1200 times less than the experimental value. Delayed magnetic dipole transitions have been studied by Graham and Bell,[13] who explained the reduction in the transition probability as being due to a forbidden change in orbital angular momentum: $\Delta l = 2$. The transition under consideration may be identified as the transition $d_{3/2} \to s_{1/2}$. A partial relaxation of the rule forbidding the transition in this case is apparently related to the meson exchange currents between nucleons.

Note added in proof. After the present article had been submitted for publication, the authors learned of the paper by Azuma[17] in which the estimate $\tau < 5\times 10^{-11}$ sec is given for the upper limit on the lifetimes of both excited states of the Ti^{46} nucleus.

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Two Limiting Momenta in Scalar Electrodynamics

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A study is made of the possibility of introducing two limiting momenta in scalar electrodynamics. It is shown that the difficulties connected with the vanishing of the renormalized charge in the limit of a point interaction appear also in the theory considered here.

KHALATNIKOV and the writer[1] have written integral equations for the Green's functions and vertex parts in the electrodynamics of charged mesons with spin zero [Eqs. (2), (4) and (7) in Ref. 1]. The last of these equations is valid only if in the photon function

$$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{d_t(k^2)}{k^2} + \frac{k_\mu k_\nu}{k^2}\frac{d_l(k^2)}{k^2}$$

one chooses $d_l = d_t$, and was introduced in the approximation $e_1^2 \ll 1$. The solution of these equations was carried through in accordance with the general scheme of Landau, Abrikosov and Khalatnikov[2] for a smeared out interaction, in which the interaction factor β_σ is replaced by

$$\beta_\sigma \to \beta_\sigma \delta_{\Lambda_p}(p)\, \delta_{\Lambda_p}(p-l)\, \delta_{\Lambda_k}(l),$$

$$\delta_\Lambda(p) = \begin{cases} 1; & -p^2 \ll \Lambda^2 \\ 0; & -p^2 \gg \Lambda^2. \end{cases}$$

In Refs. 1 and 3, it was assumed that $\Lambda_p \sim \Lambda_k$; the changes appearing in the case in which there is a decided inequality between the two limiting values have been investigated by Abrikosov and Khalatnikov[4] in the ordinary electrodynamics and the pseudoscalar meson theory. In connection with a conclusion stated by Pomeranchuk[5] about the vanishing of the charge on the passage to a point interaction, the two-limit scheme takes on particular interest. We consider this question in the electrodynamics of particles with spin zero.

Two cases are possible:

I. $\Lambda_k \gg \Lambda_p$. The integration in Eq. (4) of Ref. 1 can be taken over two regions: a) $\Lambda_p \gg p \gg k$, and b) $\Lambda_k \gg p \gg \Lambda_p \gg k$. We turn our attention to the equation for the vertex part for $p \gg l$. The integration over k involves the region $\Lambda_p \gg k \gg p$. The vertex part is equal to zero for p or $l \gg \Lambda_p$ and is the same as the expression (6) of Ref. 3 for $\Lambda_p \gg p \gg l$. Therefore the region b) makes no contribution to the expression for the polarized tensor. If $l \gg p$, the vertex part contains in the right-hand member of Eq. 7 of Ref. 1 an integration only over the region $\Lambda_p \gg k \gg l$ (because of the factor $\delta_{\Lambda_p}(p-k)$ under the sign of integration), and is equal to zero for $l \gg \Lambda_p$. Thus for $\Lambda_k \gg \Lambda_p$, only the one limit Λ_p actually has any effect in the theory.

