In this case the potential of the scalar meson is of the form

$$U(r) = -\frac{G}{A} \ln \left[1 - A\frac{f(r)}{r}\right] e^{-\chi r}. \quad (7)$$

In view of the condition $|f(r)| \leq 1$, we can expand the potential $U(r)$ in a series for $r > A$:

$$U(r) = \frac{G}{r} f(r) e^{-\chi r} \left[1 + \frac{1}{2}\left(\frac{A}{r}\right) f(r) + \frac{1}{3}\left(\frac{A}{r}\right)^2 f^2(r) + \dots\right] \quad (8)$$

For $r \gg A$, the function $f(r) \approx 1$, and we obtain the Yukawa potential

$$U(r) = Ge^{-\chi r}/r.$$

Equation (7) is a generalization of the Yukawa potential. When $r \sim A$, the gravitational field greatly alters the potential of the meson field. From (7) and (3) we obtain

$$U(0) = (G/A) \ln [1/(1 - A\alpha)].$$

Thus the theory we have here developed is in a position to give not only a finite nucleon mass, but also a finite potential well depth for nuclear forces. In this lies its attractiveness.

In conclusion I express my gratitude to Professor M. F. Shirokov for valuable advice and suggestions during the performance of this work.

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The Optical Theorem and Elastic Scattering Through Small Angles

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1. AS is known, the total interaction cross section is proportional to the imaginary part of the forward elastic scattering amplitude[1]. This general relation, which is sometimes called the optical theorem, is the result of the unitary character of the S-matrix and holds true for particles with no arbitrary spin. It may be represented thus

$$\sigma_t = (4\pi / k) \operatorname{Im} \operatorname{Tr} M(0^\circ) / (2s_1 + 1) / (2s_2 + 1), \quad (1)$$

if one introduces the matrix $M(\theta)$, which connects the wave scattered through angle θ with the incident wave, and designates the spins of the colliding particles by s_1 and s_2.

When a spinless particle collides with a particle whose spin is 1/2, the matrix M may be represented as

$$M(\theta) = a(\theta) + b(\theta)(\sigma\mathbf{n}), \quad (2)$$

where a and b are functions of invariants $(\mathbf{kk}_0)$ and $k_0^2 = k^2$, σ in the spin operator (nucleon), and $\mathbf{n} = [\mathbf{k}_0\mathbf{k}]/|[\mathbf{k}_0\mathbf{k}]|$ is the normal to the scattering surface.

When spin $s_1 = 1$ and $s_2 = 0$, M has the form

$$M(\theta) = X(\theta) + Y(\theta)(\mathbf{Sn}) + Z(\theta)(\mathbf{Sn})^2 + W(\theta)[(\mathbf{Sk}_0)(\mathbf{Sk}) + (\mathbf{Sk})(\mathbf{Sk}_0)], \quad (3)$$

where $\mathbf{s}$ is the spin-operator (deuteron) and the quantities X, Y, Z, and W play the same role as functions a and b in Eq. (2).

For the scattering of particles with a spin of $s_1 = 1/2$ on a target with an arbitrary spin of s_2, we have

$$M(0) = A + \sigma_1\mathbf{H} = A + B(\sigma_1\mathbf{n}) + C(\sigma_1\mathbf{m}) + D(\sigma_1\mathbf{l}); \quad (4)$$

and, for $s_1 = 1$ and an arbitrary s

$$s_2 \; [(\mathbf{S}, \mathbf{kk}_0) = (\mathbf{Sk})(\mathbf{Sk}_0) + (\mathbf{Sk}_0)(\mathbf{Sk}) \text{ e. t. c. }],$$

$$M(\theta) = K + L(\mathbf{Sn}) + M(\mathbf{Sl}) + N(\mathbf{Sm}) + R(\mathbf{Sn})^2 + T(\mathbf{S}, \mathbf{kk}_0) + F(\mathbf{S}, \mathbf{nl}) + V(\mathbf{S}, \mathbf{nm}) + Q(\mathbf{S}, \mathbf{ml}), \quad (5)$$

