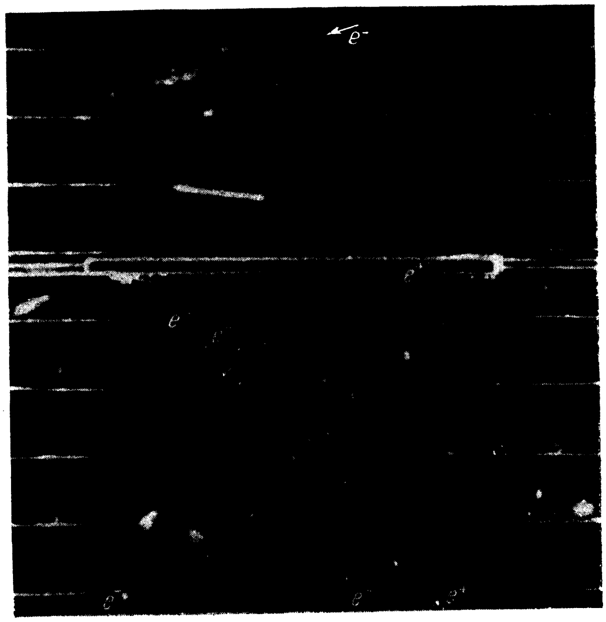

FIG. 1. Electron-positron shower of 6 particles initiated in the lead plate by an electron of momentum 360 mev/c.

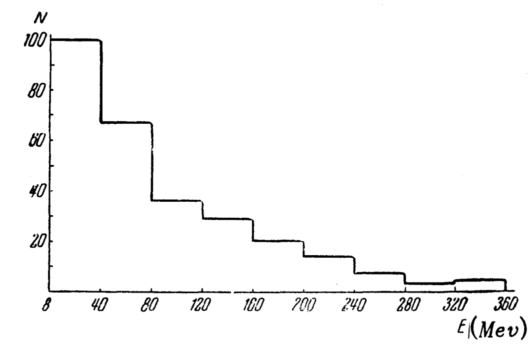

FIG. 2. Energy distribution of the secondary electrons $N(E)$.

the corresponding experimental results of reference 2, and with the theoretical value for this quantity obtained in Ref. 3 by means of a Monte-Carlo calculation of an electron cascade in lead.

Number of electrons per shower	Observed no. of showers with a given number of particles.	Observed no. of particles expressed as % of the total	Number of showers with a given no. of particles corresponding to Poisson's law, expressed in %
0	—	2—3*	17
1	88	53.6	30
2	34	20.7	26.6
3	30	18.3	15.7
4	3	1.8	6.9
5	1	0,6	2,5
6	2	1.2	0.7
7	1	0 6	0,1
8	0	0	0,01

*These data for the relative number of stoppages of primary electrons are taken from Ref. 2.

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232

Generalized Nonsingular Solutions for the Scalar Meson Field of a Point Charge in General Relativity Theory

DUAN'-I-SHI
Moscow State University

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IT has been shown that in investigating the electromagnetic and meson fields of elementary particles, the effects of gravitational interaction cannot be neglected[1-5]. The present note is an attempt to find solutions to the equations of the general relativistic gravitational and scalar meson fields of a point nuclear charge. The solution is nonsingular at all points of the gravitational and meson fields. Although it contains several functions whose form is not given, this solution can be used to obtain the potential of the scalar meson field, which is a generalization of the Yukawa potential; furthermore, it makes it possible to calculate the mass and self-energy of the nucleon, which turn out to be finite.

We take the general centrally symmetric expression for the interval in the usual form

$$ds^2 = -e^{\lambda}dr^2 - e^{\mu}r^2\{d\theta^2 + \sin^2\theta\, d\Phi^2\} + e^{\nu}dt^2,$$

where λ, μ, and ν are functions of r. The energy-momentum tensor of the scalar meson field is

$$T_i^k = \left(\frac{1}{8\pi}\right)\left\{2g^{ik}\frac{\partial U}{\partial x^i}\frac{\partial U}{\partial x^k} - \delta_i^k\left(g^{lm}\frac{\partial U}{dx^l}\frac{dU}{dx^m} - \chi^2U^2\right)\right\}.$$