II. $\Lambda_p \gg \Lambda_k$. Let us consider the equation for the vertex part in this case. Let $p \gg l$. For the vertex part and $G(p)$, we try to find a solution in the form

$$G^{-1}(p) = [\hat{p} - m_1(p^2)]/\beta(p^2),$$

$$B_\sigma(p,\ p-l,\ l) = \beta_\sigma \alpha(p^2).$$

The region of integration is bounded by $\Lambda_k \gg k \gg p$. For $\Lambda_k \gg p \gg l$ we obtain the expression (6) of Ref. 3:

$$B_\sigma(p,\ p-\bar{l},\ l) = \beta_\sigma \alpha(p^2) \tag{1}$$

$$+ (\hat{l}l_\sigma - \beta_\sigma l^2)\, p^{-2} \alpha^2(p^2)\, d(p^2)\, S_0(p^2, l^2).$$

If $p \gg \Lambda_k \gg l$, then the integral does not make a logarithmic contribution, and

$$B_\sigma(p,\ p-l,\ l) \to \beta_\sigma. \tag{2}$$

We turn to the equation for the photon Green's function. If $k \gg \Lambda_k$, then $d_t(k^2) = 1$. Suppose $k \ll \Lambda_k$. Then, as before, in the integral for the polarization tensor there are two regions of integration

$$\text{a) } \Lambda_p \gg \Lambda_k \gg p \gg k, \quad \text{b) } \Lambda_p \gg p \gg \Lambda_k \gg k.$$

In region a) one can use the expression (1) for $B_\sigma(p,\ p-l,\ l)$, and in region b) one can use Eq. (2). After some calculation we obtain:

$$d_t^{-1}(\eta) = 1 + 8\nu S(L_k, \eta) + (e_1^2/3\pi)\,\nu\,(L_p - L_k),$$

where we have at once introduced the effective number ν of different types of particles, and

$$\eta = \ln(-k^2/m^2), \qquad L_k = \ln(\Lambda_k^2/m^2).$$

For $S(\xi, \nu)$ we have

$$S(\xi, \eta) = (e_1^2/24\pi)(\xi - \eta)\,\beta(\xi).$$

But $\beta(\xi = L_k) = 1$. Therefore we get

$$d_t(k^2) = \begin{cases} [1 + (e_1^2/3\pi)\,\nu \ln(\Lambda_p^2/-k^2)]^{-1}; & \text{for } k \ll \Lambda_k, \\ 1; & \text{for } k \gg \Lambda_k. \end{cases} \tag{3}$$

Thus when two cut-off limits are introduced, the function $d_t(k^2)$ ceases to be continuous in the logarithmic scale, and takes a jump on passage through the value $-k^2 = \Lambda_k^2$. The formula for the charge will obviously be

$$e^2 = e_1^2[1 + (e_1^2/3\pi)\,\nu \ln(\Lambda_p^2/m^2)]^{-1}.$$

The function $\alpha(\xi)$ is defined by the equation[3]

$$\alpha(\xi) = 1 - \frac{e_1^2}{8\pi}\int_\xi^{L_k} \alpha(z)\, d(z)\, dz.$$

Substituting the expression for $d(z) = d_t(z)$ into this equation and integrating, we obtain

$$\alpha(p^2) = \left[\frac{1 + (e_1^2/3\pi)\nu \ln(\Lambda_p^2/\Lambda_k^2)}{1 + (e_1^2/3\pi)\nu \ln(\Lambda_p^2/-p^2)}\right]^{3/8\nu} \quad \text{for } -p^2 \ll \Lambda_k^2, \tag{4}$$

$$\alpha(p^2) = 1 \quad \text{for } -p^2 \gg \Lambda_k^2.$$

The expression for the function $\beta(p^2)$ can be obtained from an identity of Ward: $\alpha\beta = 1$; it will again be the renormalized value,

$$\beta(p^2) = [(e_1^2/e^2)\, d_t(\Lambda_k)]^{3/8\nu}\,[d_{tc}(-p^2)]^{-3/8\nu}. \tag{5}$$

