$$c = (dc_0 / dT)\, T_1. \qquad (4)$$

Here the derivative $dc_0/dT$ is taken along the appropriate equilibrium curve. Thus at the limits of the region, the concentration is proportional to the temperature, and since both the temperature and the concentration satisfy the same equation, there will be a similar relation between them everywhere inside the inclusion (irrespective of the form of the bounding surface). Therefore at the surface of separation

$$-qD\,\partial c / \partial n = -qD\,(dc_0 / dT)\,(\partial T_1 / \partial n).$$

If we now replace $\varkappa_1$ by $\varkappa_1 + q\,dc/dT$ in condition (3), we obtain the well-known problem concerning the distribution of temperature around a stationary spherical object for a constant temperature gradient at infinity (see, for example, Ref. 3). Multiplying its solution over the interior of the region by $dc_0/dT$, we obtain the distribution of concentration over the inclusion. Making use of Eq. (1), we obtain finally

$$\mathbf{v} = \frac{3\varkappa_2 D}{\varkappa_1 + 2\varkappa_2 + qD\,dc_0/dT}\,\frac{dc_0}{dT}\,\frac{\nabla T}{\rho}. \qquad (5)$$

From this equation it is obvious that the velocity of translation of the inclusion does not depend on its dimensions.

The value of v is determined by the quantity $dc_0/dT$, which under conditions of constant pressure is equal to $qc_0/kT^2$. In substances whose solubility in the material filling the inclusion does not increase with temperature, the inclusion moves in the direction of the temperature gradient. If however $dc_0/dT < 0$, the inclusion has to translate in the opposite direction. It is clear that the process under investigation is governed by Le Chatelier's principle. Indeed, for $q > 0$, the diffusion at the hot end of the inclusion and the condensation at the cold end both represent the same process, striving to bring the system back into equilibrium; this equilibrium is constantly being destroyed, however, by the source maintaining the temperature gradient.

It is not difficult to see that an analogous process takes place for the transfer of matter in a liquid medium from a crystal having a high temperture to a crystal with a lower one; that is, for the growth of one crystal at the expense of another.

The coefficient of diffusion has the order of magnitude $10^{-5}$ cm$^2$/sec; $dc_0/dT \sim 5\times 10^{-2}$ g/cm$^3$–sec–deg; $q \sim 10^2$ cal/g; $\varkappa_2 \sim 5 \times 10^{-3}$ cal/cm–sec–deg.

Consequently, $qD\,dc_0/dT \ll \varkappa_2$, and

$$\mathbf{v} = (3D / 2\rho)\,(dc_0 / dT)\,\nabla T,$$

that is, v does not depend on the thermal conductivity of the solid or of the contents of the inclusion.

If the conditions under which an inclusion exists are critical conditions for its contents, then the coefficient of diffusion will fall to zero and the inclusion will come to rest. For a temperature gradient of the order of 1deg/cm, the velocity of translation of the inclusion amounts to about 0.1 mm/day. If conditions are created whereby matter can be transferred convectively, the process ought to be noticeably accelerated.

*It is clear, for example, that there will be convection if the vectors g and $\nabla T$ are parallel, but not if they are antiparallel.

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[2] G. G. Lemmlein, Dokl. Akad. Nauk SSSR **85**, 325-328 (1952).

[3] L. D. Landau and E. M. Lifshitz, *Mechanics of Continuous Media*, GITTL, Moscow, 1953.

Translated by W. M. Whitney
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## Investigation of High Energy Electron Showers by the "Emulsion Chamber" Technique

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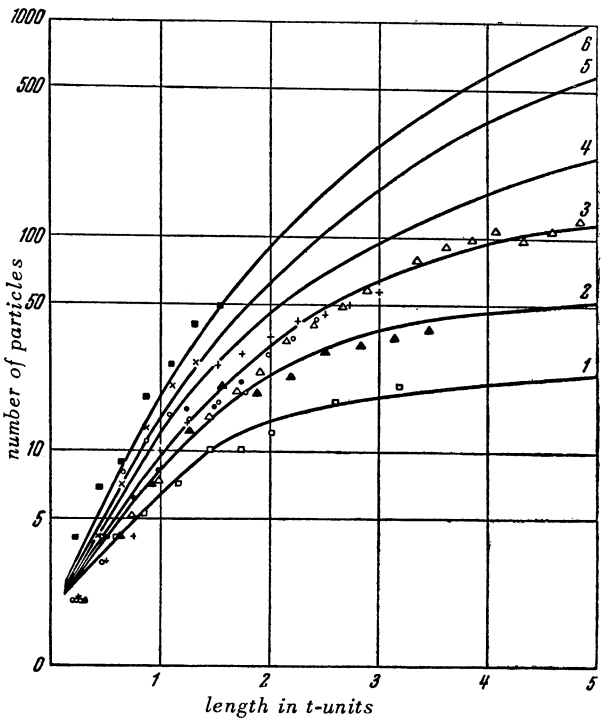
(Submitted to JETP editor June 20, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 710-712 (October, 1956)

