\ldots \mu^{3\sqrt{3/4}}\ldots$	$\ldots \mu^{3\sqrt{3/4}}\ldots$
P. 225, column 1, 3 lines above Eq. (1.1)	transversality	cross section
P. 225, column 1, 3 lines above Eq. (1.2)	transversality	cross section
P. 256, column 1, Eq. (37)	$\ldots\frac{55\sqrt{3}}{48}\ldots$	$\ldots\frac{55}{\sqrt{3}\,48}\ldots$
P. 289, column 2, Eq. (2)		$I=\sum_n \frac{1}{2n+1} A_n \sum_{\nu=-n}^{n} \frac{1}{1+i\omega\tau} Y_{n\nu}(n_1)\, Y_{n\nu}(n_2)$
P. 377, column 1, last line	$\delta_{35} = \eta - 21\times\eta^5$	$\delta_{35} - 21\,\eta^5$
P. 436–7	Figures 2 and 3 should be exchanged.	
P. 449, column 1, last Eq.	$\ldots Y_{lm}\ \varphi\sigma\ \alpha$	$\ldots Y_{lm}\ \varphi_\sigma\ \alpha$
P. 449, column 2, Eq. (12)	$\ldots W(l, j, \sigma\ 1;\ j)\ldots$	$\ldots W(l, j, \sigma\ 1; \sigma\ j)\ldots$
P. 451, column 1, Eq. (7)	$\ldots D_{\alpha\beta}^{(1)}(p, 0, \lambda'\ \lambda) = \ldots$	$\ldots D_{\alpha\beta}^{(1)}(p, \omega_0, \lambda', \lambda) = \ldots$
P. 541, column 1, Eq. (28)	$M_{++}^{*\,\mathrm{monex}}$	$M_{+}^{*\,\mathrm{monex}}$
P. 543, column 2, Eq. (35)	$\ldots \int_{\rho^2 - \tau^2 + l_0^2}$	$\ldots \int_{\rho^2 < \tau^2 + l_0^2} \ldots$