The function $m_1(p^2)$ was defined in Ref. 1 as the renormalization ratio, which in the presence of two limits is written in the following way:

$$m_1(\Lambda_k, \Lambda_p, p^2, e_1^2)/m_1$$

$$= m(-p^2/m^2, e^2)/m(\Lambda_k^2/m^2, e^2),$$

$$\lim_{-p^2 \to \Lambda_k} m_1(p^2) = m_1.$$

The correctness of such a normalization follows from the fact that in the equation for $G(p)$ the logarithmic contribution arises in the region $\Lambda_k \gg k \gg p$, and for $p \gg \Lambda_k$ the meson Green's function agrees with accuracy up to nonlogarithmic terms with its zeroth order value $(\hat{p} - m_1)^{-1}$. According to Ref. 1, we have

$$m(-p^2/m^2, e^2) = m[d_{tc}(p^2)]^{-9/8\nu},$$

from which we at once get

$$m_1(p^2) = m_1\left[\frac{1 + (e_1^2/3\pi)\nu\ln(\Lambda_p^2/-p^2)}{1 + (e_1^2/3\pi)\nu\ln(\Lambda_p^2/\Lambda_k^2)}\right]^{9/8\nu} \quad (6)$$

$$\text{for } -p^2 \ll \Lambda_k^2,$$

$$m_1(p^2) = m_1 \quad \text{for } -p^2 \gg \Lambda_k^2.$$

From Eq. (3) it can be seen that the vacuum polarization of particles with spin zero leads to a change of the photon Green's function in exactly the same way as in the case of the ordinary electrodynamics, studied in Ref. 4. Equations (4), (5) and (6) enable one to write out the expressions for the function $G(p)$ for $d_l = d_t$, according to Ref. 1. A gauge transformation to an arbitrary d_l can also be carried out.

It is now easy to convince oneself of the correctness of Pomeranchuk's result[5] for the scalar electrodynamics also. We consider the first of the diagrams for a vertex part which were not taken into account in Eq. (7) of Ref. 1 —a diagram with two intersecting virtual photon lines — and estimate the error incurred by neglecting this diagram ($d_l = d_t$). For this purpose we put in place of $G(p)$, $B_\mu(p, p-k, k)$, and $D(k)$ the functions of the basic approximation, Eqs. (3)-(5), and assume that between the two limits there holds the inequality

$$\ln(\Lambda_p/\Lambda_k) \gg 1. \quad (7)$$

Ward's identity makes it possible to reduce all the functions β and α to only a single function α, which, as can be seen from Eq. (4), always satisfies $\alpha \lesssim 1$. On the other hand, since the integration over the photon quanta is taken up to the limiting momentum Λ_k, the two photon functions d_t will be smaller than

$$[1 + (e_1^2/3\pi)\nu\ln(\Lambda_p^2/\Lambda_k^2)]^{-1}.$$

The integral under consideration contains only one logarithmic integration, so that the relative size of the contribution from this diagram to the vacuum polarization will be less than

$$\sim e_1^4\ln(\Lambda_k^2/m^2)[1 + (e_1^2/3\pi)\nu\ln(\Lambda_p^2/\Lambda_k^2)]^{-2}.$$

For not too small e_1^2, this contribution does not depend on the priming charge and will be small, if instead of Eq. (7) we require

$$\ln(\Lambda_k^2/m^2)\ln^{-2}(\Lambda_p^2/\Lambda_k^2) \ll 1$$

(say $\Lambda_k = \sqrt{m\Lambda_p}$). It can easily be seen that all the remaining diagrams together give a power series

$$\sum_{n,m} C_{nm}\ln^n(\Lambda_k^2/m^2)\ \ln^{-m}(\Lambda_p^2/\Lambda_k^2), \text{where } m > n.$$

With a reservation regarding the asymptotic character of this series, there again follows from this basic conclusion of Pomeranchuk, that the charge goes to zero in the limit of a point interaction:

$$e^2 = [(\nu/3\pi)\ln(\Lambda_p^2/m^2)]^{-1} \to 0 \quad \text{for } \Lambda_p \to \infty.$$

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