On the Equations of Motion of Finite Masses in the General Theory of Relativity*

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On the basis of the theorem of the center of mass in the general theory of relativity and the principle of equivalence, there is proved for systems of bodies of astronomical type an approximate theorem (with neglect of gravitational radiation, etc.) on the motion of the center of mass of each body along geodesic lines in the gravitational fields of the other bodies, which are regarded as point sources. This theorem gives the laws of motion of finite masses in the general theory of relativity. It provides the basis for a rather simple derivation of the relativistic equations of motion of n-body systems, by substitution into the equation for geodesic lines of the second-order solutions of the Einstein gravitational equations. The resulting equations agree with the equations of motion found in a more complicated way as the conditions for solubility of the gravitational equations in third approximation.

The theorem reveals the cause of the agreement between the work of Einstein and that of Fock and his coworkers based on different conceptions of the bodies of the system as finite or point masses, and shows the fundamental and practical insignificance of the principle of equivalence in the Einstein theory of gravitation.

1. INTRODUCTION

ONE of the most important problems of the general theory of relativity is the problem of the motion of n-bodies interacting by gravitational forces only. Under certain conditions, of which the most important is the smallness of the velocity v of their motion in comparison with the speed of light c, such bodies form systems of astronomical type, such as, for example, our planetary system.

Einstein and his co-workers[1-3] considered such systems as sets of point masses giving rise to gravitational fields surrounding them, determined by the well-known gravitational equations of the general theory of relativity,

$$R_{\mu\nu} - {}^1/_2 g_{\mu\nu} R = -kT_{\mu\nu}, \qquad (1.1)$$

where $R_{\mu\nu}$ is Einstein's curvature tensor; $T_{\mu\nu}$ is the energy momentum tensor, which is taken to be unequal to zero only at singular points of the field; $g_{\mu\nu}$ is the metric tensor; and $k = 8\pi\gamma/c^2$, where γ is the gravitational constant of Newton's theory of gravitation. Here also

$$(T^{\mu\nu})_\nu = 0. \qquad (1.2)$$

It was found that the equation of motion of such masses are contained in Eq. (1.1) as the conditions for solubility in one approximation or another; a result, by the way, which cannot be accepted as altogether natural, since Eq. (1.2) follows entirely from Eq. (1.1), and the structure of the tensor $T_{\mu\nu}$ as point singularities is not fixed by any supplementary equations.

Fock and his co-workers[4-6] approached the solution of the same problem, starting from more realistic assumptions about the bodies of the system, which they regarded as certain finite masses with their distribution and motion given by a tensor $T_{\mu\nu}$ different from zero in a certain spatial region of spherical shape. A special feature of Fock's method was the imposition of coordinate conditions

$$\partial(\sqrt{-g}\, g^{\mu\nu})/\partial x^\nu = \partial G^{\mu\nu}/\partial x^\nu = 0, \qquad (1.3)$$

fixing the choice of special coordinates, called by Fock "harmonic" coordinates. Despite the great difference in the statement of the problem and the methods of solution, the equations of motion with inclusion of relativistic corrections, as found by Petrova[5] by Fock's method, agree completely with the equations of Einstein and his co-workers[3], being obtained in both cases as conditions of solubility of the gravitational equations (1.1) in third approximation.

In Fock's opinion[4,7], the coordinates defined by Eq. (1.3) are preferred coordinates, which is of course in contradiction with the general principle of relativity. In opposition to this, Infeld[8] showed by direct calculation that the coordinate condition (1.3) is not essential for the derivation of the equations of motion. Infeld's point of view can be supported by a number of other considerations of a fundamental character.[9]

At the same time Infeld continues to treat the bodies of the system as point masses. It still

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remains unclear, however, why the treatment of the bodies as masses of finite spatial dimensions gives the same equations of motion as for point masses. We give below a new derivation of the equations of motion of a system of bodies of the astronomical type, providing an explanation of this fact and showing the deep organic connection of the gravitational equations with the general principle of relativity. Moreover, the new derivation is distinguished by great simplicity in the calculations, since it leads to the equations of motion including relativistic corrections on the basis of the solution of the gravitational equations (1.1) in only the second approximation, while in the methods of Einstein and of Fock and co-workers the equations of motion are obtained as conditions of solubility of the system (1.1) only in the third approximation.

