Interaction between a Moving Current-Carrying Wire and a Conducting Wall

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IF a current-carrying wire moves in the vicinity of any object, then a displacement current and a conduction current are induced in this object. The field of these currents will, in turn, act on the wire, causing it to be attracted to or repelled from the surface. To a large degree this is similar to what happens when a moving charge passes close to a dielectric.[1] We shall here consider the simplest case of this phenomenon, namely when a straight wire with current J moves uniformly ($v << c$) in empty space parallel to the plane surface of a conducting medium. Let l be the distance between the wire and the conductor, and σ, ϵ, and μ be the conductivity, complex dielectric constant, and magnetic permeability of the medium, so that $\epsilon = \epsilon_0 + i4\pi\sigma/\omega$; we shall neglect the dispersion ϵ_0 σ, and μ.

The x axis shall be chosen along the current, and the z axis in the direction of motion of the wire. The origin shall be located on the conductor surface. Then[1]

$$F_y = -(J/c)^2 \operatorname{Re} P; \; F_z = -(J/c)^2 \operatorname{Im} P, \tag{1}$$

$$P = 2\int_0^\infty \frac{1-\alpha}{1+\alpha} e^{-2kl} dk, \; k = \omega/v, \tag{2}$$

$$\alpha = (1/\mu)[1 - (\mu\varepsilon_0 + i4\pi\sigma\mu/kv)\beta^2]^{1/2}, \; \operatorname{Re}\alpha > 0. \tag{3}$$

As can be seen from (2) and (3), in this case the well known analog of the critical velocity for Cerenkov radiation $v = c/n$ is the quantity

$$U_1 = c^2/4\pi\mu\sigma l \tag{4}$$

(assuming that $\mu\epsilon_0\beta^2 << 1$), which we shall call the first characteristic velocity. For instance, for copper, this is $\approx 10^2/l$ cm/sec. To find the force (F_z, F_y), we need only calculate P, which we shall do for the two limiting cases $\mu = 1$ and $\mu >> 1$.

In the first case, for the condition

$$\varepsilon_0 \ll \pi^2\sigma^2 l^2/c^2,$$

which is in fact always satisfied, the wire will be repelled from the wall for any velocity, and

$$P = -\frac{1}{l} - \frac{i}{l\nu} - (e^{-i\nu}/2l)\,\pi\, H_1^{(1)}(\nu), \tag{5}$$

where $\nu = v/U_1$, and $H_1^{(1)}$ is the Hankel function. If $\nu << 1$, then

$$F_y = (J/c)^2 \nu\pi/2l, \; F_z = (J/c)^2(\nu/2l)[\ln(\nu/2) + C - 1/2],$$

and if $\nu >> 1$, then

$$F_y = (J^2/c^2 l)(1 - 1/2\sqrt{\pi/\nu}), \tag{6}$$

$$F_z = -(J^2/c^2 l)(1/2\sqrt{\pi/\nu} - 1/\nu).$$

It can be shown that for $\nu \approx 1.9$, the force F_z is a maximum, physically, this is related to the fact that an increase in v leads to a change in the relation between the ohmic and inductive impedences to the current induced in the conducting wall by the moving wire, The $\nu^{-1/2}$ dependence of F_z is explained by the skin effect.

Let us now consider the other limiting case, in which $\mu >> 1$. The case $v \lesssim U_1$ is of no particular interest, since U_1 is small, and in this case the image forces, proportional to $(\mu-1)/(\mu+1)$, are of most importance. Therefore we shall assume that $v >> U_1$. It can be shown that in this case

$$P = \frac{1}{l}\Big[1 + 2\sqrt{\pi}z + 4z^2\Big(-\frac{1}{2}e^{-z^2}\operatorname{Ei}(z^2) - \sqrt{\pi}e^{-z^2}\int_0^z e^{\alpha^2}d\alpha + \frac{i\pi}{2}e^{-z^2}\Big)\Big], \tag{7}$$

$$z = -e^{-i\pi/4}(v/U_2)^{1/2}\sqrt{2},$$

where $U_2 = \mu^2 U_1$ is the second characteristic velocity, and Ei is an integral exponential function.