We shall consider the field static and set $U = U(r)$. Then

$$T_1^1 = (1/8\pi)(-e^{-\lambda}U'^2 + \chi^2U^2); \qquad (1)$$

$$T_2^2 = T_3^3 = T_4^4 = (1/8\pi)(e^{-\lambda}U'^2 + \chi^2U^2).$$

It is easy to show, on the basis of the invariance of the Einstein equations under the transformation $r \to \sigma r$, $\chi \to \chi/\sigma$, where σ is a constant, that the solutions to these equations is of the form

$$U = U_0V(\chi r); \quad e^{\nu} = e^{\nu_0}\Phi(\chi r); \qquad (2)$$

$$e^{\lambda} = e^{\lambda_0}\Psi(\chi r); \quad e^{\mu} = e^{\mu_0}X(\chi r),$$

where U_0, e^{ν_0}, e^{λ_0} and e^{μ_0} are solutions for the case $\chi = 0$, and V, Φ, Ψ, and ψ are functions of the one argument χr.

As $\chi r \to 0$, each of these four functions approaches unity; i.e., for $r \to 0$, the solutions for the cases $\chi \neq 0$ and $\chi = 0$ are identical.

Let us set[4]

$$\mu = -2\ln f(r),$$

where $f(r)$ is an arbitrary function with no singularities in the region $0 \leq r \leq +\infty$ and satisfying the following conditions:

$$|f(r)| \leqslant 1; \quad [f(r)]_{r=0} = 0; \quad [f(r)]_{r=\infty} = 1; \qquad (3)$$

$$[f'(r)]_{r=0} = \alpha; \quad [f'(r)]_{r=\infty} = 0;$$

$$[f''(r)]_{r=0} = \beta; \quad [f''(r)]_{r=\infty} = 0.$$

Solving the Einstein equation for $\chi = 0$ and inserting the solutions for U_0, e^{ν_0}, e^{λ_0}, and e^{μ_0} into Eq. (2), we arrive at the general solution in the form

$$U_r = \frac{G}{\sqrt{A^2 + 4(kG^2/c^4)}} \times \ln\left[\frac{X + (A/2q)(1+q)}{X - (A/2q)(1-q)}\right]V(\chi r); \qquad (4)$$

$$e^{\nu} = \left[\frac{X - (A/2q)(1-q)}{X + (A/2q)(1+q)}\right]^q \Phi(\chi r);$$

$$e^{\lambda} = \frac{r^2}{f^4}\left[1 - \frac{rf'(r)}{f(r)}\right]^q \frac{1}{X}\left[\frac{X - (A/2q)(1-q)}{X + (A/2q)(1+q)}\right]^q \Psi(\chi r);$$

$$e^{\mu} = X(\chi r)/f^2(r),$$

where X is the solution to the algebraic equation

$$[X - (A/2q)(1-q)]^{1-q}[X + (A/2q) \times (1+q)]^{1+q} = r^2/f^2(r), \qquad (5)$$

$$q = A[A^2 + 4(kG^2/c^4)]^{-1/2} < 1,$$

and A is a constant of integration which can be determined from the conditions (3). From Eq. (4) and conditions (3) it is easy to prove that all the components of the metric tensor g_{ik} and of the potential U are nonsingular as $r \to 0$.

The self-energy of a particle is calculated using Tolman's formula[6]

$$W = \int (T_1^1 + T_2^2 + T_3^3 - T_4^4)\sqrt{-g}\,dV = G^2\int_0^\infty \frac{d(r/f)}{(r/f)X} = \delta. \qquad (6)$$

It can be shown that with conditions (3), the integral in Eq. (6) is finite. Then the self-mass of of the nucleon will be $m = G^2\delta/c^2$, and its classical radius is $r_0 = 1/\delta$. If we neglect small quantities of the order of $kG^2/c^4 = 10^{-68}$, then the constant $q \approx 1$.