2. ON THE LAW OF MOTION OF RELATIVISTIC CENTERS OF MASS OF BODIES IN AN ISOLATED SYSTEM OF ASTRONOMICAL TYPE

In classical mechanics this law states: the centers of mass of the bodies of the system move as if they had applied to them all the external forces that act on each body from the other bodies of the system. Since the concept of center of mass exists also in the general theory of relativity and the theorem of the center of mass is strictly valid,[10] it can be supposed that according to this theorem the centers of mass of the bodies move according to a quite definite law, containing in itself as a special case the law of classical mechanics stated above. We confine our consideration to systems of astronomical type, which we take to satisfy the following conditions[4]

$$a \ll L, \quad v \ll c, \quad R \ll L,$$

$a = \gamma m/c^2$ is the gravitational radius of a body of mass m; L is a length characterizing its linear dimensions; v is the speed of the body or of its parts; and R is the distance of the body from any other body of the system.

We note also that according to the virial theorem

$$\overline{\sum_a m_a v_a^2} = \overline{\sum_{(a\neq b)} \frac{\gamma m_a m_b}{2|r_a - r_b|}}. \qquad (2.2)$$

Here a and b are indices numbering the bodies of the system.

The total energy-momentum tensor, including also the tensor $(-gt^{\mu\nu})$ of the gravitational field,

$$S^{\mu\nu} = -g\,(T^{\mu\nu} + t^{\mu\nu}) \qquad (2.3)$$

and the corresponding angular momentum tensor

$$M^{\mu\nu\sigma} = x^\mu S^{\nu\sigma} - x^\nu S^{\mu\sigma} \qquad (2.4)$$

satisfy the conservation laws

$$\partial S^{\mu\nu}/\partial x^\nu = 0, \quad \partial M^{\mu\nu\sigma}/\partial x^\sigma = 0. \qquad (2.5)$$

Integration of the relations (2.5) over the surface of the volume ω of a body, with account taken of the fact that $T^{\mu\nu} \neq 0$ only inside it, gives

$$\frac{\partial}{\partial t}\left\{\int_\omega (-g)(T^{i0} + t^{i0})\,d\omega\right\} + \oint_f (-g)\,t^{ik} df_k = 0; \qquad (2.6)$$

$$\frac{\partial}{\partial t}\left[\int_\omega (-g)(T^{00} + t^{00})\,d\omega\right] + \oint_f (-g)\,t^{0k} df_k = 0; \qquad (2.7)$$

$$\frac{\partial}{\partial t}\left[\int_\omega M^{ik0}\,d\omega\right] + \oint_f (-g)(x^i t^{kl} - x^k t^{il})\,df_l = 0; \qquad (2.8)$$

$$\frac{\partial}{\partial t}\left(\int_\omega M^{i00} d\omega\right) + \oint_f (-g)(x^i t^{0l} - x^0 t^{il})\,df_l = 0. \qquad (2.9)$$

The second of the conditions (2.1) allows us to neglect the change of the gravitational energy, i.e., to take $t^{i0} \approx 0$. In virtue of the third of the conditions (2.1), inside a given body, the change of the gravitational field due to the other bodies can be neglected, and therefore we can choose a system of reference such that inside the body the gravitational field is produced only by itself. In such a system of reference

$$\oint_f g t^{ik} df_k \approx 0. \qquad (2.10)$$

Consequently, in this system, in virtue of Eqs. (2.6)–(2.10)

$$P^\mu = \int_\omega S^{\mu 0} d\omega = \text{const}, \qquad M^{\mu\nu} = \int_\omega M^{\mu\nu 0} d\omega = \text{const}. \qquad (2.11)$$

But then, by the theorem of the center of mass,[10] in this system of reference there will exist a four-vector of the center of mass of this body

$$Y^{\mu} = M^{\mu\nu}P_{\nu}/P_{\alpha}P^{\alpha}, \tag{2.12}$$

satisfying the relation

$$d^2Y^{\mu}/ds^2 = 0. \tag{2.13}$$

Making a transformation now to a system of reference connected with the center of mass of the system as a whole, we get instead of Eq. (2.13)

$$(d^2Y^{\mu}/ds^2) + \Gamma^{\mu}_{\alpha\beta}(dx^{\alpha}/ds)(dx^{\beta}/ds) = 0. \tag{2.14}$$

As is well known, the quantity $\Gamma^{\mu}_{\beta\alpha}$ transforms in the following way when we go from one system of reference to another:

$$\Gamma^{\mu}_{\alpha\beta} = \frac{\partial x^{\mu}}{\partial x'^{\sigma}}\left(\frac{\partial x'^{\tau}}{\partial x^{\alpha}}\frac{\partial x'^{\varepsilon}}{\partial x^{\beta}}\Gamma'^{\sigma}_{\tau\varepsilon} + \frac{\partial^2 x'^{\sigma}}{\partial x^{\alpha}\partial x^{\beta}}\right). \tag{2.15}$$

Taking the coordinates x'_{σ} to refer to the system of reference of the center of mass of our body, in which by Eq. (2.13) $\Gamma'^{\sigma}_{\tau\epsilon} = 0$, we get instead of Eq. (2.15) the simple relation

$$\Gamma^{\mu}_{\alpha\beta} = (\partial x^{\mu}/\partial x'^{\sigma})(\partial^2 x'^{\sigma}/\partial x^{\alpha}\partial x^{\beta}). \tag{2.16}$$

This shows that the symbols $\Gamma^{\mu}_{\alpha\beta}$ occurring in Eq. (2.14) owe their existence only to the transformation of coordinates leading to the appearance of the gravitational field at the center of mass of the body, produced by all the bodies of the system except the given one. Therefore, in using Eq. (2.14) in what follows, we shall substitute into it the $\Gamma^{\mu}_{\alpha\beta}$ calculated for the center of mass of the given body from the quantities $g_{\mu\nu}$ corresponding to the external gravitational field of the other bodies of the system.

The relations (2.14) also express the law of motion of the relativistic center of mass of the given body in a reference system connected to the center of mass of the system as a whole, providing a generalization of the law formulated above for the motion of the center of mass in the external force field of classical mechanics.

The new relativistic law can be formulated in the following way: the center of mass of any body of the system moves along a geodesic line in the gravitational field of the other bodies. This proposition can be used for the direct derivation of the equations of motion of finite masses from Eq. (2.14), as will be done below. For this purpose it is first of all necessary to transform Eq. (2.14) into a form closer to the usual one.

The expression for the square of the interval between two infinitely close points has the well-known form

$$ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}. \tag{2.17}$$

We shall suppose that in the Galilean approximation

$$ds^2 = c^2dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2, \tag{2.18}$$

where $x_0 = t$ is the time coordinate.

According to Eq. (2.17) we have, along the geodesic line followed by the center of mass of the body

$$ds = dt\sqrt{g_{ik}\dot{x}_i\dot{x}_k + 2g_{i0}\dot{x}_i + g_{00}} = dtA, \tag{2.19}$$

where $\dot{x}_i = dx_i/dt$ and A will be functions of the time. Then after eliminating A we can write Eq. (2.14) in the form

$$\ddot{x}_i + (\Gamma^i_{lk} - \dot{x}_i\Gamma^0_{lk})\dot{x}_l\dot{x}_k \tag{2.20}$$
$$+ 2(\Gamma^i_{l0} - \Gamma^0_{l0}\dot{x}_i)\dot{x}_l + \Gamma^i_{00} - \Gamma^0_{00}\dot{x}_i = 0.$$

3. DERIVATION OF THE RELATIVISTIC EQUATIONS OF MOTION FROM THE SOLUTION OF THE GRAVITATIONAL EQUATIONS IN SECOND APPROXIMATION, FOLLOWING FOCK[4]

For convenience in the calculations and in comparing Fock's results with ours, we shall use the notation of his paper.[4] There, in the second approximation, proceeding in terms of an expansion in powers of $1/c$, the solutions of Eq. (1.1) are

$$G^{00} = \frac{1}{c} + \frac{4U}{c^3} + \frac{7U^2}{c^5} + \ldots,$$
$$G^{0i} = \frac{4U_i}{c^3} + \frac{4S_i}{c^5} + \ldots, \tag{3.1}$$
$$G^{ik} = -c\delta_{ik} + \frac{4S_{ik}}{c^3} + \ldots$$