IN the emulsion chamber suggested in Ref. 1, we have studied the development of electron showers of high energy ($E = 10^{12}$ ev) at small depths (2–3 $\tau$–units). The results obtained are compared with cascade theory.[2,3]

The emulsion chamber consisted of 24 emulsion plates (NIKFI) type *R* with emulsion thickness of 100 $\mu$ put together into a stack and held in a special frame. Iron plates of thickness 3.5 mm were placed between emulsions. The distance between adjacent layers of emulsion was 5 mm. The glass on which the emulsion was poured was previously polished to a size of 86 × 116 mm$^2$ with an accuracy of 0.05 mm. This allowed the exact placement of adjacent plates which is essential in

following a shower.

The emulsion chamber was exposed in the stratosphere (~ 4 hrs. at height $\geq$ 20 km). After development the plates were divided into two groups for convenience during scanning. Two plates from the middle of the chamber were scanned with magnification of 20 × 7 × 1.5. During this time showers were noted which had more than 5–7 parallel tracks in a radius of 100 $\mu$ which thereafter continued into the upper and lower plate. With the available accuracy of finding the continuation of a shower (approximately 200 $\mu$) and a track background of 160 $mm^{-2}$ it was impossible to follow a single relativistic particle or shower, consisting of several tracks, through considerable gaps ($\geq$ 400 $\mu$).




Comparison of the experimental data on the development of electron showers with cascade curves. *1*- $y = 5$ ($E_0 = 1.8 \times 10^{11}$ ev); *2*–$y = 6$ ($E_0 = 7 \times 10^{11}$ ev); *3*– $y = 7$ ($E_0 = 10^{12}$ ev); *4*–$y = 8$ ($E_0 = 6 \times 10^{12}$ ev); *5*– $y = 9$ ($E_0 = 9 \times 10^{12}$ ev); *6*–$y = 10$ ($E_0 = 27 \times 10^{13}$ ev)

A shower arising in the emulsion chamber from a single photon is characterised by a gradually decreasing number of particles until two tracks are seen at very small (2–3 $\mu$) distances from each other, or a single track with double ionization appears. It is impossible to follow the shower further as indicated above. It is essential to note that in following a shower the only tracks considered as a part of the shower have direction parallel to the axis of the shower. The chief contribution to deflection of tracks from the parallel is multiple scattering. The average angle of multiple scattering for particles of energy $e$ passing through a distance $t$ is given by the relation

$$\langle \vartheta \rangle = (E_s / E) \sqrt{t},$$

where $E_s = 21$ mev and $t$ is expressed in radiation units. In the present paper the parallelism of tracks was determined to within one degree. This means that in each shower we registered particles with energy $\geq 10^9$ ev.

During the microscopic examination of the emulsion chamber 8 electron proton showers were found. For each of these the total number of particles in all emulsion plates was calculated and the dependence of the number of particles on the depth was found. This data was compared with the cascade curves obtained from the calculations of Arli[2] for depths up to $2t$–units and from the calculations of Belenkov[3] for depths greater than $2t$–units for various values of the energy of the initial electron with secondary particle energy greater than $E_1 = 10^9$ ev. Since the observed showers began with the appearance of the first electron pair the shower is considered to be caused by two primary electrons of equal energy. Curves calculated taking this into account are shown in the diagram. The experimental points corresponding to various showers are shown by the different symbols.

At distances > 1 $t$–unit from the beginning of the shower the dependence of the number of particles on the depth coincides with one of the cascade curves calculated for a particular energy of the initial electron. At depths less than one $t$–unit the number of particles in the shower fluctuates strongly.

The data given in the Figure shows that there is fairly good agreement between the experimental results and cascade theory for electron energies $E_0 \sim 10^{12}$ ev for small thicknesses of matter.

Also in the observation of the electron showers at depths $\geq 2t$–units it is possible to evaluate the initial electron energy from the cascade curves to within a factor of 2 or 3. The results of these evaluations are shown in the table.

In conclusion the author would like to express deep thanks to I. L. Rosental and D. S. Chernavskii for their consideration of the experimental results and to N. A. Dobrotin for valuable comments and to R. M. Grizunova for the scanning.

