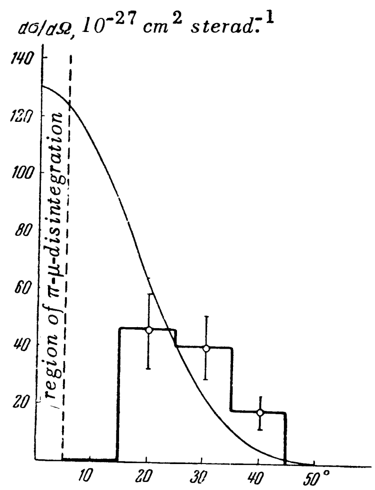
The angular distribution of the elastic scattering in a center-of-mass system is given in the Figure. Attention is drawn to the fact that not a single event of elastic scattering was observed in the angle range 5–15°. Above it was noted that the identification of elastic scattering events at small angles is associated with considerable difficulties, therefore it would be natural to assume that the latter state of affairs may be due to omissions in the analysis. In order for the differential cross section in the region 5–15° to remain at the level 50×10^{-27} cm^2/sterad, one would expect to find here 5–6 events on the basis of the existing statistical material. A thorough second examination of 40% of all the photographs did not disclose a single event of elastic scattering (additional to the first analysis). The small statistical material does not provide the possibility of drawing a completely definite conclusion regarding the course of the angular distribution in this region, however it appears probable that in the region of small angles the differential cross sections of the elastic scattering change non-monotonically.



The general character of the angular distribution of elastic scattering can be qualitatively described within the frame of the optical model of a nucleus. The calculated angular distribution is shown by the solid curve in the Figure. The mean free path of the π-mesons and the mean potential inside the nucleus V_0, used in these calculations, were determined from the total cross sections of the elastic and inelastic scattering and for a nucleus of radius $R = (\hbar/\mu c)\, A^{1/3}$ were found to be $(2.7 \pm 0.3) \times 10^{-13}$ cm and (32 ± 8) mev respectively. The angular distribution obtained in terms of the optical model shows considerable deviation from experimental data only in the region of small angles.

If non-monotonic change in the differential cross sections of elastic scattering in the region of small angles actually takes place, then for its explanation one may draw on the interference between the coulombic and the nuclear interaction. In this case it would be necessary to consider that the amplitudes of the coulombic and nuclear scatterings of the negative pions on nuclei have different signs, in contrast to the results for low energies (less than 200 mev) where the signs of the corresponding amplitudes are the same.[2]

The calculations of the energy dependence of the mean potential inside the nucleus, carried out in the investigations[3,4] on the basis of the properties of the scattering of pions on free nucleons, are in agreement with this fact.

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Note on Waves in a Homogeneous Magnetoactive Plasma

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WITHIN recent years there appeared a number of articles by Piddington[1-3] devoted to a consideration of the properties of normal waves propagated in a homogeneous plasma situated in a magnetic field H_0. In these works the calculation of thermal motion is made by the approximation method based on equations for mean particle velocities. Such a quasi-hydrodynamic method of investigation is not new; it has been repeatedly used in the analysis of similar problems by other authors (see, for example, Refs. 4 and 5). But at the same time it should be noted that many of the problems touched upon in Refs. 1-3 have been studied on the basis of the more rigorous kinetic theory method[6-11]. Most of the studies of the latter type apparently were not known to Piddington.

Here we intend to dwell on only one of the aforementioned works by Piddington[1] in which high-frequency waves* (extraordinary, ordinary, plasma) are dealt with. We shall endeavor to elucidate in greater detail than is done in Ref. 1 the question of the relationship existing between the different types of normal waves, and then to contrast very briefly the results obtained by the quasi-hydrodynamic and the kinetic methods of investigation. From the equations of electrodynamics and from the quasi-hydrodynamic equations for electron motion with due regard to electron pressure [assuming the departures from equilibrium values to be small and proportional to $e^{i(\omega t - \mathbf{kr})}$], we can get the expression