The quantities appearing in these relations satisfy the equations:

$$\Box U = -4\pi\gamma(c^2 + {}^1/_2 U)T^{00},$$
$$\Box U_i = 4\pi\gamma g T^{0i} = 4\pi\gamma(c^2 + 4U)T^{0i}, \tag{3.2}$$

$$\Box S_i = -3(\partial U/\partial t)(\partial U/\partial x_i) \tag{3.3}$$
$$+ 4(\partial U_i/\partial x_j - \partial U_j/\partial x_i)\partial U/\partial x_j,$$

$$\Delta U_{ik} = -4\pi\gamma c^2 T^{ik}, \tag{3.4}$$

$$\Delta V_{ik} = {}^1/_2\delta_{ik}(\text{grad } U)^2 - \frac{\partial U}{\partial x_i}\frac{\partial U}{\partial x_k},$$

$$S_{ik} = U_{ik} + V_{ik}. \qquad (3.5)$$

Here use has been made of the value of the determinant g, which is given in second approximation by the relation

$$g = -c^2 - 4U - (7u^2/c^2) + 4S/c^2, \qquad (3.6)$$

where

$$S = S_{11} + S_{22} + S_{33}.$$

We note that the $G^{\mu\nu}$ still have to satisfy the condition for harmonic coordinates, Eq. (1.3).

By means of Eqs. (3.1) and (3.6) and the use of the formula

$$G_{\mu\nu} = \sqrt{-g}\, g_{\mu\nu} = -\text{ min. } G^{\mu\nu} \qquad (3.7)$$

we can find the covariant components of the metric tensor with accuracy to and including the order $1/c^2$:

$$g_{00} = c^2 + 2U + (5U^2/2c^2) - 2S/c^2; \qquad (3.8)$$
$$g_{0i} = 4U_i/c^2, \quad g_{ik} = -(1 + 2U/c^2)\delta_{ik}.$$

We note that, to the required degree of accuracy

$$\sqrt{-g} = (c + 2U/c) + (3U^2/2c^3) - 2S/c^3. \qquad (3.9)$$

By means of Eqs. (3.1), (3.8), and (3.9) we find the Christoffel symbols of the second kind that are nonvanishing in our approximation; substitution of these into Eq. (2.20) gives

$$\ddot{x}_i - \frac{\partial U}{\partial x_i} = \frac{1}{C^2}\left(-\frac{9}{2} U \frac{\partial U}{\partial x_i} + \frac{\partial S}{\partial x_i} + 4\frac{\partial U_i}{\partial t} + 4\frac{\partial U_i}{\partial x_l}\dot{x}_l - 4\frac{\partial U_l}{\partial x_i}\dot{x}_l - 3\frac{\partial U}{\partial t}\dot{x}_l + \frac{\partial U}{\partial x_i}v^2 - 4\frac{\partial U}{\partial x_l}\dot{x}_l\dot{x}_i\right), \qquad (3.10)$$

where $v^2 = \dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2$. The quantities U, S, and U_i are determined by the solutions of Eqs. (3.2) — (3.8) for the point coinciding with the center of mass of the given body a, with coordinates a_1.

The energy-momentum tensor in second approximation has the form:

$$T^{ik} = (1/c^2)\rho_a(r)\dot{a}_i\dot{a}_k + (^1/_3 c^2)\psi_a(r)\delta_{ik}, \qquad (3.11)$$
$$T^{00} = (1/c^2)(1 - U/c^2)\rho_a(r) \times \{1 + (^1/_2 c^2)(v_a^2 - U^a(r))\},$$
$$T^{0i} = (1/c^2)(1 - 2U/c^2)\rho_a(r) \times \{1 + (^1/_2 c^2)(v_a^2 - U^a(r))\}\dot{a}_i + (^1/_3 c^4)\psi_a(r)\dot{a}_i.$$

Outside the body a its components are, of course, equal to zero.

The potentials $U^a(a)$ and $U_i^a(a)$ from the other bodies at the center of mass of the given body are, in virtue of Eqs. (3.2) and (3.11), given by the expressions

$$U^a(a) = \sum_b{}' \frac{\gamma m_b}{|a-b|}\left\{1 + \frac{1}{2c^2}(v_b^2 - U^b(b)\right\} + \frac{1}{2c^2}\frac{\partial^2}{\partial t^2}\left[\sum_b{}' \gamma m_b|a-b|\right], \qquad (3.12)$$

$$U_i^a(a) = \sum_b{}' \frac{\gamma m_b \dot{b}_i}{|a-b|}\left\{1 + \frac{1}{2c^2}(v_b^2 + 6U^b(b)\right\} + \frac{1}{2c^2}\frac{\partial^2}{\partial t^2}\sum_b{}' \gamma m_b \dot{b}_i|a-b| + \frac{22}{3c^2}\sum_b{}' \frac{\gamma\varepsilon_b\dot{b}_i}{|a-b|},$$

where

$$\varepsilon_b = \int_0^\infty \psi_b(r)\, d\omega \qquad (3.13)$$

is the absolute value of the potential energy of the Newtonian gravitational field of the mass of body b. The symbol Σ' denotes a sum not including the term with the index $b = a$.

The solutions of the equations (3.4) outside the masses are given by the relations

$$U_{ii} = \sum_a\left(\frac{\gamma m_a v_a^2}{|r-a|} + \frac{\gamma\varepsilon_a}{|r-a|}\right), \qquad (3.14)$$

$$V_{ii} = \sum_a\left(\frac{\gamma^2 m_a^2}{4|r-a|^2} + \sum_{b\neq a}\frac{\gamma^2 m_a m_b}{2}\frac{\partial^2 \ln S}{\partial a_j \partial b_j} - \frac{\gamma\varepsilon_a}{|r-a|}\right), \qquad (3.15)$$

in which ϵ_a is defined by Eq. (3.13) and

$$S = |r-a| + |r-b| + |a-b|. \qquad (3.16)$$

It must be noted that at distances large in comparison with the linear dimensions of the body

$$U \approx \gamma m_a/|r-a|. \qquad (3.17)$$

It is essential that, in accordance with Eqs. (3.14) and (3.15), the quantity $S = U_{ii} + V_{ii}$ defined by Eq. (3.5) does not contain terms involving ϵ_a, while at the center of mass of the given body

$$S(a)=\sum_b{}'\left(\frac{\gamma m_b v_b^2}{|a-b|}+\frac{\gamma^2 m_b^2}{4|a-b|^2}\right) \qquad (3.18)$$

$$+\sum_{b\neq c}\sum\frac{\gamma^2 m_b m_c}{2}\frac{\partial^2 \ln S}{\partial b_j \partial c_j}.$$

We now introduce the potential energy of interaction of the bodies of the system,

$$\Phi=-\frac{1}{2}\sum_a{}'\sum_b{}'\frac{\gamma m_a m_b}{|a-b|}. \qquad (3.19)$$

Multiplying Eq. (3.10) by m_a and substituting into it the quantities defined by the relations (3.12), (3.18), and (3.19), we obtain the equations of motion with relativistic corrections:

$$m_a\ddot{a}_i+\frac{\partial\Phi}{\partial a_i} \qquad (3.20)$$
$$+\frac{1}{c^2}\Big\{[v_a^2-4U^a(a)]\frac{\partial\Phi}{\partial a_i}-4\dot{a}_i\dot{a}_m\frac{\partial\Phi}{\partial dm}$$
$$+\gamma m_a\sum_b{}'\Big[\frac{7}{2|a-b|}\frac{\partial\Phi}{\partial b_i}$$
$$+\frac{(a_i-b_i)(a_m-b_m)}{2|a-b|^3}\frac{\partial\Phi}{\partial b_m}$$
$$+m_b\left(-\frac{1}{2}\dot{b}_m\dot{b}_n\frac{\partial^3|a-b|}{\partial a_i\partial a_m\partial a_n}+\frac{(a_i-b_i)}{|a-b|^3}\right)$$
$$\times\left(\frac{3}{2}v_b^2-4\dot{a}_m\dot{b}_m-U^b(b)\right)$$
$$+\frac{(a_m-b_m)}{|a-b|^3}(4\dot{a}_m\dot{b}_i-4\dot{b}_m\dot{b}_i+3\dot{b}_m\dot{a}_i)\Big]\Big\}=0.$$

They are in complete agreement with the equations obtained by Petrova[5] by Fock's method as the condition of solubility of the gravitational equations (1.1) in the third approximation. It is appropriate here to emphasize once again that Eq. (3.20) has been found on the basis of Eq. (1.1) in only the second approximation.

