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Heat Capacity of Laminar Structures at Low Temperatures

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It is well known that the heat capacities of laminar and chain structures do not obey the Debye law $C \sim (T/\theta)^3$ at low temperatures. In the work of I. M. Lifshitz^{1,2} it was shown that the deviation from the T^3 law is associated with the special role of bending waves in such structures. The anomalous form of the dispersion law (the relation between the frequency ω and the wave vector K for bending waves leads to an anomalous temperature dependence of crystal energy. In Ref. 2, the dispersion law was obtained for bending waves in strongly anisotropic media, and the corresponding heat capacity of a laminar crystal was calculated. For temperatures at which the interaction between layers may not be neglected ($T \ll \eta\theta, \zeta\theta$,

where η and ζ are small elastic moduli), the formula for the heat capacity obtained in Ref. 2 may be transformed into the form*

$$Cs^2/A = (\delta/s) + 2 \{3K(s) - s dK(s)/ds\}, \quad (1)$$

$$A = V k \eta^4 \pi^2 / 16 a^3 \nu^3 \zeta, \quad (2)$$

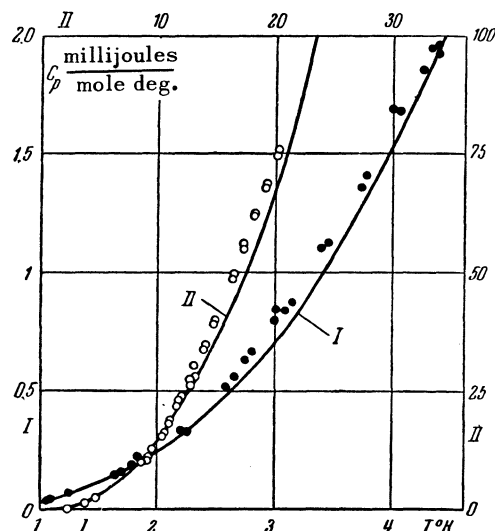
$$\delta = \zeta \eta / 15, \quad s = a' \eta^2 \theta / 4 \pi \nu a T,$$

where ν is the "transverse stiffness" of the layers ($\nu \sim 1$), a' and a are the atomic distances in the layer and normal to it, and

$$K(s) = \int_0^\infty \frac{t^2 \arctg(t/s) dt}{e^{2\pi t} - 1}. \quad (3)$$

Using an integral representation for $\ln \Gamma(s)^3$, it is easy to show that

$$\frac{dK(s)}{ds} = -\frac{1}{24} - \frac{s^2}{2} \left[\frac{d}{ds} \ln \Gamma(s) - \ln s + \frac{1}{2s} \right]. \quad (4)$$



Lattice part of the heat capacity of graphite.⁵ I—data for the region 1.0–4.4° K. II—data for the region 4.0–21.0° K. Solid line—theoretical curve.

In the region of temperatures under consideration, the term

$$\delta/s \approx \zeta T / \eta \theta \ll 1$$

and may be neglected. From Eqs. (1) and (4) we obtain the precise formula:

$$\frac{1}{A} \frac{d}{ds} (Cs^2) = s^3 \frac{d^2}{ds^2} \ln \Gamma(s) - s(s+1) - \frac{1}{6}, \quad (5)$$

where $d^2 \ln \Gamma / ds^2$ is a tabulated function.⁴ Thus, in the region of very low temperatures ($T \ll \eta\theta, \zeta\theta$) it is easy to tabulate the heat capacity of laminar crystals with the use of one graphical integration.

A comparison with experiment is possible in spite of the fact that the elastic constants in the region of temperatures under consideration are not known for laminar lattices. In fact, for

$$s \rightarrow 0: s^2 C/A \rightarrow 0.0914$$

(the region of quadratic dependence of heat capacity on temperature); and for

$$(T \rightarrow 0), s^3 C/A \rightarrow 1/30$$

(the region of cubic dependence). Determining the combinations of constants required for Eq. (2) by the limiting laws, the entire curve may be constructed.

Until very recently, the necessary experimental data was not available. The data of Keesom and Pearlman⁵, which appeared recently, allowed a comparison with experiment for graphite, as shown in

the Figure. Over the entire temperature range, the divergence does not exceed 10–15%, and may be partly due to the error attached to the exclusion of a linear term (electronic contribution to the heat capacity).

It should be noted that Eq. (1), obtained without appealing to models, but on the assumption of strong anisotropy, and yielding satisfactory agreement with experiment for graphite, cannot lay claim to a detailed agreement with experiment for lattices that are not so strongly anisotropic lattices, e.g., laminar halide salts of cadmium.⁶ However, in the same way that DeBye's interpolation formula gives good agreement with experiment in the general cases up to $T \sim \theta$, while the precise cubic law ceases to be fulfilled very early, so also in the anisotropic case it may be expected that the interpolation formula obtained by the use of the limiting law of dispersion in Ref. 2 by an integration along k , not to infinity, but to the boundaries of the wave vectors, will give better agreement with experiment at low temperatures and will be applicable to a wide class of laminar structures. This is due to the relatively great stability of the integrals expressing the heat capacity under variations of the dispersion law.⁷ and to the considerably greater influence of the upper limit of integration, which is taken into account by cutting off at the boundaries of the wave vectors.

It should be noted that in structures in which the layers differ (for example, in cadmium iodide, in which they are not monatomic and the surfaces of iodine ions facing one another have different positions with respect to the origin in a hexagonal system of coordinates), soft optical branches associated with weak interactions between layers may also contribute to the heat capacity.

The last remark was made by Prof. I. M. Lifshitz. We take this opportunity to thank him for his interest in this work.

*A direct numerical integration of the I. M. Lifshitz formula was performed by N. N. Lazarenko (diploma research, Kharkov State University, 1954). However, the accuracy attained therein is insufficient for comparison with experiment.

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Investigation of the Excitation Functions for the Reactions $C^{12}(p, pn)C^{11}$ $Al^{27}(p, 3pn)Na^{24}$ and $Al^{27}(p, 3p, 3n)Na^{22}$ in the 150-660 Mev Energy Range

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THE reaction $C^{12}(p, pn)C^{11}$ (1) is widely used for the measurement of proton flux. In connection with this, it is of interest to determine the value of the cross-section for this reaction for various proton energies. The excitation function of the reaction (1) was measured by Aamont and others¹ for energies from the threshold energy up to 340 mev. Comparison of the results of Ref. 1 with the data obtained by Soroko² (see Figure) indicate a rapid decrease of the cross-section in the 300–460 mev range. However, the measurements of the ratio of the values of the cross-section at 290 mev and 660 mev revealed³ that, in this energy range, the value of the cross-section for the reaction (1) decreases much more slowly. The mentioned-ratio was found to be

$$\sigma(670) / \sigma(290) = 0.84 \pm 0.03.$$

We therefore concluded it probable that a systematic error ($\sim 15\%$) in the determination of the absolute cross-section in one of the References 1,2 is the real cause of the discrepancy. Results similar to those obtained in Ref. 2 were soon obtained in new investigations^{4,5} in the 410–460 mev range. Finally, the cross-sections in the 170–350 mev range were measured with great accuracy by Crandall et al,⁶ (see Figure). The values found in Reference 6 are in good agreement with the data of Refs. 2-5. The cross-sections given in Ref. 1 are, evidently, systematically larger by some 15–25%.

The existence of these discrepancies led us to the investigation of the reaction (1) in the 150–660 mev range. In the course of the experiments, a graphite target was placed in the chamber of the