$$\beta_e^2 (1 - u\cos^2\alpha)\, n^6 - [1 - u - v + uv\cos^2\alpha + 2\beta_e^2 (1 - v - u\cos^2\alpha)]\, n^4 + [2(1-v)^2 - u(2 - v - v\cos^2\alpha) + \beta_e^2 (1 - 2v + v^2 - u\cos^2\alpha)]\, n^2 + (1 - v)[u - (1 - v)^2] = 0, \quad (1)$$

where $n = ck/\omega$ is the refractive index of the waves, $u = \omega_H^2/\omega^2 = (eH_0/mc\omega)^2$ (ω_H, the electron gyrofrequency, e and m, the charge and mass of the electron), $v = 4\pi e^2 N/m\omega^2 = \omega_{0e}^2/\omega^2$ (ω_{0e}, the Langmuir frequency, N, electron concentration), α is the angle between the magnetic field $\mathbf{H}_0$ and the direction of propagation $\mathbf{k}$, $\beta_e = \sqrt{\varkappa T/mc^2}$ is the ratio of the mean thermal velocity of the electrons to the velocity of light c. We add that Eq. (1) was derived and discussed in a dissertation by the author of this letter as far back as 1953[12]. Reference 12 also contains an analysis of the problem by the kinetic theory method, which analysis appeared in Refs. 10 and 11 as well.

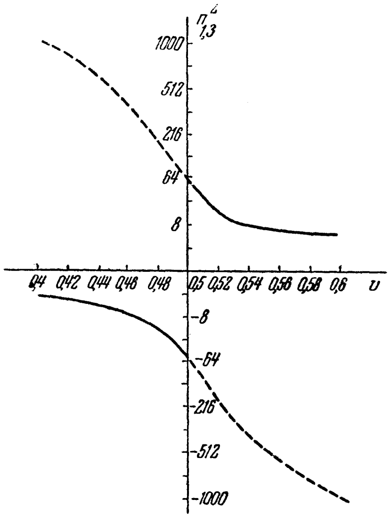
Turning to a consideration of Eq. (1) it is necessary to keep in mind that in the present nonrelativistic case, $\beta_e^2 \ll 1$. This inequality is easily satisfied in the cases that are of greatest interest from the standpoint of possible application, namely, those of the ionosphere ($\beta_e^2 \sim 10^{-7}$) and of the solar atmosphere ($\beta_e^2 \sim 10^{-4} \div 10^{-5}$). Let us examine the wave behavior in the particular cases where $\alpha = 0$ and $\alpha = \pi/2$. When propagation is in the direction of the field $\mathbf{H}_0$ ($\alpha = 0$) we obtain from (1) the expression

$$n_{1,2}^2 = (1 - v)/(1 \pm \sqrt{u}); \quad n_3^2 = (1 - v)/\beta_e^2, \quad (2)$$

in which the subscripts 1, 2, 3 indicate that the values for the refractive index refer to the extraordinary wave, the ordinary wave and the plasma wave, respectively. The plasma wave appears in the equation when thermal motion is taken into account (when $\beta_e \to 0$ $n_3^2 \to \infty$). In the case of transverse propagation ($\alpha = \pi/2$) we find from Eq. (1) that $n_2^2 = 1 - v$, while the values for n_1^2 and n_3^2 are determined from the equation

$$\beta_e^2 n^4 + [(v - 1)(1 + \beta_e^2) + u]\, n^2 + (v - 1)^2 - u = 0. \quad (3)$$

