difference in ft for the two groups mentioned would be smaller.

A direct experimental check of the accuracy of T for nuclei with a $1f_{7/2}$ shell is difficult since nuclei with $N = Z$ or $N = Z \pm 1$ are unstable. For this purpose it will probably be useful to study the reactions (γ, n) and (γ, p)[9].

Our examination shows that whenever the outer neutrons and protons are to be found in the same shell in a stable nucleus, T is a good quantum number.

When we pass from nuclei with a $1f_{7/2}$ shell to heavier nuclei, we find that the outer nuetrons and protons in stable nuclei are contained in different shells. For such cases the proton and neutron shells are considered separately and the isobaric spin is not used as a quantum number. This is easily understood, since one cannot speak of the "accuracy" of T for these nuclei.

The fact that T is still a good quantum number for nuclei which contain a large amount of Coulomb energy is associated with the character of the coulomb forces: these are long-range forces, so that the nondiagonal matrix elements are smaller by one order of magnitude than those matrix elements which are diagonal with respect to T. In the limiting case of an infinite range for the forces the nondiagonal elements (in T) would vanish because the Hamiltonian would be symmetrical with respect to permutations of the particles.

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The Theory of Slowing Down of Neutrons

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THE integral of the collisions of neutrons with the nuclei of a moderator[1] can be written as

$$\sum_\alpha \int_0^u du' \int_{-1}^{1} d\mu' \int_0^{2\pi} d\beta' f_{\alpha 0}(u-u') \frac{\lambda(u')}{\lambda_\alpha(u')} \times \delta(\mu_0 - \gamma_\alpha)\, \psi(\mathbf{r}, u', \mu', \beta') = \sum_\alpha \hat{K}_\alpha \hat{B}_\alpha \psi, \tag{1}$$

$$\hat{K}_\alpha \equiv \int_0^u du' f_{\alpha 0}(u-u') \frac{\lambda(u')}{\lambda_\alpha(u')} \times \int_{-1}^{1} d\mu' K_\alpha(\mu, \mu', u-u'), \tag{2}$$

$$K_\alpha \equiv (1 - \mu^2 - \mu'^2 - \gamma_\alpha^2 - 2\gamma_\alpha \mu\mu')^{-1/2}, \tag{3}$$

$$\hat{B}_\alpha \equiv \int_0^{2\pi} \delta(\beta' - \bar{\beta})\, d\beta', \tag{4}$$

where ψ is the distribution function for collisions between neutrons and moderator nuclei (see, for example, Refs. 1 and 2), α is an index which designates individual elements with mass number M_α contained in the moderator, ϑ and β are the spherical angles of the vector $\boldsymbol{\omega} = \mathbf{p}/p$ (where $\mathbf{p}$ is the neutron momentum and $\mathbf{r}$ is its radius vector), $u = \ln(2mE_0/p^2)$ with E_0 as the initial neutron energy and m as the neutron mass, λ_α is the partial neutron mean free path allowing for inelastic collisions with nuclei of mass M_α, and λ is the total neutron mean free path in the medium,

$$\gamma_\alpha(u) \equiv [(M_\alpha + m)e^{-u/2} - (M_\alpha - m)e^{u/2}]/2m, \tag{5}$$

$$f_{\alpha_0}(u) = \begin{cases} [(M_\alpha + m)^2/4mM_\alpha]\, e^{-u} & \text{for } u \leqslant q_\alpha, \\ 0 & \text{for } u > q_\alpha, \end{cases} \tag{6}$$

$$q_\alpha = 2\ln[(M_\alpha + m)/(M_\alpha - m)], \tag{7}$$

$$\mu_0 = \omega\omega' = \mu\mu' + \sqrt{1-\mu^2}\sqrt{1-\mu'^2}\cos(\beta' - \beta), \tag{8}$$

$\bar{\beta}$ $(0 < \bar{\beta} < 2\pi)$ is the root of the equation

$$\mu\mu' + \sqrt{1-\mu^2}\sqrt{1-\mu'^2}\cos(\bar{\beta} - \beta) - \gamma_\alpha = 0. \tag{9}$$

We note that in the one-dimensional problem which was considered in Refs. 3-6, the function ψ is independent of β, so that $B_\alpha \psi = \psi$. In this case it is easy to derive an equation for K_α in the form of a series of Legendre polynomials:

$$K_\alpha = \pi \sum_{l=0}^{\infty} (2l+1) P_l(\gamma_\alpha) P_l(\mu) P_l(\mu'). \tag{10}$$

