other means. The same cannot be said of Eq. (9), where φ_{ob} has entered in a clear fashion.

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Internal Bremsstrahlung in Electric Monopole $0^+ \to 0^+$ Nuclear Transitions

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An electric monopole transition from an excited 0^+ nuclear state to a ground state which also has a zero spin and positive parity will in all probability be accompanied by the formation of conversion electrons or electron-positron pairs. At the same time, both the conversion electron and the components of the pair may emit γ quanta of a continuous spectrum with an upper boundary equal respectively to $E - I$ and $E - 2\mu c^2$ (E is the total energy of the nuclear transition, I the ionization energy, and μ the rest mass of the electron). This investigation presents the results of calculations of the relative probability of internal bremsstrahlung emitted by the components of a pair in a $0^+ \to 0^+$ nuclear transition. As in Ref. 1*, where the same effect is examined in the case of ordinary conversion, the calculation is done in the Born approximation and is therefore applicable only to light nuclei.

The differential probability of internal bremsstrahlung by the emitted pair in an electric monopole transition is ($\hbar = c = 1$)

$$dw = \frac{e^6 Q_0^2}{72\,(2\pi)^8} \frac{d\omega}{\omega} B\delta\,(\varepsilon_- + \varepsilon_+ + \omega - E)\, p_- d\varepsilon_- p_+ d\varepsilon_+ d\Omega_{\mathbf{p}_-} d\Omega_{\mathbf{p}_+} d\Omega_{\mathbf{k}}, \tag{1}$$

$$\begin{aligned} B = &-\mu^2 (\varepsilon_- - p_- x_-)^{-2} (\varepsilon_+\varepsilon_- + \mathbf{p}_+\mathbf{p}_- + \varepsilon_+\omega + \mathbf{k}\mathbf{p}_+ - \mu^2) \\ &- \mu^2 (\varepsilon_+ - p_+ x_+)^{-2} (\varepsilon_-\varepsilon_+ + \mathbf{p}_-\mathbf{p}_+ + \varepsilon_-\omega + \mathbf{k}\mathbf{p}_- - \mu^2) \\ &+ \omega\,(\varepsilon_+\omega + \mathbf{k}\mathbf{p}_+ + \mu^2) / (\varepsilon_- - p_- x_-) + \omega\,(\varepsilon_-\omega + \mathbf{k}\mathbf{p}_- + \mu^2) / (\varepsilon_+ - p_+ x_+) \\ &+ 2\,[\varepsilon_+^2 \varepsilon_-^2 + \varepsilon_-^2\, \mathbf{k}\mathbf{p}_+ + \varepsilon_+^2\, \mathbf{k}\mathbf{p}_- - \mathbf{p}_+\mathbf{p}_-\, (\mathbf{p}_+\mathbf{p}_- + \mathbf{k}\mathbf{p}_+ + \mathbf{k}\mathbf{p}_-) \\ &+ \mu^2 (\mathbf{p}_+\mathbf{p}_- - \varepsilon_+\varepsilon_- - \omega^2)] / (\varepsilon_- - p_- (x_-)\, (\varepsilon_+ - p_+ x_+), \end{aligned}$$

where $Q_0 = \int \Psi_f^* (\mathbf{r})\, \Psi_i (\mathbf{r})\, r^2\, d\mathbf{r}$ is the matrix element of the electric monopole operator (Ψ_i and Ψ_f are the initial and final wave functions of the nucleus); $\varepsilon_-, \mathbf{p}_-; \varepsilon_+, \mathbf{p}_+, \omega, \mathbf{k}$ are the energy and momentum respectively of the electron positron, and photon; and

$$x_- = \mathbf{k}\mathbf{p}_- / \omega p_-, \quad x_+ = \mathbf{k}\mathbf{p}_+ / \omega p_+.$$

After integrating over the directions of emergence of all the particles (the nucleus is considered infinitely heavy) and over the energy of one of the components of the pair (the probability equation is symmetrical with regard to the electron and positron) we obtain

$$dw_{\pi,\gamma} = \frac{e^6 Q_0^2}{9\,(2\pi)^5} F(E;\ W, \omega)\, dW\, d\omega, \tag{2}$$

