TABLE IV

Number of "prongs" (inelastic interactions)	Number of Events		
	Experiment of Ref. 4	Theoretical (isobar states included)	Theoretical (isobar states not included)
2	14	15.1	21,5
4	16	16.3	10.5
6	2	0.6	0.5

[1] E. Fermi, Progr. Theoret. Phys. 5, 570 (1950); Phys. Rev. 81, 683 (1951).

[2] S. Z. Belen'kii and A. I. Nikishov, J. Exptl. Theoret. Phys. (U.S.S.R.) 28, 744 (1955); Soviet Phys. JETP 1, 593 (1955).

[3] V. M. Maksimenko and I. L. Rozental', J. Exptl. Theoret Phys. (U.S.S.R.) (to be published).

[4] W. Fowler *et al.*, Phys. Rev. 100, 1802 (1955).

Translated by J. G. Adashko
159

Consequences of the Renormalizability of Quantum Electrodynamics and Meson Theory

V. V. SUDAKOV

(Submitted to JETP editor July 14, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 729-731 (October, 1956)

THE consequences of the renormalizability of quantum electrodynamics and meson theory which have been obtained by Gell-Mann and Low[1] and Bogoliubov[2] are most easily formulated, in our opinion, in the following way. We shall start from the following equations of Gell-Mann and Low[1]

$$\alpha(g_0^2, \xi - L) = \frac{\alpha_c(g_c^2, \xi)}{\alpha_c(g_c^2, L)}, \qquad (1)$$

$$\beta(g_0^2, \xi - L) = \frac{\beta_c(g_c^2, \xi)}{\beta_c(g_c^2, L)}, \quad d(g_0^2, \xi - L) = \frac{d_c(g_c^2, \xi)}{d_c(g_c^2, L)}.$$

Here α_C, β_C and d_C are the asymptotic expressions for the slowly-varying factors of the renormalized vertex parts and Green's functions for the nucleon and meson*, g_C is the renormalized meson coupling constant, $\xi = \ln(-k^2/m^2)$, $L = \ln(\Lambda^2/m^2)$ (Λ is the momentum "cutoff"). The quantities α, β, d, g_0 are the nonrenormalized quantities corresponding to the cutoff momentum.

For convenience, we have introduced the logarithmic variables ξ and L from the beginning. In addition to the trivial inference that α, β and d become unity for $\xi = L$, Eq. (1) includes the statement, fundamental in what follows, that for $\xi \gg 1$, α, β and d asymptotically approach functions only if the difference $\xi - L = \ln(-k^2/\Lambda^2)$, i.e., no longer depend on the nucleon mass m.

We then introduce a quantity which may be called "the effective coupling constant"

$$g^2(\xi) = g_0^2 \alpha^2 (g_0^2, \xi - L)\, \beta^2 (g_0^2, \xi - L)\, d(g_0^2, \xi - L) \qquad (2)$$

$$= g_c^2 \alpha_c^2 (g_c^2, \xi)\, \beta_c^2 (g_c^3, \xi)\, d_c (g_c^2, \xi).$$

The second of Eqs. (2) is obtained from (1) and from the relation between the renormalized and nonrenormalized coupling constants. From Eq. (2) it is seen that the effective coupling constant g may be considered either a function of g_0^2 and $\xi - L$, or of g_C^2 and ξ.

The final formulation consists of the assertion that the logarithmic derivatives of α and α_C, etc., with respect to ξ, which are equal according to Eq. (1), depend on one variable, namely, on the effective coupling constant

$$\alpha'/\alpha = \alpha_c'/\alpha_c = F_1(g^2); \quad \beta'/\beta = \beta_c'/\beta_c = F_2(g^2); \qquad (3)$$

$$d'/d = d_c'/d_c = F_3(g^2);$$

$$(g^2)'/g^2 = 2F_1(g^2) + 2F_2(g^2) + F_3(g^2)$$

The primes here denote differentiation with respect to the arguments $\xi - L$ or ξ, whichever is appropriate. The last of Eqs. (3) follows from the first three and Eq. (2). As an example, let us prove the first of the equations. According to Eq. (2), ξ

$= \xi(g_C^2, g^2)$ and therefore the quantity α_C'/α_C, which depends on g_C^2 and ξ, can be written as a function of g_C^2 and g^2. Therefore, the ratio α'/α, which is equal to α_C'/α_C, can be written in the following form:

$$\alpha'(g_0^2, \xi - L)/\alpha(g_0^2, \xi - L) = F_1[g_c^2, g^2(g_0^2, \xi - L)].$$

We emphasize here that g^2 is considered a function of g_0^2 and $\xi - L$. If g_0^2 is held fixed, and ξ and L varied so that $\xi - L$ remains constant, then g_C^2, which depends on g_0^2 and Λ, will vary while the left side of the equation, as well as g^2 on the right side, will remain constant. This is possible only if the function $F_1(g_C^2, g^2)$ does not depend directly on g_C^2. Thus we arrive at the first of Eqs. (2). The rest of Eqs. (2) are obtained similarly.