$K_\alpha(\mu, \mu', u)$ is the angular distribution (with respect to angle ϑ) of neutrons scattered by a nucleus in the laboratory system. It is well known that its strongest dependence on μ and μ' occurs for the scattering of neutrons by hydrogen. We shall now show that even in this case the principal part is played in (10) by the sum of a few $(n_0 + 1)$ first terms of the series which we shall denote by K^0_α. If we now express K_α according to Eq. (3) the remainder of series (10) $\tilde{K}_\alpha = K_\alpha - K^0_\alpha$ can be written as a finite expression. In (2) we now replace $\hat{K}_\alpha$ by K^0_α or $\tilde{K}_\alpha$ and obtain the operators $\hat{K}^0_\alpha$ and $\hat{\tilde{K}}_\alpha$.

Regarding all functions of u and μ as points in in the metric space L_2, defining the norm of $V(u,\mu)$ as

$$\| V \| = \int_0^u du e^{-u/2} \int_{-1}^{1} d\mu V^2(u, \mu) \tag{11}$$

and denoting $\epsilon = \|\tilde{K}_\alpha\|^{1/2} / \|K^0_\alpha\|^{1/2}$, we obtain $\epsilon = 0.33 < 1$ from (10) with $M = m$ and $n_0 = 1$.

Let n be chosen so that $\epsilon < 1$. Instead of the exact kinetic equation[1]

$$\hat{L}\psi = \sum_\alpha \hat{K}_\alpha \hat{B}_\alpha \psi + S \tag{12}$$

we shall consider the equation

$$\hat{L}\psi^0 = \sum_\alpha \hat{K}^0_\alpha \hat{B}_\alpha \psi^{(0)} + S, \tag{13}$$

where S is the density of neutron sources and $\hat{L}_r(u, \mu, \beta) \equiv \lambda(u)\,\omega\,\text{grad} + 1$.

Comparing (12) and (13), we find by using Schwarz' inequality that $(\|\psi - \psi^{(0)}\|^{1/2} / \|\psi^{(0)}\|^{1/2}) \leq \epsilon$.

The effect of $\hat{\tilde{K}}_\alpha$ can be regarded as a small perturbation. For this purpose we write

$$\psi - \psi^{(0)} = \psi^{(1)} + \psi^{(2)} + \ldots + \psi^{(n)} + \ldots. \tag{14}$$

Substituting this series in (1), breaking it off for successive values of $n = 1, 2, \ldots$, taking (13) into account and each time dropping a term of the form

$$S^{(n)} = \sum_\alpha \hat{\tilde{K}}_\alpha \hat{B}_\alpha \psi^{(n)}, \tag{15}$$

we arrive at an equation for the determination of the nth correction which differs from (13) only in that S is replaced by $S^{(n-1)}$.

The solutions of these equations can be written with the aid of their Green's functions G in the form

$$\psi^{(n)}(\mathbf{r}, u, \mu, \beta) = \int S^{(n-1)}(\mathbf{r}_1, u_1, \mu_1, \beta_1) \times G(\mathbf{r}, \mathbf{r}_1, u, u_1, \mu, \mu_1, \beta, \beta_1)\, d\mathbf{r}_1\, du_1\, d\mu_1\, d\beta_1, \tag{16}$$

setting $S^{(-1)} = S$.

Recalling how the equations for $\psi^{(n)}$ were derived by the use of (13) and (16), we find that $(\|\psi^{(n)}\|^{1/2} / \|\psi^{(0)}\|^{1/2}) \leq \epsilon^n$,

$$\left(\left\| \psi - \psi^{(0)} - \sum_{i=1}^{n} \psi^{(i)} \right\|^{1/2} \Big/ \left\| \psi^{(0)} \right\|^{1/2} \right) \leq \epsilon^{n+1}, \tag{17}$$

that is, the series (14) converges to $\psi - \psi^{(0)}$ with the error defined by (17). We note that the convergence of a point sequence in space L_2, which we have used, corresponds, as is well known, to the convergence in the mean of a sequence of functions.

The Green's function G satisfies (13) when we set $S = \delta(\mathbf{r} - \mathbf{r}_1) \times \delta(u - u_1)\delta(\mu - \mu_1)\delta(\beta - \beta_1)$.