To ascertain the character of wave behavior it is quite usual and important to analyze the curves $n^2(v)$ by assigning fixed values to u and β_e for the given case. Considering the case where $\alpha = 0$, we see that for each type of wave there is a particular dispersion curve determined from the formulas given in (2). But the case $\alpha = 0$ is an exceptional one, and when $\alpha \neq 0$ it becomes impossible for all values of n^2 to distinguish by means of a single continuous curve the behavior of only one type of wave. In the accompanying diagram are shown curves representing $n^2(v)$ when $\alpha = \pi/2$. In contrast to the case $\alpha = 0$, the values for $n_3^2(v)$ are represented here not by a separate curve but by what appears to be a continuation of the curve $n^2(v)$. In the diagram we refer to that part of the curve $n^2(v)^1$, with $n^2 > 0$, where $v > 1 - u$, to the extraordinary wave, and the part where $v < 1 - u$, to the plasma wave. Such a separation is based on the premise that in the absence of thermal motion ($\beta_e = 0$) the plasma wave should disappear, but at the same time, as is known, the dispersion curve runs to the right of point $v = 1 - u$, and when $v = 1 - u$: $n_1^2 \to \infty$. Nevertheless, it should be emphasized that there is a certain artificiality about the above separation into two types of waves, the truth of which is made especially clear in the case involving the point $v = 1 - u$, where there is no reason for referring the n^2 values either to the plasma or to the extraordinary wave. In addition, it will be noted that similar peculiarities arise not only when $\alpha = \pi/2$, but also in the case of other values of $\alpha \neq 0$. The case where the values for α are small, and which is of some interest, is discussed in Ref. 13.



Solid curves—n_1^2 (extraordinary wave); Dotted curves—n_3^2 (plasma wave). $u = 0.5$; $\beta_e^2 = 10^{-4}$.

If, for $u < 1$, the previously discussed coupling between the plasma wave and the extraordinary wave is always present, then upon satisfaction of the condition $u \cos^2 \alpha > 1$, the plasma wave is characterized by a single continuous curve $n_{2,3}^2(v)$ along with the ordinary wave. Here, a study of the roots of Eq. (1) shows that in the vicinity of point $v = (u-1)/(u \cos^2 \alpha - 1)$ (when $\beta_e \ll 1$) there is a region in which the values for n^2 on the $n_{2,3}^2$ curve mentioned above are complex. If, for values $n^2 > 0$, the normal waves are characterized by the presence of propagation alone, and for the values $n^2 < 0$ they are undergoing pure decay, we have here an intermediate case where the propagation of the wave is associated with its attenuation in space even when collisions are disregarded in the equations. As for the case where $u < 1$, it can be established from Eq. (1) (provided $\beta_e^2 \ll 1$) that the n^2 values hold true for all the wave types.

An investigation of the problem dealt with in the preceding is entirely possible also on the basis of the kinetic theory method. The kinetic treatment leads to the establishment of the possibility of decay which is essentially due to the influence of particle thermal motion on wave propagation. This mechanism, however, is ineffective for the inequalities[9,12]

$$|(\beta_e^2 n^2 / u) \sin^2 \alpha| \ll 1, \quad |\beta_e^2 n^2 \cos^2 \alpha| \ll 1. \tag{4}$$

For slowly decaying waves we can obtain an equation which is analgous to the quasi-hydrodynamic Eq. (1)[10,11]:

$$\begin{aligned}\beta_e^2 v\,[A \sin^2 \alpha + B \sin \alpha \cos \alpha + (1-u)\, C \cos^2 \alpha]\, n^6 \\ - [(1-u-v+uv\cos^2\alpha) + O_1(\beta_e^2)]\, n^4 \\ + [2(1-v)^2 - u(2-v-v\cos^2\alpha) \\ + O_2(\beta_e^2)]\, n^2 + (1-v)[u-(1-v)^2] = 0,\end{aligned} \tag{5}$$

$$A = \frac{\cos^2\alpha\,(1+3u)}{(1-u)^2} + \frac{3\sin^2\alpha}{1-4u},$$

$$B = \frac{4\sin\alpha\cos\alpha}{1-u}, \quad C = 3\cos^2\alpha + \frac{\sin^2\alpha}{1-u}$$

The values for the roots of this equation correspond to the correct solution of the problem, provided the inequalities given in (4) are fulfilled. The magnitudes of $O_1(\beta_e^2)$, $O_2(\beta_e^2)$ in (5) are of the order of β_e^2 and are of small consequence; the fundamental difference between Eqs. (1) and (5) is the fact that the expressions preceding n^6 in both are different. This difference in particular accounts for the fact that in the kinetic treatment, the values for n^2 can be complex also when $u < 1$, and not only in the case where $u \cos^2 \alpha > 1$.