The functions F_1, F_2, F_3 of Eq. (3) can be determined by considering values of ξ close to L. Then $\ln(\Lambda^2/-k^2) = L - \xi$ is small, and if $g_0^2 \ll 1$, then $g_0^2(L - \xi)$ is also small, i.e., usual perturbation theory is applicable, and in the second order for the symmetric pseudoscalar theory this leads to the following results (all the calculations are carried out to logarithmic accuracy; i.e., we consider only the largest logarithmically divergent part of the integrals):

$$\alpha = 1 - (g_0^2/4\pi)(\xi - L); \quad \beta = 1 + (3g_0^2/8\pi)(\xi - L);$$
$$d = 1 + (g_0^2/\pi)(\xi - L).$$

From this we obtain for $\xi \to L$

$$\alpha'/\alpha = -g_0^2/4\pi, \quad \beta'/\beta = 3g_0^2/8\pi, \quad d'/d = g_0^2/\pi.$$

From Eq. (2) for $\xi \to L$, $g^2 \to g_0^2$, and to first order in g^2, we obtain

$$F_1(g^2) = -g^2/4\pi; \; F_2(g^2) = 3g^2/8\pi; \; F_3(g^2) = g^2/\pi; \qquad (3')$$
$$(g^2)'/g^2 = 5g^2/4\pi.$$

Integrating this last equation between the limits ξ and L, and remembering that $g^2(L) = g_0^2$, we have

$$g^2(\xi) = g_0^2/Q; \quad Q = 1 + (5g_0^2/4\pi)(L - \xi). \qquad (4)$$

From Eqs. (3), (3′) and (4), we obtain the asymptotic expressions for the vertex parts and Green's functions of the nucleon and meson[3,4]

$$\alpha = Q^{1/5}; \quad \beta = Q^{-3/10}; \quad d = Q^{-4/5}. \qquad (5)$$

Finally, we shall show for the simpler case of quantum electrodynamics how, if we know the perturbation theory series for the photon Green's function

$$d_t = \sum_{m \geqslant n} c_{mn} e_0^{2m} (\xi - L)^n, \qquad (6)$$

we can obtain the asymptotic expression for d_t to arbitrary order in e^2. On the basis of Ward's theorem, Eqs. (2) and (3) can, for quantum electrodynamics, be written in the form

$$e^2(\xi) = e_0^2 d_t(e_0^2, \xi - L) = e_c^2 d_{t,c}(e_c^2, \xi),$$
$$d_t'/dt = d_{t,c}'/d_{t,c} = F(e^2), \quad (e^2)'/e^2 = F(e^2). \qquad (7)$$

Considerations similar to the above give

$$F(e^2) = \sum_{m=1}^{\infty} c_{m1} e^{2m} \Big/ \sum_{m=0}^{\infty} c_{m0} e^{2m}. \qquad (7')$$

It should be emphasized that in obtaining the series (6) one must make use of the cutoff indicated by Gell-Mann and Low[1], thus assuring that the condition $d_t(e_0^2, 0) = 1$ is satisfied.

In conclusion, I should like to express my gratitude to K. A. Ter-Martirosian for many valuable comments and to I. Ia. Pomeranchuk, V. B. Berestetskii and B. L. Ioffe for fruitful discussions.

---

* As an example, we are considering the symmetric pseudoscalar meson theory with pseudoscalar coupling.

[1] M. Gell-Mann and F. F. Low, Phys. Rev. **95**, 1300 (1954).

[2] N. B. Bogoliubov and D. V. Shirokov, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 77 (1956); Soviet Phys. JETP **3**, 57 (1956).

[3] Abrikosov, Galanin and Khalatnikov, Dokl. Akad. Nauk SSSR **97**, 793 (1954).

[4] J. J. C. Taylor, Proc. Roy. Soc. (London) **234**, 296 (1956).

Translated by E. J. Saletan
160