In this case the potential of the scalar meson is of the form

$$U(r) = -\frac{G}{A}\ln\left[1 - A\frac{f(r)}{r}\right]e^{-\chi r}. \qquad (7)$$

In view of the condition $|f(r)| \leq 1$, we can expand the potential $U(r)$ in a series for $r > A$:

$$U(r) = \frac{G}{r}f(r)\,e^{-\chi r}\left[1 + \frac{1}{2}\left(\frac{A}{r}\right)f(r) + \frac{1}{3}\left(\frac{A}{r}\right)^2 f^2(r) + \ldots\right] \qquad (8)$$

For $r \gg A$, the function $f(r) \approx 1$, and we obtain the Yukawa potential

$$U(r) = Ge^{-\chi r}/r.$$

Equation (7) is a generalization of the Yukawa potential. When $r \sim A$, the gravitational field greatly alters the potential of the meson field. From (7) and (3) we obtain

$$U(0) = (G/A)\ln[1/(1 - A\alpha)].$$

Thus the theory we have here developed is in a position to give not only a finite nucleon mass, but also a finite potential well depth for nuclear forces. In this lies its attractiveness.

In conclusion I express my gratitude to Professor M. F. Shirokov for valuable advice and suggestions during the performance of this work.

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234

The Optical Theorem and Elastic Scattering Through Small Angles

L. I. Lapidus
The United Institute of Nuclear Research

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1. AS is known, the total interaction cross section is proportional to the imaginary part of the forward elastic scattering amplitude[1]. This general relation, which is sometimes called the optical theorem, is the result of the unitary character of the S-matrix and holds true for particles with no arbitrary spin. It may be represented thus

$$\sigma_t = (4\pi / k)\,\mathrm{Im}\,\mathrm{Tr}\,M(0^\circ) / (2s_1 + 1) / (2s_2 + 1), \qquad (1)$$

if one introduces the matrix $M(\theta)$, which connects the wave scattered through angle θ with the incident wave, and designates the spins of the colliding particles by s_1 and s_2.

When a spinless particle collides with a particle whose spin is 1/2, the matrix M may be represented as

$$M(\theta) = a(\theta) + b(\theta)(\sigma \mathbf{n}), \qquad (2)$$

where a and b are functions of invariants $(\mathbf{kk}_0)$ and $k_0^2 = k^2$, σ in the spin operator (nucleon), and $\mathbf{n} = [\mathbf{k}_0\mathbf{k}]/|[\mathbf{k}_0\mathbf{k}]|$is the normal to the scattering surface.

When spin $s_1 = 1$ and $s_2 = 0$, M has the form

$$M(\theta) = X(\theta) + Y(\theta)(\mathbf{Sn}) + Z(\theta)(\mathbf{Sn})^2 + W(\theta)[(\mathbf{Sk}_0)(\mathbf{Sk}) + (\mathbf{Sk})(\mathbf{Sk}_0)], \qquad (3)$$

where $\mathbf{s}$ is the spin-operator (deuteron) and the quantities X, Y, Z, and W play the same role as functions a and b in Eq. (2).

For the scattering of particles with a spin of $s_1 = 1/2$ on a target with an arbitrary spin of s_2, we have

$$M(0) = A + \boldsymbol{\sigma}_1\mathbf{H} = A + B(\boldsymbol{\sigma}_1\mathbf{n}) + C(\boldsymbol{\sigma}_1\mathbf{m}) + D(\boldsymbol{\sigma}_1\mathbf{l}); \qquad (4)$$

and, for $s_1 = 1$ and an arbitrary s

s_2 $[(\mathbf{S}, \mathbf{kk}_0) = (\mathbf{Sk})(\mathbf{Sk}_0) + (\mathbf{Sk}_0)(\mathbf{Sk})$ e. t. c.],

$$M(\theta) = K + L(\mathbf{Sn}) + M(\mathbf{Sl}) + N(\mathbf{Sm}) + R(\mathbf{Sn})^2 + T(\mathbf{S}, \mathbf{kk}_0) + F(\mathbf{S}, \mathbf{nl}) + V(\mathbf{S}, \mathbf{nm}) + Q(\mathbf{S}, \mathbf{ml}), \qquad (5)$$

where $\mathbf{m} = (\mathbf{k}_0 - \mathbf{k})/|\mathbf{k}_0 - \mathbf{k}|$, $\mathbf{l} = (\mathbf{k}_0 + \mathbf{k})/|\mathbf{k}_0 + \mathbf{k}|$, and the quantities $A, B, C, \ldots, K, L, M, \ldots$ are linear combinations of the total assembly $(2s_2 + 1)^2$ of the operators in the spin space