$$\overline{\varphi^2_\mu} = {}^1/_2\,\mathrm{tg}^2\phi\,\mu\,\frac{ceu\sin\phi}{E_0L}\,\Omega_0^2 f^{1/2}(x)\int\limits_{x_0}^{x}\left[f^{1/2}(x) + \frac{\Omega_0^2\tau^2}{(1+x^2)^3}\,f^{-1/2}(x)\right]\frac{dx}{(1+\Omega^2\tau^2)^2}. \qquad (77)$$

In Eqs. (75) through (77), it is assumed that the values of the spectral intensities ν, η and μ do not depend on the frequency. These formulas determine $\overline{\varphi^2}$ in the adiabatic region. Inasmuch as the effect of noises decreases on approaching the critical region, they can be used during the entire cycle of acceleration.

The mean square deviation of the momentum in the critical region is of interest. It can be calculated by Eqs. (34) and (39) on substituting for $\overline{\varphi^2}$ from Eqs. (75) through (77).

In conclusion, we present graphs of the functions which enter in Eqs. (75) through (77). These functions are evaluated for several positive- and for one negative-value of a^2. Negative values of a^2, characteristic for strong focusing accelerators without a critical energy, were proposed by Vladimirskii and Tarasov[3].

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A Comparison of the Fermi-Landau Theory with Some Experimental Data on Cosmic Rays

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The energy spectra of the secondary particles originating in nuclear collisions of high-energy particles ($10^{12}-10^{18}$ ev) have been calculated on the basis of the theory of L. D. Landau. A calculation of the altitude dependence of the radioactive particles in the atmosphere, as well as of the number of high-energy μ-mesons at sea level, has been carried out on the basis of the spectra obtained. The results of the calculations are compared with experimental data.

UP to the present time there have been few direct experimental data concerning collisions of super-high energy nucleons ($> 10^{12}$ ev) with nucleons or light nuclei. There exists practically no knowledge of the distribution of energy between the secondary particles of various types formed as a result of such collisions. Hence, in spite of the considerable time since the publication of the Landau theory[1], there have been, up to the present, almost no papers devoted to a comparison of the results of this theory with experimental data. However, although direct experimental data relative to the energy distribution of secondary particles in the energy region $E > 10^{12}$ ev are lacking, we nevertheless have indirect experimental data relative to the formation of secondary particles of super-high energy. For example, in the works of Ryzhkova and Sarycheva[2], and also Kaplon *et al.*[3], the coefficient of absorption is measured in the atmosphere of radioactive particles with energy $\gtrsim 10^{12}$ ev. Moreover, there exist data relative to the number of μ-mesons[4] penetrating a depth of the earth of as much as 3 km of water equivalent, i.e., possessing energies of the order of 3×10^{12} ev. As the calculations show, these data are very sensitive to the mechanism of the elementary act of collision, especially to the form of the energy spectrum of the particles produced in nuclear collisions. Hence, the calculation of the absorption curve of radioactive particles of high energy and the number pf μ-mesons produced in the atmosphere on the basis of any given theory of the elementary act of nuclear collision, allows the verification of the correctness of the results of that theory.

The basic initial data on which the carrying out of the calculations just mentioned is dependent are: 1) the primary particles of the cosmic rays, mainly protons (and we assume below that they are all protons); 2) secondary particles arising as a result of collisions of a nucleon of given energy with a nucleus of an atom of the air. The energy spectrum of the primary cosmic rays has been determined as a result of many experimental investigations, and we have taken it to be in accordance with Ref. 4. The energy spectra of the secondary particles were calculated on the basis of the Landau theory, the results of which may be satisfactorily reconciled with a number of experimental results obtained by the method of photoemulsions[5] and which, in the case of an elementary analysis of the properties of wide atmospheric showers[5,6], do not display the sharp contradictions with these properties which are shown, for example, by the Fermi theory[5,7].

ENERGY SPECTRUM OF THE PARTICLES ARISING IN NUCLEAR COLLISIONS ACCORDING TO THE LANDAU THEORY

In a nucleon-nucleon collision the total number of secondary particles $N(E_0)$ is equal to

$$N(E_0) = K(E_0/2M_{H}c^2)^{1/4}, \qquad (1)$$

where M_H is the mass of the nucleon, E_0 is the energy of the generating particle, and $K \approx 2$ (Ref. 1). In the collision of a nucleon with a quiescent nucleus of an "air atom," the number of particles produced is greater since, according to the Landau theory,

$$N \sim E_c^{3/4} V_c^{1/4}. \qquad (2)$$

where E_c is the energy of the colliding particles in their center-of-mass system and V_c is the volume of the system formed immediately after the collision.

We have assumed that the interaction occurs between a nucleon and a mass of nuclear material contained in a cylinder of radius corresponding to the radius of the nucleon $r_0 = \hbar/M_H c$ (Ref. 5). The entire calculation was carried out for a cylinder of average height $\bar{H}$, which was taken equal to the average chord of a sphere of radius $R = r_0 A^{1/3}$, where A is the atomic weight of the nucleus, i.e.,

$$\bar{H} = {}^4/_3 R = {}^4/_3 r_0 A^{1/3}. \qquad (3)$$

The mass of such a cylinder is equal to

$$M_{cyl} = A^{1/3} M_H \qquad (4)$$

An average value was taken for the atomic weight of the nucleus of an atom of air.