The new derivation of Eq. (3.20) shows that in the method of Fock there is essentially a replacement of extended bodies by masses located at their centers of mass. We also get an understanding of why the relativistic equations of motion turn out the same when obtained from different hypotheses about the bodies, either as finitely extended masses[4,5] or as point masses,[3] since in both cases it is a question of the centers of mass of the bodies, in which their masses are, as it were, concentrated. The correctness of what has been said can also be verified by direct calculation.

4. THE RELATIVISTIC EQUATIONS OF MOTION AS A CONSEQUENCE OF THE SOLUTION OF THE GRAVITATIONAL EQUATIONS IN SECOND APPROXIMATION BY THE METHOD OF EINSTEIN AND HIS COWORKERS[3]

In this work the bodies are treated as point masses, for which the tensor $g_{\mu\nu}$ of the external field is to be found in the form

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}, \qquad (4.1)$$

where $\eta_{\mu\nu}$ are the elements of the matrix

$$\eta^{\mu\nu}=\begin{pmatrix}+1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}. \qquad (4.2)$$

Here we have set $x_0=ct$; the $h_{\mu\nu}$ are expanded in series

$$h_{00}=c^{-2}\underset{2}{h}_{00}+c^{-4}\underset{4}{h}_{00}+\ldots;$$
$$h_{0n}=c^{-3}\underset{3}{h}_{0n}+c^{-5}\underset{5}{h}_{0n}+\ldots, \qquad (4.3)$$
$$h_{nm}=c^{-2}\underset{2}{h}_{nm}+c^{-4}h_{nm}+\ldots$$

Use is also made of the quantities

$$\gamma_{\mu\nu}=h_{\mu\nu}-{}^1/_2\,\eta_{\mu\nu}\eta^{\sigma\rho}h_{\sigma\rho}, \qquad (4.4)$$

which are also expanded in series analogous to Eq. (4.3).

For the case of two bodies values of $h_{\mu\nu}$ and $\gamma_{\mu\nu}$ have been found in second approximation that made it possible to obtain the value of the metric tensor $g'_{\mu\nu}$, with, as before, $x_0=t$. At the location of the particle with mass m_a, or, with our approach, at the center of mass of the body with mass m_a, these values $g'_{\mu\nu}$ have the form

$$g'_{00}=c^2-\frac{2\gamma m_b}{|a-b|}-\frac{3\gamma m_b v_b^2}{c^2|a-b|}$$
$$+\frac{4\gamma^2 m_b^2}{c^2|a-b|}-\frac{\gamma m_b}{c^2}\frac{\partial^2}{\partial t^2}|a-b|, \qquad (4.5)$$
$$g'_{0n}=\frac{4\gamma m_b\dot{b}_n}{c^2|a-b|},\quad g'_{mn}=-\delta_{mn}\left(1+\frac{2\gamma m_b}{c^2|a-b|}\right).$$

It is not hard to verify that the relations (4.5) agree precisely with (3.8), if in these latter relations the values of U, U_i and S are substituted and the system is taken to consist of only two bodies. It is obvious that substitution of (4.5) into the equation of motion (2.20) and replacement of x_i by a_i lead to the relations written for a system of two bodies.

1 A. Einstein and J. Grommer, Sitzber. Berl. Acad., p. 2, 1927.
2 A. Einstein, Sitzber. Berl. Acad. p. 235, 1927.
3 Einstein, Infeld, and Hoffman, Annals of Math. 39, 55 (1938).
4 V. A. Fock, J. Exptl. Theoret. Phys. (U.S.S.R.) 9, 375 (1939).
5 N. M. Petrova, J. Exptl. Theoret. Phys. (U.S.S.R.) 19, 989 (1949).
6 I. G. Fikhtengol'ts, J. Exptl. Theoret. Phys. (U.S.S.R.) 20, 233 (1953).
7 V. A. Fock, Priroda 12, 1953.
8 L. Infeld, Bull. Pol. Acad. Sci., Sect. III, 2, 4 (1954).
9 M. F. Shirokov, J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 180 (1956); Soviet Phys. JETP 3, 197 (1956).
10 M. F. Shirokov, J. Exptl. Theoret. Phys. (U.S.S.R.) 27, 251 (1954).

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