Employing (18), we write the integral in the form

$$\int \exp\{-i\varphi_s(k_2 + ma_1/\alpha_0^2) + i\varphi_r(k_2) + \varphi_{1s}(k_2 + ma_1/\alpha_0^2) + \varphi_{1r}(k_2)\}\, dk_2 \qquad \text{(A–16)}$$

Since [see Eq. (22)]

$$\varphi_r(k_2) = \alpha_0^2 \int_c^{k_2} \varkappa_{1r} dk_2,$$

$$\varphi_s\left(k_2 + \frac{ma_1}{\alpha_0^2}\right) = \alpha_0^2 \int_c^{k_2+ma_1/\alpha_0^2} \varkappa_{1s} dk_2,$$

then the imaginary part of the exponent is very large ($\alpha_0 \to \infty$) and the integral can be solved by the method steepest descents. A saddle point will exist if there is a point of the plane $(\varkappa_1, k_2)$ in which $\varkappa_{1r}(k_2) = \varkappa_{1s}(k_2 + ma_1/\alpha_0^2)$.
Since $a_1/\alpha_0^2 \ll \pi/a_2$, then the conditions given above will be satisfied only if the curves $\varkappa_{1r}(k_2)$ and $\varkappa_{1s}(k_2)$ intersect (more precisely, if they come very close to one another, or intersect). In this case we can employ the method of steepest descents. It is easy to convince oneself of the fact that the integral in this case will be of order ϵ while the entire matrix elements will be of order ϵ^3.

If $\varkappa_{12}$ and $\varkappa_{12}$ (which refer to states with identical n and k_3 in the bands r and s) do not intersect, the saddle points will not exist and the integral will be of order ϵ^2, while the matrix element will be of order ϵ^4.

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SOVIET PHYSICS JETP VOLUME 5, NUMBER 2 SEPTEMBER, 1957

The Acceleration of a Plasma by a Magnetic Field

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(Submitted to JETP editor November 28, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 305–310 (February, 1957)

Acceleration of a current–conducting plasma jet by a magnetic field is considered. The nature of the processes is preliminarily elucidated by examining the case of motion of conductors possessing some resistance and inductance. Furthermore, the motion of ions and electrons is studied under conditions where collisions, magnetic interaction of the particles and excitation of waves are negligible. The existence of a critical charge density in accelerators has been established for this case. Peculiarities of acceleration of very dense jets are also considered.

If a current is flowing in a plasma jet situated in a magnetic field, there will be a force of the volume density $f = [\mathbf{jH}]/c$ acting upon the jet and imparting to it an acceleration. As an example, we might consider the case of the motion of an electric arc, burning between two electrodes connected to a current source, in a magnetic field. Such a process was experimentally studied by Bron[1,2] at atmospheric pressure. In his experiments, the velocity of the arc attained several hundred meters per second despite air resistance.

It is of interest to consider the motion of the arc in the absence of the resistance of the surrounding medium, i.e., in a vacuum. In that case, the process of divergence of the jet can be prevented by several means. In particular, we shall assume that the time during which the jet is accelerated is sufficiently small, or that the jet is contracted by its proper magnetic field. We shall not consider the processes of jet formation (consult, for instance, Ref. 3)

1. ON THE MOTION OF CONDUCTORS

We can obtain a rudimentary picture of the motion of a current–conducting plasma jet in a magnetic field if we consider it as a conductor with resistance R and inductance L. We shall examine the simplest case of a straight conductor of length l, which makes contact with conducting rails and moves perpendicularly to a homogeneous magnetic field $H(t)$. A voltage $U(t)$ is applied to the rails. The equations for this case are

$$m\frac{dv}{dt} = \frac{1}{c} IHl, \qquad (1)$$

$$\frac{1}{c^2}\frac{dLI}{dt} = -\frac{1}{c} lHv - IR + U,$$

where I is the current intensity in the circuit.*

Let R, L, m, H, and U be constant. For these conditions, and for a relatively small resistance of the circuit, i.e., for $Q = \omega L/R >> 1$, we have

$$I = I_0 \sin \omega t; \qquad (2)$$

$$I_0 = c^2 U / L\omega = cu\sqrt{m/L}; \quad u = cU/lH;$$

$$x = u(t - \omega^{-1}\sin \omega t); \qquad (2a)$$

$$v = u(1 - \cos \omega t); \quad \omega = Hl/\sqrt{Lm}.$$

We shall call u the drift velocity and shall assume it to be much smaller than the velocity of light. Formula (2a) indicates that the motion of the conductor can be represented as a drift with a constant velocity u, upon which oscillations with the amplitude u/ω are superimposed. The maximum velocity $V = 2u$ will be attained by the conductor after the time $T_0 = \pi/\omega$, which we shall call the acceleration time. The path traversed during this time is $a = uT_0$.