The volume formed after the collision of the system was taken equal to the volume of the cylinder of nuclear substance modified in accordance with the Lorentz contraction for the nucleon-cylinder center-of-mass system. On the basis of Eq. (2), the total number of particles in this case is equal to

$$N(E_0) = KA^{1/4}(E_0/2M_{H}c^2)^{1/4}, \qquad (5)$$

where K has the same magnitude as in (1). Thus, taking account of a collision with a nucleus leads to an increase by a factor of $A^{1/4}$ in the number of particles given rise to.

In the calculation of the energy spectra of the particles by the Landau theory, a basic parameter is

$$L = -\ln(\Delta/a), \qquad (6)$$

where Δ is the longitudinal measureof the system formed immediately after the collision and a is the transverse dimension of the same system. In a nucleon-nucleon collision

$$L = {}^1/_2 \ln(E_0/2M_{H}c^2). \qquad (7)$$

We have assumed that in a nucleon-nucleus collision Δ and a correspond to the dimensions of the pre-collision cylinder in the cylinder-nucleon center-of-mass system. Then

$$L = {}^1/_2 \ln(9E_0/8AM_{H}c^2). \qquad (8)$$

The energy distribution of the secondary particles in the laboratory system of coordinates is given by Landau in the parametric form

$$dN = c_1 \exp(\sqrt{L^2 - \lambda^2})\, d\lambda, \qquad (9)$$

$$E = B_1 M_{H}c^2 \exp({}^5/_6 L + \lambda + {}^1/_3\sqrt{L^2 - \lambda^2}),$$

where $\lambda = -\ln \mathrm{tg}(\theta/2)$ (θ is the angle of departure of the secondary particle in the center-of-mass system). The coefficients C_1 and B_1 in Eq. (9) are found from the normalization conditions

$$\int dN = N; \int E dN = E_0. \qquad (10)$$

Introducing the parameter $\nu = \lambda/L$, we obtain for C_1 and B_1 the expressions

$$C_1 = 2A^{1/4}(E_0/2M_H c^2)^{1/4} / L \int_{-1}^{0,87} e^{L\sqrt{1-\nu^2}} d\nu; \qquad (11)$$

$$B_1 = E_0 \left[M_H c^2 C_1 L e^{5L/6} \times \int_{-1}^{0,87} \exp L\left(\nu + \frac{4}{3}\sqrt{1-\nu^2}\right) d\nu \right]^{-1},$$

where L is to be calculated according to Eq. (7) in the case of a nucleon-nucleon collision and according to Eq. (8) in the case of a nucleon-nucleus collision.

For the solution of practical problems, it is convenient to have the energy spectrum in graphical rather than parametric form. For this purpose it is expedient to choose the representation $dN/d\ln E$, since in this case the form of the curves describing the spectrum is symmetrical, almost Gaussian. Thus, letting $y_0 = \ln E_0$ and $y = \ln E$, we obtain

$$f(y_0, y) = \frac{dN}{d\ln E} \qquad (12)$$
$$= C_1 L e^{L\sqrt{1-\nu^2}} / (1 - \nu/3\sqrt{1-\nu^2}),$$
$$y = \ln E = \ln(B_1 M_H c^2) + L\left(5/6 + \nu + 1/3\sqrt{1-\nu^2}\right).$$

Here, as in what follows, the energy is expressed in electron bolts.

It is clear that Eq. (12) is not applicable for large values $\nu \sim 1$, since for $\nu = 0.95$, when $\nu/3\sqrt{1-\nu^2} = 1$, the function $dN/d\ln E$ breaks off, but the energy of the particles attains a maximum for this value of ν and subsequently falls off.

However, the energy spectrum of the particles calculated on the basis of the Landau theory is characterized by a slow fall-off with increase in energy in the region of applicability of Eq. (12), so that a significant portion of the energy is carried off by particles the energy of which corresponds to the region of the spectrum where $\nu \approx 0.9$, that is, near the limit of applicability of the Landau formula.

From the basic concepts of the theory there is no reason to expect that the spectrum should break off sharply at $\nu \approx 0.9$, since the singularity at $\nu = 0.95$ occurs as a result of the form in which the calculations of Landau are represented. Hence, we have carried out a smooth extrapolation into the region of energies near E_0, for which the Landau formulas are no longer applicable, of the Landau spectra which we previously calculated up to energies corresponding to $\nu = 0.9$.

If we make graphs of the energy spectra, taking $\ln f(y_0, y)$ as ordinate and $\ln E$ as abscissa, then, designating the abscissa value corresponding to the spectral maximum as y_{max}, and the difference $y - y_{max}$ as z, we may write our chosen approximating function in the form

$$\varphi(z) = A - z^2(a + bz^k)$$

for each curve with a given value of the parameter y_0. The constant A corresponds to the spectral maximum, lying at the value y_{max}. The number a is found from the curvature of the Landau curves for $y = y_{max}$, while the numbers b and k are chosen so that the approximating function coincides exactly with the Landau function for the energy corresponding to the maximum in the energy current of the secondary particles and for the energy corresponding to the upper limit of applicability of the Landau formula (near $\nu = 0.87$). The functions $\varphi(z)$ obtained for various values of E_0 correspond with great exactness to the Landau curves for the entire region $0 < \nu < 0.9$.

The extrapolation of the curves of the energy distribution of the number of particles and the corresponding curves for the energy current $Ef(y_0, y)$ beyond the point $\nu = 0.87$ essentially changes the normalization of these curves, so that the area under these curves gives the total number of secondary particles N and the total energy of the secondary particles E_0, respectively, whereas normalization in accordance with Eq. (11) takes into account only the area to the left of the break-off point.

The values of the renormalized constants CL and $BM_H c^2$ are given in Table I for certain values of the energy. Values of the parameters A, a, b, k for the function $\varphi(z)$ are given in Table II for the same values of the energy . Figure 1 gives the dependence of $f(y_0, y)$ on y, that is, the dependence of $\ln(dN/d\ln E)$ on $\ln E$ for given values of

$y_0 = \ln E_0$, obtained on the basis of the Landau theory by the method set forth above for cases of collision of a nucleon with a nucleus of an air atom for $E_0 \geq 10^{12}$ ev. The portions of the curves lying below the dashed line were obtained by extrapolation. The integral $\int_0^{y_0} f(y_0, y)dy = n_{tot}$. Values of n_{tot} are given in Table III.

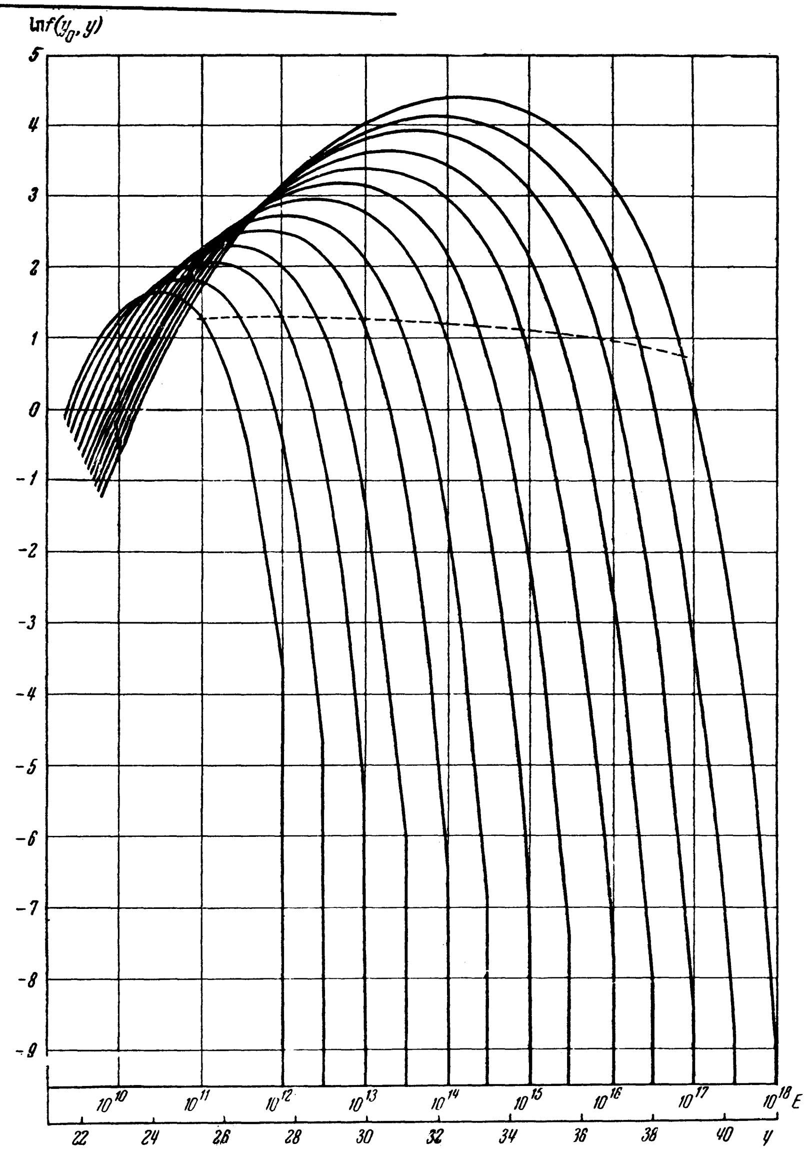

FIG. 1. Energy spectra of the secondary particles according to Landau for the case of a collision of a nucleon with the nucleus of an atom of the air. The energy of the primary particle (10^{12} ev, 3×10^{12} ev, 10^{13} ev, etc.) for each curve corresponds to the break-off of the curve at the upper limit of the spectrum.

Figure 2 gives curves showing the dependence of the current of energy carried away by the secondary particles on the $\ln E$ of these particles for the case of a collision of a nucleon with a nucleus of an air atom. The integral $\int_0^{y_0} Ef(y_0, y)\,dy = E_0$, where E_0 is the energy of the primary particle. A comparison of Figs. 1 and 2 shows that a principal part of the energy is carried away by particles with energy significantly greater than the energy possessed by particles more representative of the energy spectrum of the secondary particles.

TABLE I. Values of the finally renormalized coefficients CL and BMc^2.

Type of collision	E_0 (ev)				
	Coeff.	10^{12}	10^{14}	10^{16}	10^{18}
nucleon-nucleon	$CL\cdot 10^3$	106	23,3	6,11	1,74
	$BMc^2\cdot 10^{-9}$	0,770	0,987	1,18	1,32
nucleon-nucleus	$CL\cdot 10^3$	551	134	33,2	9,00
	$BMc^2\cdot 10^{-9}$	1,85	2,41	2,73	2,88

TABLE II. Values of the parameters of the function $\varphi(Z)$ approximating the curves of the energy spectrum of the secondary particles.

Type of collision	Parameter	E_0 (ev)			
		10^{12}	10^{14}	10^{16}	10^{18}
nucleon-nucleon	A	2,555	5,565	14,42	40,74
	$a\cdot 10^2$	16,14	9,32	6,53	5,02
	$b\cdot 10^4$	316,0	85,3	3,67	1,89
	k	1,358	1,577	3,062	2,873
	z_0	24,91	27,70	30,39	33,08
nucleon-nucleus	A	5,152	12,36	30,34	81,47
	$a\cdot 10^2$	22,67	11,25	7,43	5,55
	$b\cdot 10^4$	388,4	72,7	11,7	3,23
	k	1,413	2,170	2,564	2,749
	z_0	23,97	27,10	29,80	32,62

Table IV gives the share of the energy, in percent of E_0, carried away on the average by a single very fast secondary particle. It should be noted that this share contains a rather large amount of the generating particles right up to very high energies, and agrees very well with the calculations of Gerasimova and Chernavskii[8], which considered hydrodymanic problems corresponding to the collision of nucleons and took into account the presence of traveling waves. We note that the probability of emission of a particle having concentrated in itself a large portion of the energy of the primary particle which is obtained from our graphs for a collision of the nucleon-nucleon type, exceeds the estimates obtained in the work of Podgoretskii, Rosental' and Chernavskii[9].

For the calculation of energy spectra by the method of successive slopes it is necessary to calculate the number of particles of given energy, integrating the probability of their formation from primary particles with energy above the given. The representation of the function $f(y_0, y)$ in the form of a family of curves for fixed values of y_0 is inconvenient for this purpose. Hence the function $f(y_0\, y)$ in Fig. 3 is represented in the form of a family of curves for fixed y and varying y_0.

In many calculations connected with nuclear-cascade processes in the atmosphere (for example, Refs. 10-12), it is assumed that the energy of the incident particle is divided equally among the secondary particles. In this case each particle receives the average energy $\bar{E} = E_0/n_{tot}$, where n_{tot} is the total number of secondary particles occurring. But from the curves which we have obtained for the energy current of the secondary particles and the data of Table IV, it follows that the greater part of the energy is carried away by a much smaller number of particles than n_{tot}. This number of secondary particles possessing the largest energies is actually determined in the basic development of the nuclear-cascade process in the atmosphere. The effective number of such particles may be determined[6] as $n_{eff} = E_0/E_{1/2}$, where $E_{1/2}$ is the value of the energy of the secondary particles determined by the magnitude of $y_{1/2}$, from the equation

$$\int_{-\infty}^{y_{1/2}} Ef(y_0, y)\,dy = \int_{y_{1/2}}^{y_0} Ef(y_0, y)\,dy.$$

$E_{1/2}$ coincides approximately with the energy value at the maximum in the curve of the energy current.

The dependence of n_{tot}, n_{eff} and n_{tot}/n_{eff} on E_0 is given in Table III for two types of collisions. According to the Landau theory $n_{tot} \sim E_0^{0.25}$. It follows from the data of Table III that, for both types of collisions, $n_{eff} \sim E_0^{0.16}$, approximately.

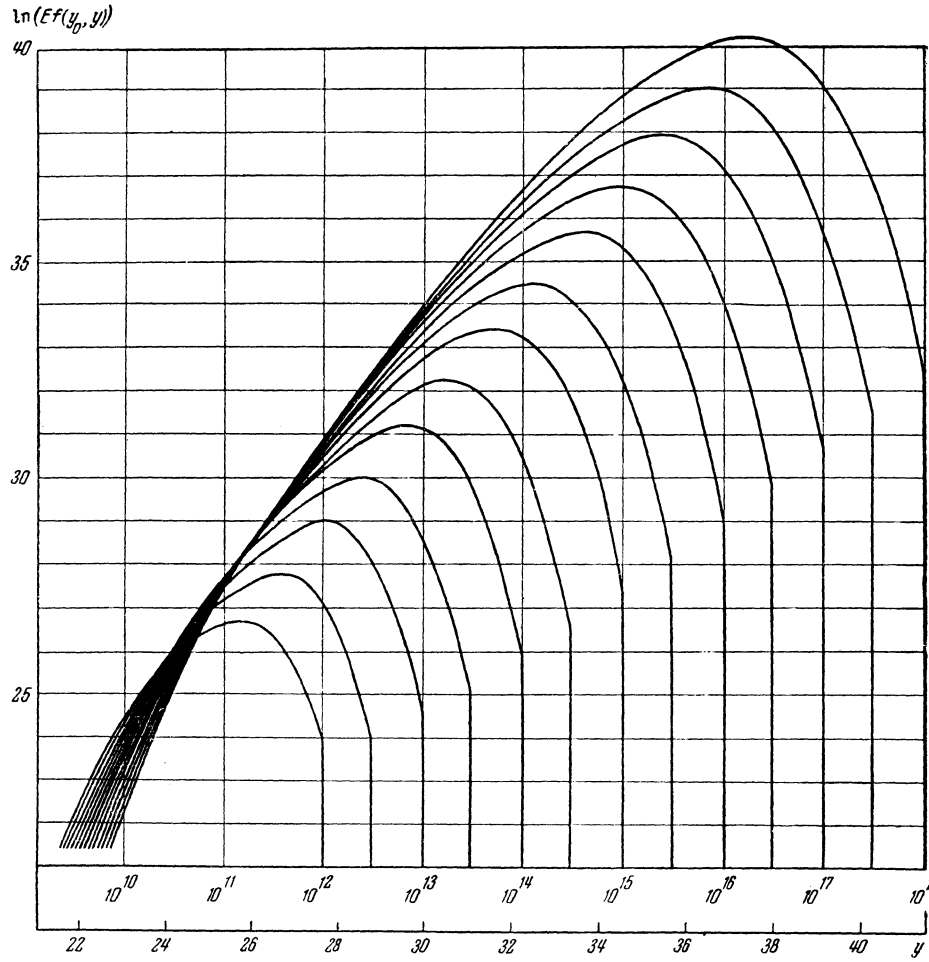

FIG. 2. Currents of energy carried away by the secondary particles, for the case of a collision of a nucleon with the nucleus of an atom of the air. E is expressed in electron volts. The energy of the primary particle for each curve corresponds to the break-off of the curve at the upper limit of the energy current.

CALCULATION OF THE ALTITUDE DEPENDENCE OF RADIOACTIVE PARTICLES OF HIGH ENERGY BY THE METHOD OF SUCCESSIVE SLOPES

In a calculation of the altitude dependence of radioactive particles in the region of high energies, for which the Landau theory is applicable ($E_0 > 10^{12}$ ev), we may neglect the ionization losses of the particles and their disintegration. In the calculations we have assumed that the result of a collision of a radioactive particle with a nucleus does not depend on the nature of the generating particle. The effective cross section for collision was assumed constant and corresponding to the free path length $\lambda_0 = 65$ g/cm^2. In this case the change in the number of radioactive particles in a given energy interval with depth x into the atmosphere may be described by the equation

$$\frac{1}{\mu_0}\frac{\partial F(y,x)}{\partial x} = -F(y,x) + \alpha\int_y^\infty F(y',x)\,f(y',y)\,dy', \tag{13}$$

where $F(y, x)dy$ is the numberof radioactive particles at depth x, the energy of which lies in the interval corresponding to y, $y + dy$. The function $f(y', y)\,dy$ has the sense of the probability of formation of a secondary particle in the interval y, $y + dy$ from a primary of corresponding y'. It is represented graphically in Fig. 4. α is the fraction of radioactive particles contained in the total number of secondary particles formed in the act of collision; $\mu_0 = 1/\lambda_0$.

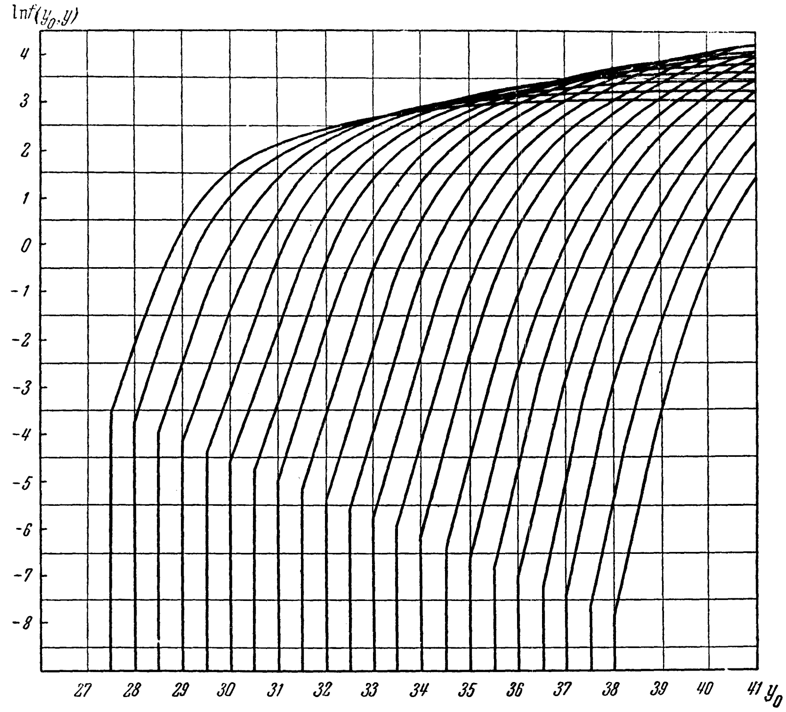

FIG. 3. Values of $\ln f(y_0, y)$ as a function of $y_0 = \ln(E/1\text{ ev})$. Each curve relates to a definite value of $\ln y$ of the secondary particle, which is equal to the value y_0 corresponding to the break-off of the curve at the upper limit.

As is well known[6,13,14], Eq. (13) is satisfied by the function $F(y, x)$ taken in the form of the series

$$F(y, x) = e^{-\mu_0 x} \sum_{i=0}^{\infty} \alpha^i \frac{(\mu_0 x)^i}{i!} F_i(y), \; F_i(y) = \int_y^{\infty} F_{i-1}(y') f(y', y)\, dy'. \quad (14)$$

Figure 4 shows the dependence of $F_i(y)$ for $0 \leq i \leq 5$. It is not necessary to take into account the subsequent terms of the sum with $i > 5$, since they make a negligible contribution even at sea level. $F_0(y')\,dy'$ represents the differential energy spectrum of the primary particles, calculated in accordance with the integral spectrum adduced in Ref. 4.

Substituting the functions $F_i(y)$ into Eq. (14), we obtain the altitude dependence of particles of various energies, from which we may calculate the coefficient of absorption at various altitudes.

The graph of Fig. 5 gives the absorption curve for particles with $E = 10^{12}$ ev. Curves *1* and *2* represent the result of calculations wherein it is assumed that $\alpha = 2/3$ and $\alpha = 1$, respectively. Curve *3*, which was observed experimentally, represents the absorption curve of particles with $E = 10^{12}$ ev. In its construction the experimental results of Kaplon *et al.*[3] were used for the interval of depth less than 700 g/cm^2. According to these authors, the coefficient of absorption in this interval corresponds to $1/\mu = 120$ g/cm^2. Similarly, the experimental results of Ryzhkova and Sarycheva[2] were used in the interval of depth from 650 to 100 g/cm^2. These results gave $1/\mu = (112 \pm 9)$ g/cm^2 in this interval of depth. Curve *4* gives the absorption curve which would be observed if each act of collision with a nucleus led to complete absorption of the particle ($1/\mu_0 = 65$ g/cm^2).

From the results given in Fig. 5 it is clear that experiment gives a significantly smaller magnitude for the coefficient of absorption than does calculation. The coefficient of absorption of the particles may be represented in the form

$$\mu(y, x) = \mu_0 [1 - \Delta(y, x)],$$

whence, according to Eq. (14),

$$\Delta(y, x) = \int_y^\infty F(y', x) f(y', y) dy' \,/\, F(y, x).$$

As a consequence of the fact that $F(y, x)$ falls off quickly with increasing y ($F \sim e^{-\gamma y}$, $\gamma \approx 1.5$ to 2), the magnitude of Δ depends essentially on the value of $f(y', y)$ for $y \approx y'$, that is, on the probability of formation of a secondary particle with energy near the energy of the primary. Thus, the magnitude of Δ is extremely sensitive to the mechanism of the elementary act of the nuclear collision.

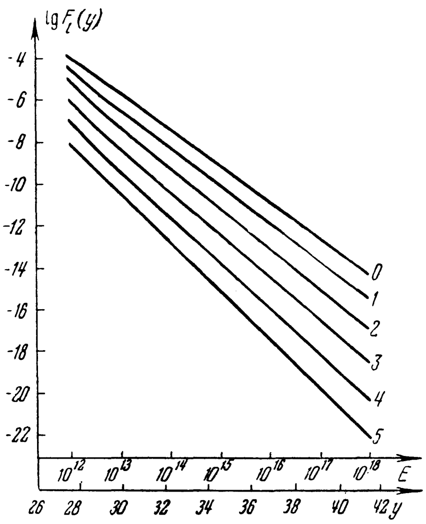

FIG. 4. Energy spectra of successive slopes of radioactive particles in the atmosphere. $F_0(y)\,dy$ represents the differential energy spectrum of the primary cosmic ray particles, expressed in units of cm^{-2} sec^{-1} $sterad^{-1}$. The numbers on the curves indicate the value of i.

From Eq. (14) we obtain

$$\Delta(y, x) = \sum_{i=0}^{\infty} \alpha^{i+1} \frac{(\mu_0 x)^i}{i!} \times F_{i+1}(y) \Big/ \sum_{i=0}^{\infty} \alpha^i \frac{(\mu_0 x)^i}{i!} F_i(y).$$

Figure 6 shows the calculated dependence $\Delta(x)$ for particles with $E = 10^{12}$ ev. The value of $\Delta(y, x)$ in Fig. 6 does not exceed 0.25 and falls off quickly with depth into the atmosphere, while according to the experimental results $\Delta(x)$ depends weakly on the depth, remaining equal to about 0.5 or 0.4.

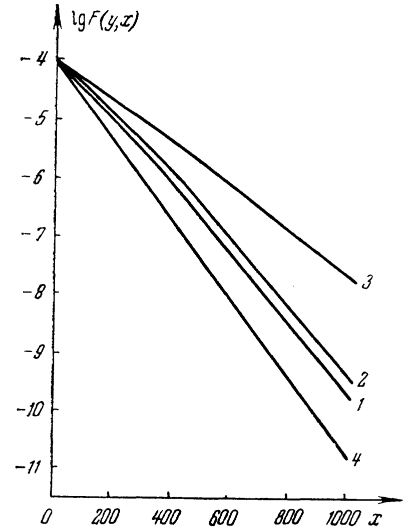

FIG. 5. Altitude dependence of the particles with energy 10^{12} ev. The atmospheric depth in g/cm^2 is plotted as abscissa, and the common logarithm of the function $F(y, x)$ is plotted as ordinate.

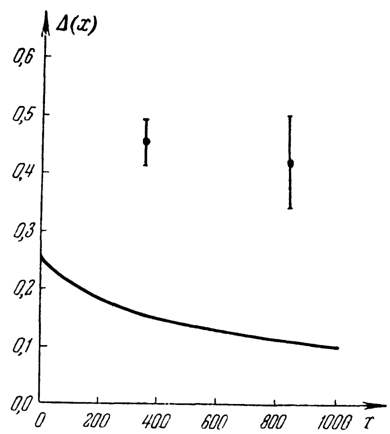

FIG. 6. Dependence of Δ on x for $E = 10^{12}$ ev. The points correspond to the experimental results given in Refs. 2 and 3. The continuous curve gives the results of calculations according to the Landau theory.

Thus the spectra of the secondary particles which we have calculated on the basis of the Landau theory do not correspond to that which actually occurs. The character of the lack of correspondence shows that for nuclear collisions of high energy the probability of having a great part of the energy ($E/E_0 > 0.5$) concentrated in a single particle is significantly greater than is obtained from the spectra of Fig. 2 (Table IV). Moreover, the small change in the experimental value of Δ with depth shows that this probability decreases more weakly with increasing energy of the generating nucleon than is called for by the Fermi-Landau theory.

TABLE III.

Type of collision	E_0 (ev)	10^{12}	10^{14}	10^{16}	10^{18}
nucleon-nucleon	n_{tot}	9,5	30,1	95,4	302,0
	n_{eff}	3,7	8,0	17,1	36,6
	n_{tot}/n_{eff}	2,57	3,76	5,58	8,25
nucleon-nucleus	n_{tot}	18,7	59,2	187,0	592,0
	n_{eff}	8,9	18,4	37,7	76,8
	n_{tot}/n_{eff}	2,10	3,22	4,96	7,72

TABLE IV. Average fraction, in percent, of the energy of the primary particle which is found in a single very fast secondary particle.

Type of collision	E_0 (ev)			
	10^{12}	10^{14}	10^{16}	10^{18}
nucleon-nucleon	48,6	35,0	24,8	17,2
nucleon-nucleus	30,0	19,8	14,3	01,5

CALCULATION OF THE NUMBER OF μ-MESONS OF HIGH ENERGY AT SEA LEVEL

The method of successive slopes allows us to write the intensity of μ-mesons at arbitrary depth in the atmosphere in the form of a series, each term of which represents the μ-mesons arising from a disintegration of π-mesons of the corresponding generation.

In calculations for the region of high energy, $E > 10^{12}$ ev, only the first term of the series, corresponding to the disintegration of $\pi^{\pm}$-mesons in the very highest layers of the atmosphere, contributes a significant amount to the energy. All the other members of the series contribute an insignificant amount, since practically no disintegration of $\pi^{\pm}$-mesons of high energy occur deep in the atmosphere. Thus for the region $E \geq 10^{12}$ ev it is sufficient to calculate only the μ-mesons arising from the disintegration of $\pi^{\pm}$-mesons originating in the first collisions of the primary protons with nuclei of atoms of the air.

In the Landau theory, it is customary to assume that all the secondary particles originating in nuclear collisions are π-mesons. Hence, in the calculation we have used the results shown in Fig. 4, assuming that $\pi^{\pm}$-mesons comprise 2/3 of the total number of particles, corresponding to curve $F_1(y)$.

The disintegration of μ-mesons during their passage through the atmosphere and their ionization losses may be neglected. Then the number of μ-mesons of given energy can be written in the form

$$F^{(\mu)}(E)\,dE = \frac{2}{3}\int\limits_{E}^{EM_\pi^2/M_\mu^2} \frac{F_1^{(\pi)}(E')\,dE'}{1+\frac{E_\pi}{E}}\,\frac{E_\pi}{E} \times \frac{dE'}{E'(1-M_\mu^2/M_\pi^2)},$$

where M_μ and M_π are the masses of the μ- and π-mesons and $F_1(\pi)(E')dE'$ is the spectrum of the first generation of π-mesons generated. The coefficient E_π/E_x characterizes the probability of disintegration of a $\pi^{\pm}$-meson with energy E in the interval of path x, $x + dx$. According to Refs. 6 and 14,

$$E_\pi/E = l_0\lambda_0 T/T_0\rho_0 (E/M_\pi c^2)\tau_0 = 1.17 \times 10^{11}\ \text{ev}/E.$$

Here T is the temperature of the air (we assume it constant and equal to 225° K); ρ_0 is the density of the air in g/cm^2 at sea level and at temperature T_0; τ_0 is the average lifetime of a $\pi^{\pm}$-meson at rest.

The total number of μ-mesons with energy higher than the given is obtained as

$$\Phi^{(\mu)}(>E) = \int\limits_{E}^{\infty} F^{(\mu)}(E)\,dE.$$

We obtain as a result $\Phi^{(\mu)}(> 10^{12}\ \text{ev}) = 4.4 \times 10^{-7}$ particles cm^{-2} sec^{-1} sterad^{-1}. The calculated magnitude of the current of μ-mesons may be compared with the experimental magnitude obtained on the basis of Ref. 15, according to which $\Phi^{(\mu)}(> 10^{12}\ \text{ev}) = 1.8 \times 10^{-7}$ particles cm^{-2} sec^{-1} sterad^{-1}. We see that the calculated magnitude of the intensity of the μ-meson current exceeds the experimentally observed magnitude by approximately 2.5 times.

As a consequence of the fact that the number of μ-mesons of given energy is determined by the number of $\pi^{\pm}$-mesons of approximately the same energy , it follows that the number of μ-mesons which we have calculated exceeds by at least 2.5 times the number of π-mesons actually formed. Since the primary spectrum of the cosmic rays falls rapidly with increasing energy, a basic contribution to the number of π-mesons of given energy is made by mesons which are formed with energy near the energy of the nucleons which generate them. Thus the result which we have obtained shows that the probability of forming π-mesons with energy near the energy of the nucleon which generates them is at least 2.5 times less than that which follows from the curves of Fig. 1.

DISCUSSION OF THE RESULTS

The calculations just carried out show that the energy spectra of the secondary particles which we have obtained on the basis of the Landau energy (by use of the assumptions on the magnitude of the effective collision cross section and on the composition of the secondary particles) leads to a discrepancy with experimental results both with respect to the absorption of radioactive particles and with respect to the number of observed μ-mesons of high energy. The fact that we allowed a certain arbitrariness in the extrapolation of the energy spectra of the secondary particles into the region of energy near the energy of the generating particle does not seem essential for qualitative results, due to the character of the deviations of the experimental results from the theoretical.

Actually, to produce agreement of the theoretical and experimental results with respect to the absorption curve for radioactive particles, it would be necessary to change the energy spectra in such a way that the probability of formation of secondary particles with energy near that of the primary would be greater. On the other hand, in order to reconcile the observed number of μ-mesons with the calculated number, it is necessary to suppose that the probability of formation of $\pi^{\pm}$-mesons with theory near the the energy of the primary particles is less than that obtained from the spectra found.

Although the assumption that the effective interaction cross section of nucleons of high energy differs essentially from the geometrical cross section (being less than the geometrical by about 1.5 times) may do away with the discrepancies we have obtained, it can nevertheless not be assumed, since it is contradicted by many experimental results. Clearly, the only possible cause of the contradictions we have found is the assumption which we made, in agreement with Landau's work, that all the secondary particles are π-mesons.

The essential difference (by approximately a factor of 2) between the experimental coefficient of absorption of the radioactive particles and the reciprocal magnitude of the free path of these particles shows that after a collision with an atom of the air, a significant portion of the energy (of the order of 0.6 to 0.7) of the primary radioactive particle is concentrated on the average in a single secondary particle[6]. On the other hand, the small number of observed μ-mesons shows that these secondary particles cannot be $\pi^{\pm}$-mesons or other particles disintegrating with a lifetime less than that of $\pi^{\pm}$-mesons and giving μ-mesons as a result of their decay. Thus these secondary particles in which the larger portion of the energy is concentrated cannot be the K-mesons found in stars produced by primary particles of high energy. Hence, of all the particles known at the present time, only nucleons (and hyperons) satisfy the stipulated conditions.

The small number of μ-mesons with high energy in comparison with the number of primary cosmic ray particles with such energy shows that the energy spectra which we have obtained on the basis of the Landau theory gives too great a probability of formation of $\pi^{\pm}$-mesons with energy near the energy of the generating nucleon.

In the calculations we assumed that $\pi^{\pm}$-mesons comprise 2/3 of the total number of particles produced. Certain experimental results show that a significant portion of the particles produced are not π-mesons but K-mesons. However, taking account of these K-mesons which make up a part of the secondary particles can lead only to an increase in the observed discrepancy.

Actually, the lifetime of K-mesons is significantly smaller than that of $\pi^{\pm}$-mesons. In the region of energy $E > 10^{12}$ ev, the number of μ-mesons produced is inversely proportional to the lifetime of the particle from the disintegration of which they are produced (if this lifetime is not too small). Hence, the assumption that a portion of the secondary particles are K-mesons leads only to a still greater number of expected μ-mesons. Since on the basis of experimental results with photoemulsions we may consider that $\pi^{\pm}$-mesons and K-mesons make up the basic number of the secondary particles, the result which we have obtained shows that the energy spectrum of the mesons produced in nuclear collisions is weaker than that calculated on the basis of the Landau theory.

On the basis of the calculations which we have carried out, we may draw the following conclusions:

1. The theory of nuclear collisions of very high energy, in the form in which it is given in Landau's work[1], does not allow one to carry out a quantitative calculation of cascade processes, because of the indeterminacy of the form of the energy spectrum of the secondary particles near the upper limit;

2. Experimental results show that the energy distributions of secondary particles of various types are essentially different from each other. However, we can at least affirm that in the case of nucleons (or hyperons) emitted as a result of nuclear collisions, there is a great probability that a significant portion of the energy of the incident nucleon is concentrated in one of them. On the other hand, the probability of concentration of a significant amount of energy in individual $\pi^{\pm}$-mesons or K-mesons is small.

After the calculations reported above had been carried out, it was shown by Belen'kii[16,17] that, in spite of the opinion expressed earlier, a systematic consideration of nuclear collisions on the basis of the Landau concepts leads to a notable probability of the appearance of particles heavier than π-mesons. Belen'kii showed further that the hydrodynamic character of the expansion of the system formed after the collision of the nucleons leads to a dependence of the energy concentration on the mass of the particles considered, with the concentration increasing with increasing mass. Thus, the further development of the Landau theory carried out by Belen'kii must lead to a smaller discrepancy between the theoretical and experimental results. Detailed calculations; analogous to those carried out in the present work, are needed in order to determine how well the Landau-Belen'kii theory agrees with present results.

It seems to us, however, that even the detailed theory cannot be reconciled with experimental results, since even it must clearly lead to an essential increase in the cœfficient of absorption of radioactive particles with increasing energy of the particles and increasing depth into the atmosphere. In our opinion, consequently, serious consideration should be given to the development of a theory of nuclear collisions which will be based on new premises[18], taking account of the possible "stucturedness" of the nucleon, and which will provide for the concentration of a large portion of the energy in a single nucleon and a small concentration of energy in mesons.

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