Multiplying this equation on the left by the operator L^{-1} and using g to denote the Green's function of the operator L which satisfies the boundary conditions of the problem, we put it into the form

$$G = \sum_\alpha \int d\mathbf{r}' \hat{K}^0_\alpha \hat{B}_\alpha g(\mathbf{r}, \mathbf{r}', \mu, \beta) \times G(\mathbf{r}', \mathbf{r}_1, u', u_1, \mu', \mu_1, \beta', \beta_1) + g(\mathbf{r}, \mathbf{r}_1, u, \mu, \beta)\,\delta(u - u_1)\,\delta(\mu - \mu_1)\,\delta(\beta - \beta_1). \tag{18}$$

The right-hand side of (18) actually contains G^0, the sum of the first $\sum_{n=0}^{n_0}(2n+1)$ (in the one-dimensional case n_0+1)] spherical harmonics of the Green's functions. Therefore, the obtaining of G is reduced to the obtaining of G^0 by the method of spherical harmonics, that is, to the solution of a small set of integral equations which can be solved with consideration of the dependence of λ and λ_α on u. The Green's function G and the functions $\psi^{(0)}$ and $\psi^{(n)}$ are then determined according to (18) and (16) in finite form.

This is especially important for calculating neutron distributions in media which contain hydrogen (water, petroleum, etc.) since the usual expansion of the distribution in spherical functions[1-6] converges poorly in this case.

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The Effect of Finiteness of Target-Nucleus Mass on Angular Distribution in (d, p) and (d, n) Reactions

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In analyzing the angular distribution of the products of (d, p) and (d, n) reactions, it is customary to use the formulas derived by Butler[1] for an infinitely heavy target-nucleus. We have calculated the angular distribution with allowance for the finiteness of the nuclear mass by the same method of smooth connection of functions and subject to the same assumptions as in Ref. 1 without considering the question of the legitimacy of these assumptions. The results of the calculation are given below.

The angular distribution is given by the following formula:

$$S(\theta) = |(Z_1^2+\alpha^2)^{-1} - (Z_1^2+(\alpha+\beta)^2)^{-1}|^2 \times \sum_l N^J_{l k_n^s} \left| \left(\sum_{q=0}^{l} \frac{(l+q)!}{(l-q)!\, q!\, (2k_n^s R_0)^q} \times [l+q+k_n^s R_0] \right) f_l(Z_2R_0) - \left(\sum_{q=0}^{l} \frac{(l+q)!}{(l-q)!\, q!\, (2k_n^s R_0)^q} \right) (Z_2R_0)\, f_{l+1}(Z_2R_0) \right|^2, \tag{1}$$

where l is the orbital angular momentum of a nucleon captured in an orbit of the ultimare nucleus; k_n^s is the wave number of this nucleon in the final state. The values of N are the same as in Ref. 1. R_0 is the "reaction radius" and is obtained by setting in agreement the theoretical and experimental curves for the angular distribution of the products of stripping. The wave numbers α and β which describe the state of the deuteron, are taken to be $\alpha = 0.23\times10^{13}$ cm^{-1} and $\beta = 1.4\times10^{13}$ cm^{-1}; $f_l(x)$ are spherical Bessel functions: $f_l(x) = \sqrt{\pi/2x}\, J_{l+1/2}(x)$. This formula differs from that of Butler only in that K is replaced by Z_1, Z by Z_2 (and r_0 by R_0). Here Z_1 and Z_2 are given by

$$Z_1 = \frac{1}{\hbar}\left| \left(\frac{M_a}{M}\right)^{1/2} \left(\frac{M_b}{M_b+M_c}\right) \sqrt{2(M_b+M_c)W_1}\,\mathbf{n}_1 - \left(\frac{M_a+M_c}{M}\right)^{1/2} \sqrt{2M_bW_2}\,\mathbf{n}_2 \right|, \tag{2}$$

$$Z_2 = \frac{1}{\hbar}\left| \left(\frac{M_a}{M}\right)^{1/2} \sqrt{2(M_b+M_c)W_1}\,\mathbf{n}_1 - \left(\frac{M_a^2}{M(M_a+M_c)}\right)^{1/2} \sqrt{2M_bW_2}\,\mathbf{n}_2 \right|,$$

where $(\mathbf{n}_1, \mathbf{n}_2) = \cos\theta$, θ is the angle between the directions of the escaping nucleon and the incident deuteron in the center-of-mass system, W_1 and W_2 are the kinetic energy of the deuteron and nucleon with respect to the initial and final nucleus; M_a, M_b and M_c are the masses of the target-nucleus, the