In the second extreme case, where the inductance is small ($Q << 1$), we have

$$x = ut - T_0 u(1 - e^{-t/T_0}); \qquad (3)$$

$$I = (U/R)e^{-t/T_0}; \quad T_0 = c^2 Rm/H^2 l^2,$$

that is, the velocity asymptotically approaches the drift velocity.

It is useful to note that, for $R = 0$, the system of equations (1) is analogous to the equations of motion of a single charged particle in mutually perpendicular electric and magnetic fields. If the fields E and H are constant, the motion of the particle, as is well known,[9] consists of a drift with the velocity $u = cE/H$ with superimposed uniform circular motion with the angular velocity $\omega = eH/mc$.

Apart from the case of the direct contact coupling between the conductor and the current source, inductive coupling is of considerable interest. This case can be realized if a closed ring is placed within within a changing magnetic field. In this case, an induced current flows in the ring and under the influence of the magnetic field, the ring will start moving.

A similar case of gas discharge, the so-called ring electrodeless discharge, is known. It is conspicuous by the fact that, due to the absence of any electrodes, the current in such discharge can attain a very large value, even at quite low pressures. For instance, at a pressure of 1.7×10^{-4} mm Hg and for an EMF of the rotor field in the circuit equal to 8V, the current in the experiments of Smith[4] using mercury vapors attained 450A. The generator frequency was of the order of 900 c/s.

If we assume the jet to be sufficiently thin and axially symmetric, then we can write the system of equations, describing the displacement of the ring permeated by the flux Φ perpendicularly to its plane (along the z-axis) and the expansion of its radius, making use of the Lagrangian

$$\mathscr{L} = (m/2)(\dot{r}^2 + \dot{z}^2) + (LI^2/2c^2) + (\Phi I/c) - W(r), \qquad (4)$$

where $W(r)$ is the potential of the elastic stresses acting in the cross-section of the jet, and I can be regarded as a generalized velocity q. It should be noted that, for the densities encountered in modern accelerators, $W(r) = 0$ and $L = c^2 l^2 m_1^2/e^2 m$, a case which we shall treat in greater detail below [see Eq. (14)].

It can be seen from Eq. (4) that, for the case where both the resistance and $W(r)$ are small, the kinetic energy of the ring leaving the field will be of the order of $\Phi_0^2 / 2L$, under the obvious condition that the current intensity at this moment will drop to zero. Φ_0 and L_0 denote certain values of the magnetic flux and of the inductance, characteristic for the process. The mean current will be of the order of $c\Phi_0/L_0$. In the simplest case, for example, where an ideally conducting ring of a constant radius and inductance moves in the direction of the z-axis, with the ring permeated by the flux

$$\Phi = \Phi_0 (t/T)(1 - z/a), \qquad (5)$$

the velocity of ejection and the current are, res-

*U includes the EMF induced by the changing magnetic field.

pectively,

$$V = 2\Phi_0 / 3\sqrt{Lm}, \qquad (6)$$

$$I = (c\Phi_0 t / LT_0)(1 - z/a).$$

The initial velocity and current are assumed to be zero.* The acceleration time and the corresponding ing path a are connected by the relation

$$T_0 = 4a\sqrt{mL} / \Phi_0. \qquad (7)$$

In the second extreme case of practical interest, namely that of plane motion of a ring that contracts to a point, the results obtained correspond to the estimate given above.

Similarly to the fact that the equations of motion for a particle in a homogeneous field are identical with the equations of motion for a straight conductor, the Lagrangian (4) is identical with the Lagrangian describing the motion of a charged particle in an axially-symmetric magnetic field.

2. MOTION OF E-CONNECTED PARTICLES

We shall consider a neutral jet of ions and electrons of sufficiently low density that we can neglect the collisions between the particles as well as their magnetic field, and we shall account for the electric interaction between the particles only. Such a cloud of particles we shall call an E- plasma, and the particles, E-connected.

The behavior of the plasma can be described by the system of two kinetic equations of Vlasov,[6] written for the electrons and the ions respectively, taking into account the self-consistent field. For simplicity, we shall consider the plane case, and assume that the plasma jet has the shape of an infinite (in y and z) layer of finite thickness, perpendicular to the x-axis. In order to avoid the necessity of having to account for the divergence of the plasma, we shall assume that the thickness of the layer is much larger than $v_T^i\, T_0$, where v_T^i denotes the mean thermal velocity of ions. In addition, in the present treatment we shall limit ourselves to the solutions with constant density, which correspond to $R = 0$, being equivalent to neglecting the terms proportional to $\partial f/\partial r$ in the kinetic equations.

