Accuracy of Isobaric Spin for the $1f_{7/2}$ Shell

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RADICATY[1] has calculated the accuracy of the isobaric spin T for the lightest nuclei on the assumption that the inaccuracy of T is caused by Coulomb forces alone. We have carried through analogous calculations for nuclei with $1d$ and $1f_{7/2}$ shells. These calculations, which followed the method of Racah[2,3], are characterized as follows:

1. The matrix elements of the Coulomb interaction can be of several kinds.

a) $\langle j^n T(\sigma) JM_T | G^c | j^n T'(\sigma') JM_T \rangle$,

$$G^c = \sum_{i>j}^{n} g_{ij}^c, \quad g_{ij}^c = \frac{(1/2 - \tau_i)(1/2 - \tau_j)}{|\mathbf{r}_i - \mathbf{r}_j|} e^2,$$

τ_i is the operator of the z component of the isobaric spin; (σ) is the symbol of the irreducible representation of the symplectic group[3]; $M_T = 1/2 \times (N - Z) = 1/2(A - 2Z)$; j^n is the nucleonic configuration of the unfilled outer shell. The inaccuracy of the isobaric spin of the lowest nuclear states which is associated with matrix elements of this form is $10^{-4} - 10^{-6}$ (squares of the amplitudes of the additional terms) and is relatively independent of configurations and types of coupling (LS or jj coupling).

b) Proton density of distribution in a closed spherically symmetrical shell. $P(r)$ denotes the combined density of protons in all closed shells. Neglecting exchange interaction[4], we can represent the Coulomb interaction operator of nucleons in the outer unfilled shell with the filled shells as

$$\sum_i f_i (1/2 - \tau_i), \quad f_i = e^2 \int \frac{P(r)\, d\tau}{|\mathbf{r} - \mathbf{r}_i|}.$$

A matrix element which is nondiagonal with respect to T:

$$\langle j^n T(\sigma) JM_T | \Sigma_i f_i (1/2 - \tau_i) | j^n T'(\sigma') JM_T \rangle$$

vanishes because of the well-known orthogonality of the genealogical coefficients[2,3]. This conclusion is not changed when the exchange interaction is taken into account and the matrix element is written in rigorous form, using the formulas given by Elliott and by Janagawa[5].

c) $$\langle (N_j)^n T(\sigma) JM_T \Big| \times \sum_i (1/2 - \tau_i) f_i \Big| (N_j)^{n-1} (N_j') T'(\sigma') JM_T \rangle$$

N and N' ard nonidentical principal quantum numbers of the states of individual nucleons. These matrix elements are the most important.

2. The calculations were carried through with oscillator wave functions, whose parameters were determined mainly by the differences in the binding energies of mirror nuclei, assuming charge independence of nuclear forces. The values of the parameters were obtained analogously in Ref. 6. In particular, we used for Sc_{21}^{42}: $\nu^{-1/2} = 2.0 \times 10^{-13}$ cm, for F_9^{18}: $\nu^{-1/2} = 1.7 \times 10^{-13}$ cm, for Li_3^6: $\nu^{-1/2} = 1.4 \times 10^{-13}$ cm, where ν is a parameter in the wave function; for example, for the $1s$ state: $\psi = \text{const}\, e^{-\nu r^2/2}$.

Calculation of matrix elements (c) gives the following values of the square of the amplitude of the added term for the ground states:

Nucleus	Li_3^6	F_9^{18}	Sc_{21}^{42}
$\alpha_{TT'}^2 =$	$5\cdot10^{-5}$	$4\cdot10^{-4}$	$1\cdot10^{-2}$

$T = 0, \quad T' = 1.$

These values are typical for nuclei of the corresponding shells.

Extensive experimental data are available for the lightest nuclei[7-9]. For nuclei with a $1f_{7/2}$ shell there are almost no data. Some conclusions, however, can be drawn from the following fact. The resolved β-transitions in odd nuclei with a $1f_{7/2}$ shell between the states $J_{in} = J_{fin} = j = 7/2$ are divided into two distinct groups: for non-mirror transitions ($T_{in} = T_{fin} \pm 1$): $\lg ft = 5 - 6$, and for mirror transitions ($T_{in} = T_{fin}$): $\lg ft \approx 3.5$. Examples are:

$Ti_{22}^{43}(\beta^+)Sc_{21}^{43}$: $\lg ft = 3{,}4$; $\quad Ca_{20}^{45}(\beta^-)Sc_{21}^{45}$: $\lg ft = 5.9$.

