μ-Pair Production in Nuclei by Gamma Rays

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IN a recent article by Masek and Panofsky[1], data were given on the electromagnetic production of muon pairs, which had been observed for the first time. It is appropriate to note that in the interaction of high-energy gamma rays ($h\omega > 200$ mev) with nuclei, in addition to the "ordinary" production of free muon pairs, there is another effect which consists of the direct production of the μ-meson of a pair in the K shell of a mesic atom.

A distinguishing characteristic of the problem of μ-pair production in nuclei by gamma rays is the necessity for taking into account the finite size of the nucleus*. In the production of free muon pairs this fact leads, first, to a departure from the Coulomb law because the electrical charge of the nucleus is considerably smeared out at small distances[2] and, second, to the total or partial destruction of coherence in the nuclear process.

With regard to the direct production of the μ-meson of a pair in the K shell of a mesic atom, the necessity for taking into account the finite size of the nucleus follows from the closeness in size of the K-shell "radius" of the mesic atom a and the nuclear "radius" R_0. However, in this case, as will be seen from the following, the question of coherence does not appear in the formulation of the problem. Therefore, consideration of nuclear size reduces to taking into account the weakining of the electrical interaction and can be accomplished approximately by replacing the atomic number Z in the ordinary hydrogen-like wave functions with an effective atomic number $Z_{\text{eff}} (Z_{\text{eff}} < Z)$.

Such an approximation is clearly suitable only when $a > R_0$, i.e., for not too heavy nuclei, since the employment of hydrogen-like wave functions is otherwise meaningless. The transition matrix element for the production of a μ-mesic atom and a μ^+-meson by a gamma ray on a nucleus can be represented (assuming $\hbar = c = 1$) as:

$$S_{if} = -e\left(\frac{2\pi}{\omega}\right)^{1/2}\int_0^\infty \Psi_K^*(\alpha\varepsilon)\, e^{-i\omega \mathbf{r}}\Psi_p\, d^3\mathbf{r}. \qquad (1)$$

In the relativistic Born approximation for the differential cross section of the effect we have

$$d\sigma_K = 2\pi Z_{\text{eff}}^5\, \alpha^4 r_0^2 \frac{\mu^6}{\omega^{7/2}}(\omega - 2\mu)^{3/2} \qquad (2)$$

$$\frac{\sin^2\theta\, d(-\cos\theta)}{[\omega - \mu - \sqrt{\omega(\omega - 2\mu)}\cos\theta]^4},$$

where θ is the angle between the momenta of the gamma ray and the positive muon; ω is the gamma frequency: $r_0 = e^2/\mu$, $\alpha = 1/137$. Thus at the threshold, the emitted μ^+-mesons must be emitted mainly at the angle $\pi/2$ to the incident gamma-ray beam.

The total cross section is

$$\sigma_K = (8\pi/3)\, r_0^2 Z_{\text{eff}}^5\, \alpha^4 \mu^2 \omega^{-7/2}(\omega - 2\mu)^{3/2}. \qquad (3)$$

At the threshold for $\omega \to 2\mu$

$$\sigma_K = (\pi r_0^2/3\sqrt{2})\, Z_{\text{eff}}^5 \alpha^4\, [(\omega - 2\mu)/\mu]^{3/2}. \qquad (4)$$

Departure from the Born approximation is taken into account by inserting in (3) and (4) the factor $f(\zeta)$:

$$\sigma'_K = f(\zeta)\,\sigma_K, \quad f(\zeta) = 2\pi\zeta/(e^{2\pi\zeta} - 1), \qquad (5)$$
$$\zeta = \alpha Z'_{\text{eff}}\,[1 + \mu^2/\omega(\omega - 2\mu)]^{-1/2}.$$

A more general expression than (3) can be obtained in the Born approximation by using the relativistic functions Ψ_k and Ψ_p. In this case

$$\sigma_K = {}^3/_4\, \sigma_0 \alpha^4 Z_{\text{eff}}^5 (\eta^2 - 1)^{3/2}(\eta + 1)^{-5} \qquad (6)$$
$$\times\left\{\frac{4}{3} + \frac{\eta(\eta - 2)}{\eta + 1}\left[1 - \frac{1}{2\eta\sqrt{\eta^2 - 1}}\ln\frac{\eta + \sqrt{\eta^2 - 1}}{\eta - \sqrt{\eta^2 - 1}}\right]\right\},$$
$$\sigma_0 = 8\pi r_0^2/3, \qquad \eta = (\omega - \mu)/\mu.$$

When $\omega \to 2\mu$ (6) goes over into (3) and (4).