In view of the complexity of Eq. (7), we shall write out the components of the force for two limiting cases. In the case $U_1 << v << U_2$,

$$F_y = (J^2/c^2l)\,[1-2\,(v/U_2)^{1/2}],$$

$$F_z = -(2J^2/c^2l)\,(v/U_2)^{1/2},$$

and in the case $v >> U_2$

$$F_y = (J^2/c^2l)\,[1-{}^1/_2\sqrt{\pi}\,(U_2/v)^{1/2}];$$

$$F_z = -(J^2\sqrt{\pi}/c^2l2)\,(U_2/v)^{1/2}.$$

From these equations it is seen that for $v \sim U_2$, the component F_y changes sign and F_z changes from an increasing function of velocity to a decreasing one.

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1 A. I. Morozov, Vestn. Moscow State University (to be published).

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On the Damping Theory of Particle Scattering by a Fixed Center

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DAMPING theory, which is a stage beyond perturbation theory, enables us to calculate a cross section σ not only for long de Broglie waves ($\sigma < \lambda^2$) but also for small wave lengths ($\sigma >> \lambda^2$).

Damping theory was developed in Refs. 1–3 in the investigation of meson scattering by nucleons; Sokolov[1] established the relation

$$C^+C + \sum_{\mathbf{k}'} C'^+C' = 1, \tag{1}$$

which states that the sum of incident and scattered particles at any instant of time remains unchanged. Subsequently we applied damping theory to an investigation of particle scattering by a fixed center.[4–5]

It is well known that the exact formula for the cross section of elastic scattering of particles with momentum hk ($k = 2\pi/\lambda$) is

$$\sigma = \frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin^2\eta_l.$$

Perturbation theory enables us to determine the phase η_l when $\eta_l << 1$. Damping theory gives the following more exact approximation for the phase shift[5]

$$\operatorname{tg}\eta_l = -\frac{\pi K}{c\hbar}\int_0^{\infty} rV(r)\,J^2_{l+1/2}(kr)\,dr, \tag{2}$$

(hk is the momentum and chK is the energy of the particle), which for $\eta_l << 1$ becomes the expression established for the phase by perturbation theory.

For the elastic scattering differential cross section, damping theory gives the following expression:

$$\frac{d\sigma}{d\Omega'} = \frac{1}{k^2}\sum_{l,l'}\frac{\operatorname{tg}\eta_l\operatorname{tg}\eta_{l'}(1+\operatorname{tg}\eta_l\operatorname{tg}\eta_{l'})(2l+1)(2l'+1)P_l(\cos\theta')P_{l'}(\cos\theta')}{(1+\operatorname{tg}^2\eta_l)(1+\operatorname{tg}^2\eta_{l'})} \tag{3}$$

These formulas enable us to investigate the scattering of spinless particles acted on by short-range forces. In particular, we made a detailed study[5] of particle scattering by Yukawa forces. We have been able to extend our results to Dirac particles (i.e., with spin) only for a δ-function interaction.[4]

We shall now investigate particle scattering by damping theory when the interaction potential is of the form

$$V(r) = \begin{cases} B = 3V_0/4\pi a^3, & r < a, \\ 0, & r > a. \end{cases}$$

Then according to (2) we have for the phase η_l

$$\operatorname{tg}\eta_l = -\frac{\pi BK}{c\hbar k^2}\int_0^{ka} J^2_{l+1/2}(y)\,y\,dy.$$

In the one limit $ka >> 1$ scattering is practically determined by the s phase:

$$\sigma = \frac{4\pi}{k^2}\begin{cases} 4B^2K^2a^6k^2/9c^2\hbar^2 & \text{for } 2BKa^3k/3c\hbar \ll 1, \\ 1 & \text{for } 2BKa^3k/3c\hbar \gg 1, \end{cases}$$

i.e., damping will play a decisive part only for