$$\begin{aligned} F(E;\ W,\ \omega) = (1/\omega)\, \{&[5W^2 + 5W\omega + 3\omega^2 - 5\,(E-2)\, W - (3E-5)\,\omega - 5E + 9] \\ &\times \sqrt{(W+1)^2 - 1}\, \sqrt{(E-1-W-\omega)^2 - 1} + \sqrt{(W+1)^2 - 1}\, [2W^3 + 2W^2\omega + W\omega^2 \\ &- 2\,(2E-3)\, W^2 - 2\,(E-2)\, W\omega + \omega^2 + (2E^2 - 8E + 9)\, W - (2E-5)\,\omega \\ &+ 2E^2 - 6E + 5] \ln (E - 1 - W - \omega + \sqrt{(E-1-W-\omega)^2 - 1}) \\ &+ \sqrt{(E-1-W-\omega)^2 - 1}\, [-2W^3 - 4W^2\omega - 3W\omega^2 - \omega^3 + 2\,(E-3)\, W^2 \\ &+ 2\,(E-4)\, W\omega + (E-3)\,\omega^2 + (4E-9)\, W + 2\,(E-2)\,\omega \\ &+ 3E - 5] \ln (W + 1 + \sqrt{(W+1)^2 - 1}) \\ &- (1 + 2\omega^2) \ln (W + 1 + \sqrt{(W+1)^2 - 1}) \cdot \ln (E - 1 - W - \omega + \sqrt{(E-1-W-\omega)^2 - 1})\}, \end{aligned}$$

where W is the kinetic energy of the electron or positron (the energy is expressed in units of μc^2).

The obtained equation is applicable to the presently known electric monopole transitions between a 7.68 mev state and the ground state in C^{12} and between a 6.06 mev state and the ground state in O^{16} [2]. In both cases the transition is in the main accompanied by the emission of electron-positron pairs. The Born approximation used in the calculation should be correct, because C^{12} and O^{16} are light nuclei, and the transition energy is considerable. The relative probability of bremsstrahlung will be

$$dW^{(0)}_{\pi,\gamma} = \frac{e^2 \cdot 10^{-3}}{(2\pi)^2 I(E)} F(E;\ W,\ \omega)\, dW\, d\omega, \tag{3}$$

$$I(E) = \int_0^{E-2} [E-2+(E-2)x - x^2] \sqrt{(E-1-x)^2-1}\sqrt{(x+1)^2-1}\, dx$$

$(e^4 Q_0^2 I(E) / 9(2\pi)^3$ is the known[3] expression for the total probability of the formation of an electron-positron pair in an electric monopole transition).

Numerical integration of Eq. (3) permits one to obtain the energy spectrum of the γ rays and a total relative probability of the process. For O^{16} these are

$$N_1(\omega) = 2.64 \cdot 10^{-8} \int_0^{9.86-\omega} F(E = 11.86;\ W,\ \omega)\, dW,\ N_1$$

$$= \int_\varepsilon^{9.86} N_1(\omega)\, d\omega = 3.30 \cdot 10^{-3}$$

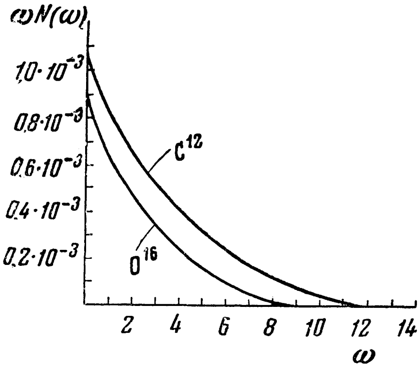
and for C^{12}

$$N_2(\omega) = 0.79 \cdot 10^{-8} \int_0^{13.03-\omega} F(E = 15.03;\ W;\ \omega)\, dW;$$

$$N_2 = \int_\varepsilon^{13.03} N_2(\omega)\, d\omega = 3.96 \cdot 10^{-3},$$

where ϵ is a small constant.

The graphs show functions $\omega N_1(\omega)$ and $\omega N_2(\omega)$ (in units of μc^2). The values N_1 and N_2 present the ratio of the number of γ quanta emitted simultaneously with electron-positron pairs to the number of pairs not accompanied by bremsstrahlung in unit time. They are equal in order of magnitude (10^{-3}) to the integral relative probability of bremmstrahlung in an ordinary conversion for an electric monopole nuclear transition. We also evaluated the contribution to the continuous spectrum of γ rays by a process in which the energy of the $0^+ \to 0^+$ transition goes into the formation of a conversion electron and a γ quantum, with the γ quantum being emitted by the nucleus itself. As might have been expected, the role played by this process is insignificantly small, especially in the region of small γ ray energies. Its total contribution is about 10^4 times less than those made by the process examined here and the one dealt with in the investigation conducted by I. S. Shapiro and this author[1].



In conclusion I express my sincere gratitude to I. S. Shapiro for the interest he has shown in this investigation and to V. V. Turevtsev, who called my attention to this effect.

* In this paper a factor 1/2 must be introduced in the equation for the matrix element and correspondingly, a factor 1/4 in the probability equations. Then the order of magnitude of the integral relative probability must be 10^{-3}.

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The Numerical Calculation of Path Integrals

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It is known that we can write down the answers to many physical problems in the form of so-called paths integrals – integrals in function