In the other limiting case where $\omega \gg \mu$:

$$\sigma_K = {}^3/_4 \sigma_0 \alpha^4 Z_{\text{eff}}^5 \mu/\omega. \qquad (7)$$

The maximum of (6) corresponds to $\omega \approx 7.5 \approx 770$ mev.

The values of Z_{eff} can be obtained by using Wheeler's semi-empirical formula[3] for the lifetime of a muon in a K shell with respect to capture by the nucleus:

$$\tau_{cap} = \tau_{dec}\,'Z_{eff}/Z_0)^{-4}. \qquad (8)$$

According to Wheeler,

$$Z_{eff} = Z[1 + (Z/37.2)^{1.54}]^{-1/1.54}, \qquad (9)$$

whence it follows in particular that

	Al	Fe	Zn	Ag
$Z =$	13	26	30	47
$Z_{eff} =$	11.58	19.4	21.1	26.4

For these values of Z_{eff} the cross section of the effect under consideration does not exceed $\approx 10^{-34}$ cm^2. The relative role of pair plus mesic-atom production is significant near the threshold for the production of μ pairs. At the threshold, the cross section for the production of a pair of free muons is

$$\sigma \leqslant (^1/_3)\, r_0^2 \alpha Z\, [(\omega - 2\mu)/\mu]^3. \qquad (10)$$

The linear dependence of (10) on Z results from the destruction of coherence of the nuclear effect because of the transfer of large momenta q ($q \approx 2u >> 1/R_0$). As follows from a comparison of (4) and (10), for Z_{eff} not extremely small, the production of a pair plus a μ-mesic atom can predominate over the production of free muon pairs.

However, the smallness of the cross sections at the threshold makes the experimental investigation of pair production very difficult. It must be emphasized that the present estimates are approximate and give cross sections which are too small. A rigorous theory based on exact wave functions in the field of a finite nucleus will yield results which are more favorable to experiment and will be published hereafter.

In conclusion I wish to express my sincere thanks to Professor A. I. Alikhanian and to G. M. Garibian for very valuable discussions of a number of questions involved in the present note.

* Excluding the case for $h\omega/\mu c^2 >> 1$, where the main contribution to the cross section for the production of free pairs comes from collisions at distances such that the size of the nucleus can be neglected by comparison.

1 G. Masek and W.K.H. Panofsky, Phys. Rev. **101**, 1094 (1956).

2 G. H. Rawitscher, Phys. Rev. **101**, 423 (1956).

3 J. A. Wheeler, Rev. Mod. Phys. **21**, 133 (1949)

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231

Showers in Lead Produced by 360 ± 30 mev Electrons

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THE data on electron showers communicated in this note have been obtained in the course of analyzing results of experiments in which the interaction of negative π-mesons with lead nuclei was studied. The experiments were carried out using the synchrocyclotron of the Laboratory of Nuclear Problems with the aid of a Wilson cloud chamber of 400 mm diameter in a magnetic field of intensity 10^4 oersted.

The beam of π-mesons which passed through a lead plate of thickness 4.6 g. cm^{-2} situated inside the cloud chamber contained $(2 \pm 1)\%$ of electrons. Therefore in addition to the events produced by the interaction of π-mesons with nuclei, the photographs also recorded cases of the formation of electron showers in lead. In the course of the analysis of the photographs, 159 showers were recorded which were produced by electrons whose energy lies in the interval from 330 to 390 mev. An example of one such shower is shown in Fig. 1. This number does not include a few cases in which the primary electron was stopped in the lead plate, since it is practically impossible to distinguish them from the large number of cases of stopping of π-mesons. In counting up the number of particles in the showers only secondary electrons of energy $E \leq 8$ mev were taken into account. By means of this criterion of selection of secondary electrons, we excluded errors related to the presence inside the chamber of a background of low energy electrons.

The experimentally obtained distribution of showers with respect to the number of particles is given in the Table. For comparison the last column of this Table gives the distribution of showers with respect to the number of electrons according to Poisson's law. The average number of electrons per shower in accordance with the data given in the Table is equal to 1.77.

The energy distribution of the secondary electrons is shown graphically in Fig. 2. The value for the average number of secondary electrons in the shower obtained as a result of our measurements agrees within experimental error both with