As for the plasma waves, they undergo slow decay only in the vicinity of the point $v = (u-1)/(u\cos^2\alpha - 1)$. The plasma waves are rapidly damped, however, when $[1-u-v+uv\cos^2\alpha] \gg \beta_e^2$.

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* The assumption of high-frequency is equivalent to a disregard of ionic motion.

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On the Motion of Inclusions in a Solid

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CONSIDER a foreign inclusion in a solid body of infinite extent. Let the inclusion be spherical in shape, and filled with substance (liquid or gas) in which the material of the solid under existing conditions has a marked solubility. Let a constant but infinitesimal temperature gradient ∇T be maintained in the solid; we will investigate the translational motion of the inclusion under the influence of this gradient.[1,2]

It is obvious that, in a solid, the translation of an inclusion can take place only by means of the transfer of matter into the inclusion, but a hydrodynamic mechanism similar to that involved in the rise of bubbles in a liquid is excluded (we do not consider viscous flow of the crystal). In the presence of only a thermal field, the indicated transfer is connected with the difference in saturation concentrations of the solution at the cold and hot ends of the inclusion, and takes place purely by diffusion. The presence of other fields leads, generally speaking, to the appearance of other flows, (for instance, the presence of a gravitational field of intensity g can lead to convection*). Thus each element of surface surrounding the inclusion will have a velocity

$$\mathbf{v} = (D/\rho)\,\nabla c, \tag{1}$$

where ρ is the density of the substance comprising the solid, ∇c is the concentration gradient of this substance in the material filling the inclusion, taken near the portion of the surface under consideration, and d is the diffusion coefficient.

The concentration c, which depends, generally speaking, on the coordinates and on the time, is determined from the equations of diffusion and thermal conduction, with suitable boundary conditions:

$$\begin{gathered} \partial c/\partial t = D\,[\Delta c + (k_T/T)\,\Delta T]; \\ \partial T_1/\partial t - (k_T/c_p)\,(\partial\mu/\partial c)_{p,T}\,\partial c/\partial t = \chi_1 \Delta T_1; \\ \partial T_2/\partial t = \chi_2 \Delta T_2, \end{gathered} \tag{2}$$

where k_T is the coefficient of thermal diffusion, μ the chemical potential of the contents of the inclusion, c_p their specific heat, and χ_1, T_1 and χ_2, T_2 respectively the thermal conductivity and temperature inside and outside the inclusion. Leaving the boundary conditions out of the picture for the moment, we go over in these equations to a coordinate system in which the inclusion is at rest. Terms proportional to $\mathbf{v}\nabla T$ and $\mathbf{v}\nabla c$, appearing as a result of this transformation, will be of second order in ∇T (since $v \sim \nabla T$). For $\nabla T =$ const, $\mathbf{v}$ does not depend explicitly on the time, and the partial derivatives of the temperature and concentration with respect to time will be at most of second order. Consequently, to the approximation being considered here, both the temperature and the concentration satisfy Laplace's equation. We now return to the conditions at the surface of separation. These will have the form:

$$T_1 = T_2, \quad \varkappa_1\,\partial T_1/\partial n - \varkappa_2\,\partial T_2/\partial n = -qD\,\partial c/\partial n, \quad c = c_0, \tag{3}$$

$\varkappa_1$ and $\varkappa_2$ are the respective coefficients of thermal conductivity, and $\partial/\partial n$ is an operator denoting the derivative along the normal to the surface. The right-hand side of the second condition makes allowance for the evolution (or absorption) of latent heat of crystallization q at the boundary, and the third conditions requires that the solid solution be saturated at this surface. Since the gradient ∇T is small, the change in temperature along the surface of separation will not be great, but if one reckons the temperature and concentration with respect to their values at the center of the inclusion, the last condition in (3) will be written thus: