between these levels.

Gamma-quanta with energies $E_1 = 276$ and $E_2 = 226$ kev and approximately equal intensities of about 20 γ-quanta per 100 neutron capture have been observed in the measurement of the spectra of the γ-rays from the radiative capture of thermal neutrons by Co nuclei. The energies of the γ-quanta E_1 and E_2 are in agreement with the data from the work of Reier and Shamos[5].

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Concerning a Certain Possibility in Quantum Field Theory

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THE author has attempted to overcome the well-known difficulties of contemporary relativistic quantum field theory which result, first, from the appearance of irremovable infinities in many "unrenormalizable" variants of the theory (among which are evidently all variants with an interaction Hamiltonian which either consists of the product of more than three field operators or contains the derivatives of these operators) and, secondly, from the divergence of series that are obtained after the elimination of all infinities in the "renormalizable" variants (particularly in electrodynamics).

It would seem that the explanation of these difficulties must be sought not in a deficiency of the theory itself but in a short-coming of the method that is used to solve the equations of the theory (the Lomonoga-Schwinger equations), namely, the perturbation method (the decomposition of the desired solution in a MacLaurin series of powers of the field binding constant g). In other words, the series which is obtained formally in the course of the solution, because of the illegitimacy of this very expansion within the framework of ordinary mathematical analysis, does not provide the desired solution in the usual sense of a sum (the limit of the sum of n terms as $n \to \infty$)*.

It can reasonably be asked, however, whether it is possible in some manner to derive from the formally obtained series the solution which was improperly expanded in this very series. In other words, what mathematical operation should be understood by the "sum" of the series so that this "sum" would give the solution formally expanded in the series even when the ordinary concept of a sum leads to an obviously incorrect result ($\equiv \infty$). If the S matrix $S(g, p)$ of a process is selected as the solution of the quantum equations (where p denotes the set of momenta of "real" particles), then, assuming that this S matrix is an analytic function of p, our question can be answered by introducing the following generalizations of the usual concept of the sum of an infinite series, which satisfy general axiomatic requirements (see, for example, Ref. 1).

If the series $\sum_i U_i(\xi)$ converges in some interval of ξ to the analytic function $S(\xi)$, then by the generalized sum $\sum_{i\,\xi}$ (with respect to ξ) of this series (in the intervals where it diverges) we shall understand the analytic continuation of $S(\xi)$ in these intervals. Given a series of the form $\sum_i \int_{(l)} U_i(l, \xi)$, where each of the l-integrals can diverge**, then by the l-sum $\sum_i {}^l_\xi$ (for ξ) of this series, if it exists, we shall understand the l-integral of the generalized sum with respect to l and ξ of the integrands:

$$\sum_i{}^l_\xi \int_{(l)} U_i(l, \xi) = \int_{(l)} \sum_i{}_{l,\xi} U_i(l, \xi).$$

The proposed method is applicable to the pair theory of interactions of spinor fields and of a spinor and pseudoscalar field. The entire infinite set of series terms, each of which corresponds to a Feynman diagram, is divided in a definite manner into (infinite) subsets and the l-summation of the terms in each subset is performed (this l-summation can be accomplished in closed form). It results (as can be shown by the inductive method which the author used previously to solve another problem[2]) that the modified series which results from this operation, consisting of subsets summed in the l sense is not only renormalizable, but (after renormalization) converges in the usual sense and converges absolutely in each case for large momenta of "real" particles. Each term of the renormalized modified series is a complicated function of g which cannot be reduced to an integral power; for small g the series decomposes the S matrix in increasing powers of $g \ln g$, which reveals the origin of the irremovable infinities in the original perturbation series.

A natural result is that the modified series is equivalent to the original series in the following sense. If in the original series the l-integration is limited by some l_{max}, then such a "cutoff" series A can be compared with another series A' such that the formal expansion of each term A' in a Maclaurin series with respect to g (which converges to give the expanded term only for small values of l_{max}) leads to series A. For small g and large l_{max}, series A' gives an expansion in increasing powers of $g \ln(g + c/l_{max}^2)$. In the limit $l_{max} \to \infty$, A' coincides with our modified series, which we obtained without this intervening "cutting-off" procedure. This fact is extremely important for an understanding of the relationship between modified and ordinary series, since it emphasizes the difference in principle between the proposed method and the usual methods such as the introduction of cut-off factors, for which such equivalence obviously does not exist.

One of the peculiarities of the foregoing modification is the fact that now the interaction potential of two spinor particles in any approximation of pair theory has a singularity at the origin which is not stronger than $1/r \ln r$ (compare the results in Ref. 3), which makes it possible to use pair theory for the solution of problems concerning nuclear forces. An analysis of the potential of the first nonvanishing approximation in one of the variants of pair theory shows that attraction at large distances which decreases exponentially as $r \to \infty$, becomes repulsion at small distances (with a $1/r$ singularity at the origin. This is the result required by experiment.

It should be noted, in conclusion, that the proposed method can be used in electrodynamics and in the variants of the theory containing derivatives, although the latter possibility is obviously associated with certain difficulties.

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* The origin of the difficulty resulting from "irremovable" infinities can be illustrated by the formal expansion of the function $\ln g$ in a Maclaurin series; the difficulty resulting from series divergence can be illustrated by expansion of the function $(1 - g^2)^{-1}$ for $g^2 \geq 1$.

** Keeping in mind that l can mean any set of virtual momenta and ξ another set of both virtual and real momenta.

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