Figure 2 gives the angular distribution of secondary protons. It can be seen that a considerable fraction of the particles are emitted in directions close to the line of motion of the deuteron. About 90% of the fast protons are emitted in the forward hemisphere; 30% of these are in a narrow cone of 30° apex angle. The half width of the angular distribution is 18°, which exceeds the calculated half width (9.5°) for the angular distribution of protons resulting from "stripping". The gray tracks are distributed symmetrically to the right and left of the direction of the incident deuteron.

The energy distribution of the protons (Fig. 3) was obtained by counting the grains in the tracks. The energy spectrum covers the range from 50 to 210 mev and possesses a sharp maximum at 80-90 mev. The half width of the energy distribution of all protons is 70 mev.

The dashed line in Fig. 3 is a histogram which represents the energy distribution of protons whose emission angle was not greater than 10°. The peak of this distribution is at about 110 mev, which is half of the initial deuteron energy. The half width of the distribution is 40-50 mev, which agrees with the calculation of $\Delta E_{1/2} = 2(\epsilon_d/E_d) = 45$ mev for the transparent nucleus model and with $\Delta E_{1/2} = 34$ mev for an opaque nucleus. A comparison of the theoretical and experimental results shows good agreement. Therefore the protons included in the above energy distribution resulted predominantly from the disintegration of deuterons by nuclei.

Our analysis of the angular and energy distributions of the protons enables us to state that two processes are mainly responsible for the emission of fast protons; these are "stripping" and a cascade process.

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A Static Solution of the Nonlinear Meson Equation

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IN order to explain the phenomenon of saturation of nuclear forces and to provide a basis for a nuclear shell model Schiff proposed the simplest nonlinear generalization of the Klein-Gordon equation for meson theory[1]

$$\Box \varphi - k_0^2 - \lambda\varphi^3 = 0, \qquad (1)$$

where k_0 and λ are constants. The renormalized equation of the pseudoscalar theory with pseudoscalar coupling has the same form in the case of weak interaction.[2]

If we restrict ourselves to the static approximation we shall obtain for the spherically symmetrical case the equation

$$d^2u / dx^2 - (u^3/x^2) - u = 0, \qquad (2)$$

in which the variables $x = k_0 r$, $u = \sqrt{\lambda r}$ have been introduced.

This equation has been discussed by many authors largely in connection with the phenomenon of nuclear saturation, and in such cases the equation was discussed for the case of a certain given nucleon source density.[3-8] In the present note we shall obtain asymptotic solutions of Eq. (2), and shall also integrate the equation numerically.

According to a theorem due to Hardy,[9-10] every rational function $R(x, u, u')$ is necessarily monotonic along the solution $u(x)$ of the differential equation of the form

$$u'' = P(x, u) / Q(x, u),$$

where Q, P are polynomials in u, x. The application of this statement to the ratio of any two arbitrary terms of equations $Qu' - P = 0$ allows one to find asymptotic solutions of the differential equation for $x \to \infty$. The limit of such a ratio may be equal to $\pm\infty$, 0, or to a constant different from zero, and it is guaranteed that there must exist at least one ratio which tends to a constant different from zero. A similar result may be shown to hold for an equation of the type

$$u'' = P(u, x) / Q(u, x).$$

That solution of Eq. (2) is of physical interest which diminishes as $x \to \infty$. The asymptotic behavior of such a solution for large values of x may be represented in the form

$$u = -g\sqrt{\lambda}\, e^{-x}, \qquad (3)$$

where g is an arbitrary constant.

For small distances, Eq. (2) can be replaced by the asymptotic souation

$$d^2u / dx^2 - u^3 / x^2 = 0. \qquad (4)$$

Equation (4) is an analog of the Emden-Fowler equation, and it can be reduced to an equation of the first order.[10] Indeed, as a result of making the substitution $x = e^{-5}$ and of the introduction of $y = du/dt$, we shall obtain

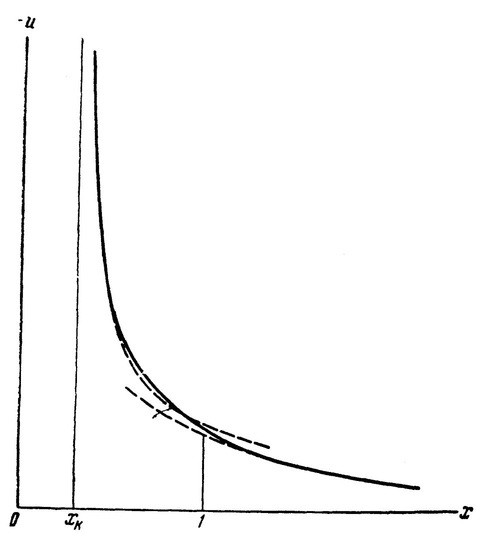
$$dy / du = u^3 / y - 1. \qquad (5)$$

A qualitative investigation of the behavior of phase trajectories in the (y, u) plane shows that a characteristic feature of all the solutions of Eq. (4) is the existence of a singular point whose position is not fixed, but depends on the constant of integration.

Applying Hardy's theorem to Eq. (5), we obtain for the asymptotic solution (for small distances)

$$u = -\sqrt{2} / \ln(x / x_k), \qquad (6)$$

where x_k is an arbitrary constant.



In addition, Eq. (2), was integrated numerically. The integration was carried out by starting with the asymptotic solution at large distances. To each value of the quantity g corresponds a definite value of the quantity x_k.

As may be seen from the diagram the solution obtained by numerical integration (solid curve) may be roughly approximated by the functions (3) and (6) matched at the point $x = 1$ (dotted curves).

The author hopes to give the interpretation of the result obtained above and its application to the description of the properties of a system of two nucleons at low energies in a subsequent article.

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On the Hydrodynamic Description of Plasma Oscillations

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SOMETIMES for the theoretical study of plasma one uses in place of the kinetic equation for the distribution function of the particles the simpler "transport" equations for the moments of this function (see, for example, Refs. 1,2). In so doing, in order to obtain a closed system of equations, one usually assumes that the distribution function is Maxwellian. Such an approximation cannot be applied to the description of high frequency plasma oscillations since in this case the electron distribution deviates appreciably from the Maxwellian