*The expression for z, under the condition (5) and for $z(0) = 0$, $\dot{z}(0) = 0$, $I(0) = 0$ is of the following form:[5]

$$z = a\left[\frac{\alpha}{3\cdot 4}\left(\frac{t}{T_0}\right)^4 - \frac{\alpha^2}{3\cdot 4\cdot 7\cdot 8}\left(\frac{t}{T_0}\right)^8 + \cdots\right],$$

$$\alpha = \frac{c^2\Phi_0^2 T_0^2}{mLa^2}.$$

Under these conditions, the above-mentioned system of equations can, by averaging over the velocities, be written in the following way, valid for the whole region of constant concentration:

$$m_e\, d\mathbf{v}_e / dt = -e(\mathbf{E}_0 + \mathbf{E}') - \frac{e}{c}[\mathbf{v}_e\mathbf{H}]; \qquad (8)$$

$$m_i\, d\mathbf{v}_i / dt = e(\mathbf{E}_0 + \mathbf{E}') + \frac{e}{c}[\mathbf{v}_i\mathbf{H}]. \qquad (8a)$$

where $\mathbf{v}_i$ and $\mathbf{v}_e$ denote the velocities of the ordered motion of ions and electrons respectively. In the following discussion, we shall call $\mathbf{v}$ simply the particle velocity and, correspondingly, talk about its trajectory and coordinates, having in mind the line of current and the coordinates of a point on it. It can be assumed that, if the displacement of particles with respect to each other will be sufficiently small compared to the layer thickness, then the solution of the above system of equations will correctly describe the behavior of the cloud.

Since the plasma as a whole is neutral and the problem is one-dimensional, we have, in the region under consideration,

$$E' = 4\pi e \int_{-\infty}^{x} (n_e - n_i)\, dx = 4\pi e n (x_e - x_i) \qquad (9)$$

independently of the state of the plasma on the surface (Fig. 1).* x_i and x_e are the coordinates of two fixed particles which coincided at the initial moment when the plasma was non-polarized.

Substituting (9) into (8), we obtain a system of ordinary linear equations. For convenience, we shall introduce the notation

$$\omega_0^2 = 4\pi e^2 n / m_e, \quad \mu = m_e / m_i,$$

$$\omega_e = eH / m_e c, \quad \lambda^2 = \omega_e^2 / \omega_0^2$$

and assume the fields to be constant, and the plasma to be at rest and $x_i = x_e = 0$ for $t = 0$.

a) We shall consider the motion of particles in the direction of the x-axis. The characteristic equation of the system (8) has two roots

$$\Omega_1^2 \approx \mu\omega_e^2(1 + \mu\lambda^2)/(1 + \lambda^2), \quad \Omega_2^2 \approx \omega_e^2 + \omega_0^2. \qquad (10)$$

*We shall obtain a formula analogous to (9) also in the case where the cross-section of the jet is, and remains in the process of acceleration, of the form of an ellipse or a circle.

In these formulas, we neglected those of the terms $\sim\mu^2$ which do not qualitatively change the relation between Ω and λ. The root Ω_1 equals the angular frequency of ion rotation in a magnetic field for $\lambda\to\infty$, that is, for diminishing concentration of particles, and for $\lambda\to 0$ tends to $\omega_e\sqrt{\mu}$. The root Ω_2 gives the electron rotation frequency for $\lambda\to\infty$, and equals the Langmuir frequency for $\lambda\to 0$. Evidently, the root Ω_1 for $\lambda\to 0$ pertains to the movement of the jet as a whole, while Ω_2 characterizes its polarization vibration.

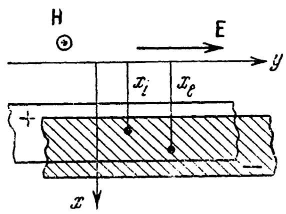

FIG. 1

The motion of particles in the x direction, for our initial conditions, will be as follows:

$$x_e = u\left[t - \frac{1}{\Omega_1^2(1+\lambda^2)}\sin\Omega_1 t - \frac{\lambda^2}{\Omega_2(1+\lambda^2)}\sin\Omega_2 t\right]; \tag{11}$$

$$x_i = u\left[t - \frac{1}{\Omega_1^2}\sin\Omega_1 t + \frac{\lambda^2\mu}{\Omega_2(1+\lambda^2)}\sin\Omega_2 t\right]. \tag{11a}$$

From these formulas it is clear that, for small λ, the electrons lead the ions for the first half of the acceleration time (that is, for $t < \pi/2\Omega_1$), while during the second half of that period the ions catch up with them.