TABLE

| No. of Shower | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Total length in $t$-units . . | 2.4 | 3,4 | 1,7 | 3,2 | 3 | 4.6 | 1.6 | 1.5 |
| $y = \ln(E_0/E_1)$ . . . . . | 7 | 6 | 7 | 5 | 7 | 7 | 9 | 10 |

1 Kaplon, Peters, Reynolds and Ritson, Phys. Rev. 85, 295 (1952).

2 B. Rossi and K. Greisen, *Interaction of Cosmic Rays with Matter.*

3 S. Z. Belenkii, *Avalanche Processes in Cosmic Rays*, Gostekhizdat, 1948.

Translated by G. L. Gerstein
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## Dispersion Relations for Pions Scattered by Deuterons

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(Submitted to JETP editor June 27,1956)

J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 712-713 (October, 1956)

CONSIDER zero-angle elastic scattering of pions by deuterons. The dispersion relations corresponding to this process differ substantially from the dispersion relations for the scattering of pions by free nucleons: first, the dispersion relations depend on the polarization of the deuterons; second, if the Coulomb interaction is neglected, there is only one dispersion relation for the sum of the scattering amplitudes of the positive and negative pions.

Let us denote by $D_m(\omega)$ and $A_m(\omega)$ the real and imaginary parts of the amplitude of the scattering of pions with energy $\omega$ by deuterons having a spin projection m along the direction of motion of the pions. Using the relationship between the imaginary part of the zero-angle scattering amplitude and the total cross section, $A_m(\omega) = (k/4\pi)\,\sigma_m(\omega)$ (where $k^2 = \omega^2 - \mu^2$, and $\mu$ is the pion mass), and using the usual procedure for obtaining the dispersion relations[1,2] we obtain

$$D_m(\omega) - D_m(\mu) = \frac{2k^2}{\pi}\int_0^{\mu}\frac{\omega' A_m(\omega')\,d\omega'}{k'^2(\omega'^2-\omega^2)} + \frac{k^2}{2\pi^2}\int_0^{\infty}\frac{\omega'\sigma_m(\omega')\,d\omega'}{(\omega'^2-\omega^2)k'}. \quad (1)$$

To determine the contribution from the region $0 \le \omega \le \mu$ we employ the following expressions for $A_m(\omega')$, obtained readily from Ref. 1:

$$A_m(\omega') = \pi\sum_f |M_m(\omega', \mathbf{f})|^2\,\delta\left[\omega' - \varepsilon_0 - \frac{k'^2}{4M} - \frac{f^2}{M}\right], \quad (2)$$

where $M_m(\omega', \mathbf{f})$ is the matrix element corresponding to the capture of a pion by a deuteron in state $m$ and the formation of two identical nucleons with a relative momentum $\mathbf{f}$:

$$M_m(\omega', \mathbf{f}) = (\sqrt{2}\,g/M)\,\langle\Phi_m^*\sigma_1\mathbf{k}'F_i(\mathbf{k}', \rho)\,\Psi_{\mathbf{f}}^-\rangle. \quad (2')$$

here $\Phi_m$ is the deuteron wave function, $\Psi_{\mathbf{f}}^-$ the wave-function of two identical nucleons in the final state, g the pion to nucleon coupling constant, $M$ the nucleon mass, and $\epsilon_0$ the coupling energy of the deuteron.

The function $F_i(\mathbf{k}', \rho)$ equals $\sin(\mathbf{k}'\rho/2)$ if the two forming nucleons are in the triplet state and $\cos(\mathbf{k}'\rho/2)$ in the case of a singlet state.

Integrating with respect to $\omega'$, we get

$$\frac{2k^2}{\pi}\int_0^{\mu}\frac{\omega' A_m(\omega')\,d\omega'}{k'^2(\omega'^2-\omega^2)} = 2k^2\sum_{\mathbf{f}}\frac{\mu^2/4M - (f^2/M + \varepsilon_0)}{\tilde{k}^2(\omega^2 - \tilde{\omega}^2)}\,|M_m|^2. \quad (3)$$

Generally speaking, $\tilde{\omega}$ is a function of $\mathbf{f}$, determined by the conservation laws. However, taking into account the fact that the matrix element differs substantially from zero only in the region $\mathbf{f} \sim \mathbf{k}/2$, we shall assume hereinafter $\tilde{\omega} \approx \mu^2/2M - \epsilon_0$. Next, since small changes in $\omega'$ correspond to large changes in $\mathbf{f}$, we extend the summation with respect to $\mathbf{f}$ to infinity. Using the completeness of the set of functions $\Psi_{\mathbf{f}}^-$, we have

$$\sum_f |M_m|^2 = \frac{2g^2}{M^2}\tilde{k}^2\left\{\delta_{m1}\int\varphi_0^2F_t^2\,d\rho + \delta_{m0}\int\varphi_0^2F_s^2\,d\rho\right\}, \quad (4)$$