where $\mathbf{m} = (\mathbf{k}_0 - \mathbf{k})/|\mathbf{k}_0 - \mathbf{k}|$, $\mathbf{l} = (\mathbf{k}_0 + \mathbf{k})/|\mathbf{k}_0 + \mathbf{k}|$, and the quantities $A, B, C, \dots, K, L, M, \dots$ are linear combinations of the total assembly $(2s_2 + 1)^2$ of the operators in the spin space of a particle with a spin of s_2. When $s_2 = 1/2$ we have

$$A = \alpha + \gamma(\sigma_2 n),\ B = \gamma' + \beta(\sigma_2 n), \qquad (6)$$

$$C = \delta(\sigma_2 m),\ D = \varepsilon(\sigma_2 l)$$

and analogous expressions for the quantities $K, L, M, \ldots$

2. Differential elastic scattering cross sections are expressed[2] by $M(\theta)$ in the following manner:

$$\sigma(\theta) = \mathrm{Tr}\, M^{+}(\theta) M(\theta) / (2s_1 + 1)(2s_2 + 1), \qquad (7)$$

which, in case of an arbitrary s_2, becomes

$$\sigma(\theta) = \mathrm{Tr}\,(AA^{+} + HH^{+}) / 2(2s_2 + 1) \qquad (8)$$

when $s_1 = 1/2$ and an analogous equation when $s_1 = 1$.

The equations presented above indicate quite convincingly that when particles with arbitrary spins collide, a simple inequality is applicable

$$\sigma(0^\circ) \geqslant k^2\sigma_t^2 / 16\pi^2. \qquad (9)$$

When the collision involves a spinless particle and a particle with a spin of $s_2 = 1/2$, the function $b(\theta)$ in Eq. (2) becomes zero for $\theta = 0^\circ$, and then, just as in the case of a collision of spinless particles, it is easy to arrive directly at proof of the validity of Eq. (9), viz.,

$$\sigma(0^\circ) = [\mathrm{Im}\, a(0^\circ)]^2 + [\mathrm{Re}\, a(0^\circ)]^2 \geqslant [\mathrm{Im}\, a(0^\circ)]^2 = k^2\sigma_t^2 / 16\pi^2.$$

Proof of the validity of Eq. (9) for the case of a collision of two fermions with spins of $s_1 = s_2 = 1/2$ can be provided in the following manner. From Eqs. (1), (4) and (6) we have $k\sigma_t = 4\pi\, \mathrm{Im}\, \alpha(0^\circ)$. On the other hand, $(\gamma(0^\circ) = \gamma'(0^\circ) = 0)$

$$\sigma(0^\circ) = |\alpha(0^\circ)|^2 + |\beta(0^\circ)|^2 + |\delta(0^\circ)|^2 + |\varepsilon(0^\circ)|^2 \geqslant |\alpha(0^\circ)|^2,$$

and therefore,

$$\sigma(0^\circ) \geqslant |\alpha(0^\circ)|^2 \geqslant [\mathrm{Im}\, \alpha(0^\circ)]^2 = k^2\sigma_t^2 / 16\pi^2.$$

When $s_2 = 0$, $s_1 = 1$, the expression for the total cross section and for $\sigma(0^\circ)$ $(Y(0^\circ) = Z(0^\circ) = 0)$ can be represented as

$$k\sigma_t = 4\pi\, \mathrm{Im}\, [X(0^\circ) + {}^2/_3\, W(0^\circ)];$$

$$\sigma(0^\circ) = |X(0^\circ) + {}^2/_3\, W(0^\circ)|^2 + {}^2/_9\, |W(0^\circ)|^2,$$

whence the validity of Eq. (9) follows directly.

For the more general case where the spin of one of the particles is 1/2 or 1 and the other is arbitrary, a similar argument can be applied.

3. In the exceptional case, as in $(\pi\text{-}N)$ scattering as observed by Karplus and Ruderman[3], if $\sigma_t(E)$ is known over a wide energy interval, the dispersion relation can be used to calculate the forward elastic scattering cross section. In other cases, for example, that of $(n - \alpha)$ scattering, one can thus far, specify only the lower limit of the value of this cross section.