The maximum displacement of the one type of particles with respect to the other equals, in the central region of the layer,

$$\Delta = u\lambda^2/\Omega_1 = E/4\pi en\sqrt{\mu}. \tag{12}$$

Of the same order of magnitude is the region of the diffuse boundary of the plasma, due to the difference in the motion of ions and electrons. Consequently, it is necessary for the validity of our laws that the layer thickness be $\ll\Delta$. For instance, for an external field of 3000 V/cm and $n\sim 10^{12}$ cm^{-3}, we have $\Delta\sim 0.1$ cm. It is interesting to note that the maximum value of the polarization field is $1/\sqrt{\mu}$ times larger than the external field,

b) we shall now consider the displacement of particles in the direction of the y-axis. From (8) and (11), we have, for $\lambda\to 0$,

$$(v_i)_y \approx \sqrt{\mu}\,u\sin(\sqrt{\mu}\,\omega_e t); \tag{13}$$

$$(v_e)_y \approx (u/\sqrt{\mu})\sin(\sqrt{\mu}\,\omega_e t).$$

Integrating over t, we find that the displacement of ions in the y-direction is smaller by a factor of $1/\mu$ than that of electrons. This is exactly the opposite of what takes place for free particles, and resembles the situation in metals. Ions in an E-connected jet accelerate because of the electron current. The trajectories of the particles are shown schematically in Fig. 2.

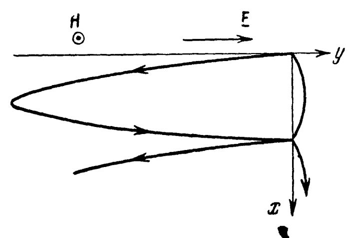

FIG. 2

It is important to mention that the above calculation does not take into account the abrupt stopping of particles on the electrodes and, therefore, pertains in fact to a ring-shaped jet.

It can be shown, making use of the conservation of energy, that the character of the results obtained for the idealized one-dimensional case, including formula (12), will not change for the case of a ring-shaped jet of finite cross-section, if the magnetic field safeguards the stability of the cloud in the direction perpendicular to the direction of motion [see, for instance, (19)].

c) it is not difficult to find the coefficient of self-induction for a jet of E-connected particles, under some limiting conditions fulfilled for $\lambda\to 0$, making use of the equations of motion of the particles, containing the interaction forces acting between the particles. This coefficient of self-induction L_I arises due to the inertia of particles, and therefore we denote it as inertial. For a cloud of similar particles with mass m_1 it equals

$$L_I = c^2 l^2 m_1^2 / e^2 m,$$

and for a cloud consisting of ions and electrons (see Ref. 10)

$$L_I = c^2 l^2 m_i m_e / e^2 m, \qquad (14)$$

where m is the total mass of the jet and l its length. Taking into account that $m_e << m_i$, we come to the conclusion that in this case L_I is determined by the inertia of electrons. Formula (14) is correct for thecase of a ring-shaped plasma as well.

It can be seen from (14) that L_I decreases without limits for increasing concentration and, therefore, for a considerable value of concentration the coefficient of self–induction of the jet will not any more equal L_I but L_E – the electromagnetic coefficient of self–induction of the jet,* which may be calculated from the distribution of currents according to well known rules of electrodynamics.

Since, in the absence of magnetic materials in the immediate vicinity of the jet, L_E for a straight conductor or a ring equals, in Gaussian system of units,

$$L_E = \Lambda l, \qquad (15)$$

where Λ is a value of the order of unity,[7] then the concentration n_L for which $L_I = L_E$, is given by the formula

$$n_L = (4 / \Lambda S) \cdot 10^{12} \ cm^{-3}, \qquad (16)$$

where S is the cross-section of the jet.

d) The influence of the proper magnetic field of the jet is not limited only to the change of the coefficient of self–induction, but causes a contradiction of the jet as well. This takes place because the forward motion of the front particles is decelerated by the proper magnetic field of the jet, while the motion of the rear particles is accelerated.

In orderto neglect this effect, as it has been done in the preceding calculation, it is necessary that the jet density be smaller than or of the order of

$$n_m = \alpha n_L \Lambda (b / a)^2 \qquad (17)$$

(see reference 11), where α is a value of the order of unity depending on the field structure, the form of the jet cross–section etc. and b is the half–width of the jet. If the jet density is much larger than n_m, then the jet will be strongly contracted in the process of acceleration under the influence of the proper magnetic field. As a result, the velocity of ejection may be considerably decreased. This is connected both with the increase in the resistance of the ring and the increase of the influence of the electromagnetic inductance.