Without attempting to explain this fact, whose causes are still unknown[8], we shall note that the existence of such a clear difference between the values of ft for mirror and non-morror transitions enables us to say that the squares of the amplitudes of the added terms for the isobaric spins does not exceed a few percent; otherwise, the difference in ft for the two groups mentioned would be smaller.

A direct experimental check of the accuracy of T for nuclei with a $1f_{7/2}$ shell is difficult since nuclei with $N = Z$ or $N = Z \pm 1$ are unstable. For this purpose it will probably be useful to study the reactions (γ, n) and (γ, p)[9].

Our examination shows that whenever the outer neutrons and protons are to be found in the same shell in a stable nucleus, T is a good quantum number.

When we pass from nuclei with a $1f_{7/2}$ shell to heavier nuclei, we find that the outer nuetrons and protons in stable nuclei are contained in different shells. For such cases the proton and neutron shells are considered separately and the isobaric spin is not used as a quantum number. This is easily understood, since one cannot speak of the "accuracy" of T for these nuclei.

The fact that T is still a good quantum number for nuclei which contain a large amount of Coulomb energy is associated with the character of the coulomb forces: these are long-range forces, so that the nondiagonal matrix elements are smaller by one order of magnitude than those matrix elements which are diagonal with respect to T. In the limiting case of an infinite range for the forces the nondiagonal elements (in T) would vanish because the Hamiltonian would be symmetrical with respect to permutations of the particles.

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The Theory of Slowing Down of Neutrons

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THE integral of the collisions of neutrons with the nuclei of a moderator[1] can be written as

$$\sum_\alpha \int_0^u du' \int_{-1}^{1} d\mu' \int_0^{2\pi} d\beta' f_{\alpha 0}(u-u') \frac{\lambda(u')}{\lambda_\alpha(u')} \times \delta(\mu_0-\gamma_\alpha)\,\psi(\mathbf{r}, u', \mu', \beta') = \sum_\alpha \hat{K}_\alpha \hat{B}_\alpha \psi, \quad (1)$$

$$\hat{K}_\alpha \equiv \int_0^u du' f_{\alpha 0}(u-u') \frac{\lambda(u')}{\lambda_\alpha(u')} \times \int_{-1}^{1} d\mu' K_\alpha(\mu, \mu', u-u'), \quad (2)$$

$$K_\alpha \equiv (1-\mu^2-\mu'^2-\gamma_\alpha^2-2\gamma_\alpha\mu\mu')^{-1/2}, \quad (3)$$

$$\hat{B}_\alpha \equiv \int_0^{2\pi} \delta(\beta'-\bar{\beta})\, d\beta', \quad (4)$$

where ψ is the distribution function for collisions between neutrons and moderator nuclei (see, for example, Refs. 1 and 2), α is an index which designates individual elements with mass number M_α contained in the moderator, ϑ and β are the spherical angles of the vector $\boldsymbol{\omega} = \mathbf{p}/p$ (where $\mathbf{p}$ is the neutron momentum and $\mathbf{r}$ is its radius vector), $u = \ln(2mE_0/p^2)$ with E_0 as the initial neutron energy and m as the neutron mass, λ_α is the partial neutron mean free path allowing for inelastic collisions with nuclei of mass M_α, and λ is the total neutron mean free path in the medium,

$$\gamma_\alpha(u) \equiv [(M_\alpha+m)e^{-u/2}-(M_\alpha-m)e^{u/2}]/2m, \quad (5)$$

$$f_{\alpha_0}(u) \equiv \begin{cases} [(M_\alpha+m)^2/4mM_\alpha]\,e^{-u} & \text{for } u \leqslant q_\alpha, \\ 0 & \text{for } u > q_\alpha, \end{cases} \quad (6)$$