Inequality (9) represents a useful restriction that may be applied in the treatment of experimental data on elastic particle scattering. For the most part, the region of small angles is not very accessible in experimental research on the angular distribution of scattered particles. In view of this fact, the presence of a lower limit for a cross section is useful; often one can, within the limits of error, pass several curves through the experimental points in the region of large angles which curves can be differentiated only at small angles. As numerical evaluation shows, the lower limit of the magnitude of forward elastic cross sections can in certain cases be rather high. Let us give a few examples. When the neutron energy is 400 mev, the cross section of neutron scattering by protons is, according to Eq. (9), $\sigma_{pn}(0^\circ) \geq 3.4 \times 10^{-27}$ cm^2/ster, and the value of the cross section $\sigma_{np}(13^\circ)$ amounts[4] to about 4×10^{-27} cm^2/ster. Correspondingly, for neutrons with energies of 590 and 630 mev, $\sigma_{np}(0^\circ)$ is not less than 5.8×10^{-27} and 6.6×10^{-27} cm^2/ster. respectively. This shows that in the 400-600 mev energy region one should expect an increase of the cross section in the small-angle region in addition to a further tendency toward an increase with a rise in the energy of the colliding particles (total cross sections of $(n - p)$ interaction remain almost constant or even increase in the energy interval from 300 to 2500 mev.

Another example of the use of the restriction of the optical theorem may be afforded by the case of meson scattering in nuclei. The angular distribution of 300 mev π^--mesons elastically scattered by helium nuclei[5] decreases (in comparison with the large-angle region) perceptibly in the region of angles of 10° - 15°, where not a single case of scattering was found. At 15°, one finds $\sigma_{\pi\alpha}(15°) \approx 50 \times 10^{-27} cm^2/ster$. From Eq. (12) and the known value of the total cross section, which is equal to 150×10^{-27} cm^2, we obtain $\sigma_{\pi\alpha}(0°) \geqslant 75 \times 10^{-27}$ cm^2/ster, which means that there is a minimum in the angular distribution of mesons scattered by helium nuclei. The presence of a minimum in the cross section is indicative of a different sign for the amplitudes of nuclear and Coulomb scattering, and this reveals a change in the sign of the nuclear amplitude in a $(\pi - \alpha)$ interaction in comparison with what took place at energies below 200 mev. This may be connected with the change in the sign of the amplitude for $(\pi - N)$ scattering in the same energy region. The inequality (9) is near equality at high energies. For the region of solid angles $\Delta\omega$, where the differential cross section increases, it is possible to obtain the value $\Delta\omega \leqslant (4\pi / k)^2 \sigma_s / \sigma_t^2$, where σ_s is the elastic cross section.

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On Light Emission by a Shock Wave Front

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IN experiments using a ballistic type of apparatus, emission of light by the leading front of a shock wave in various gases was observed[1] although the temperature behind the shock wave in a polyatomic gas was insufficient in these cases to cause light emission. The following hypothesis may be advanced to explain this phenomenon.

As a result of molecular collisions at the front of a shock wave the energy of directed motion is converted into random heat energy. Zener's calculations[2] have shown that after approximately ten collisions Maxwellian velocity distribution of the molecules is established, while the rotational and vibrational degrees of freedom remain practically unexcited ("frozen"). In the above process all the energy is transferred only to the translational degrees of freedom, and the local gas temperature becomes much higher than the temperature corresponding to thermodynamic equilibrium which is established later.

Electronic energy levels and rotational degrees of freedom are excited only subsequently to the excitation of the translational degrees of freedom. Depending on the individual properties of the molecules the electronic levels may be excited before the rotational levels, or the two may be excited simultaneously. In both cases the local temperature remains higher than the equilibrium value. It is just this nonequilibrium distribution of energy that may be used to explain the observed emission of light, particularly since the light is emitted by the front of the shock wave where the vibrational degrees of freedom have not yet been excited; for their excitation $10^4 - 10^5$ collisions are needed[3].

The subsequent excitation of rotational and vibrational degrees of freedom leads to a lowering of the gas temperature which tends to the equilibrium value, and consequently leads to the cessation of light emission. A more rapid rate of excitation of the internal degrees of freedom will lead to a narrower zone of light emission; therefore in gases with polyatomic molecules the region of light emission will be narrower than in monatomic gases in which the temperature decreases only as a result of light emission.

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