Consequently, the largest velocities of ejection are attainable for thecase of an E-plasma. Since for that case $L \approx L_I$, it follows from (6) that

$$V \approx 2\Phi_0 / 3 \sqrt{Lm} \approx 2{,}7 \cdot 10^5 \, \Phi_0 / \sqrt{A} \, (2\pi r_0), \qquad (18)$$

where A is the atomic weight of the ion. It follows that the ejection velocity will is, in this case, independent of the total mass of the ring.

For $n >> n_L$, the ejection velocity will decrease as $\sim m^{-1/2}$ even in the absence of resistance if the external flux Φ_0 remains constant. Something similar takes place for an induction accelerator with immobile jet of the betatron type if we take into account that the current intensity is Φ_0 / L and the beam energy equals $\Phi_0^2 / 2L$.

Finally, in connection with all that was said above, we want to make two additional comments:

1) Condition (17) indicates that the concentrations for which the magnetic interaction between the particles can be neglected are relatively low, and it can be therefore assumed that, in this region, the resistance of the jet will be relatively low.

2) It is necessary for the existence of a ring of E-plasma that the magnetic field safeguards its stability in the direction perpendicular to the acceleration. The necessary conditions for that, which have to be imposed on the field, can be obtained from equation (4). The stability conditions both for the ions and the electrons are then fulfilled.* As an example of such a field for which the radius r_0 of the ring remains constant we may take

$$A_\varphi = \tfrac{1}{2} H_0(t)(a - z)(r / r_0 + r_0 / r). \qquad (19)$$

3. EJECTION VELOCITY OF A DENSE JET

Let us consider a dense jet, prevented from

*This corresponds to taking into account the magnetic interaction between the particles.

*With the exception, possibly, of the initial moment, since the depth of the potential well[8] in which the oscillations of particles take place is of the order of $\sim 1/m$ and the progressing plasma can be in a state of thermal equilibrium.

diverging by its proper magnetic field. Let the state of the plasma in it be stationary, so that the condition $\rho = \gamma p / c_T^2$, where C_T is the sound velocity in the plasma, be fulfilled. Assuming $p \sim H^2 / 4\pi$ and taking into the account that for the absence of resistance we have $I \sim V\sqrt{m/L}$, we obtain, after a simple calculation, the following expression:

$$V = kc_T \sqrt{\Lambda},$$

Λ has been determined above [see Eq.(15)] and $k \sim 1$. The ejection velocity, therefore, is found to be of the order of the maximum velocity of sound in the jet during the acceleration stage. The formula is independent of the nature of the coupling with the current source (contact or induction) but since the flux Φ_0 in the ring is known and $V \sim \Phi_0 / \sqrt{Lm}$, the relation between m and c_T follows.

The author wishes to express his gratitude to Prof. A. A. Sokolov for the interest shown in the present work, and to G. M. Strakhovskii for the discussion of problems connected with it.

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SOVIET PHYSICS JETP VOLUME 5, NUMBER 2 SEPTEMBER, 1957

Investigation of U^{235} Fission γ Rays in the Energy Region up to 250 kev

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(Submitted to JETP editor September 24, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 256-262 (February, 1957)

A scintillation spectrometer with a NaI(Tl) crystal and a proportional counter connected in coincidence with a fission chamber was used to measure the U^{235} fission γ spectrum in the energy range up to 250 kev. Gamma rays with energies of 30 ± 1.5 and 210 ± 10 kev were detected, as well as a broad distribution in the 110 to 150 kev region, which apparently consists of several lines. It has been found that in the investigated energy region most of the fission γ rays are emitted by fission fragments with life times between 0.5×10^{-9} and 2.5×10^{-9} sec.

The measurements made with the proportional counter showed that the line at 30 kev is not monochromatic and apparently corresponds to x radiation (K line) from heavy fission fragments with different Z.

1. INTRODUCTION

FISSION γ rays have thus far been the subject of very few investigations, and there have been only brief reports on the results of these. Rose and Wilson[1] have measured the angular distribution of fission γ rays relative to the direction in which the fission fragments emerge. Voitovetskii, Levin, and Marchenko[2] have discovered several monochromatic γ rays in the low energy region. Other papers[3-5] deal with measurements of either the average energy of fission γ rays or their spectrum.

The measurement of a fission γ ray spectrum by itself furnishes no basis for any conclusions as to the mechanism of the radiation. Theoretically one cannot rule out the possibility that some or all fission γ rays are connected with the proc-