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Scattering of Fast Protons on Protons

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It is shown that the main term in the interaction of two protons with $J = 0$ should have a singularity of the order $1/r^3$

AS is known, the cross section for fast proton-proton scattering is characterized by isotropy and energy independence.[1] This isotropy occurs in the energy range from 100 mev to 400-500 mev; beyond that the isotropy is violated in account of meson production inelastic processes.[2,3]

Another characteristic of proton elastic scattering is the appreciable magnitude of the elastic cross section in comparison with its limiting value for S—wave scattering $-\lambda^2$. Actually, the cross section amounts to $2\lambda^2$. This indicates the essential contribution to the proton scattering of states with total angular momentum $J = 0$, i.e., of the states ${}^1S_0$ and ${}^3P_0$ 1,2 and the suppression of other waves with respect to these. The spin orbit coupling in the interaction ought to be small.

In this paper, we did not intend to go into the physical nature of such a character of proton—proton interaction. The interaction between fast protons does not reduce to any sort of potential. We will restrict ourselves to ask a somewhat phenomenological question: what should be the singularity in the main part of the interaction between two protons with $J = 0$, if this interaction is considered in the coordinate (space) representation?

For a suggestive treatment, we will start from the Born approximation. The phase δ_l of the scattered wave is determined by the known relation

$$\sin\delta_l = -\frac{1}{k}\int_0^\infty g_l(kr)\,V(r)\,u_l(kr)\,dr, \qquad (1)$$

where k is the vector wave-number relative to the nucleon motion; $g_l(kr)$ is the outgoing wave of orbital angular momentum 1, in the absence of interaction [it has the asymptotic form $\sin(kr - \pi l/2)$] ; $u_l(kr)$ is the same but with the interaction present [it has the asymptotic form $\sin[kr - (\pi l/2) + \delta_l]$] $V(r)$ is the interaction energy for $J = 0$ (or its main term) multiplied by $2m^*/\hbar^2$ (m^* is the nucleonic reduced mass).

Let us first consider a $V(r)$ of the form

$$V(r) = 2m^* U_0 \hbar^{-2}(a/r)^n, \qquad (2')$$

where a is a certain length and V_0 is the value of the interaction energy at $r = a$.

Let us now introduce the dimensionless variable $\rho = kr$. Then (1) can be written in the form:

$$d_l = \frac{\sin\delta_l}{k} = -aV_0(ak)^{n-3}\int_0^\infty g_l(\rho)\,\rho^{-n}u_l(\rho)\,d\rho. \qquad (2)$$

Here d_l is the scattering length (the differential scattering cross section is proportional to $\sim d_l^2$). $V_0 = 2m^* U_0 a^2/\hbar^2$ is a dimensionless quantity characterizing the value of the interaction energy.

In the Born approximation, this integral depends on the nucleon energy, only through the cutoff at the lower integration limit, which is necessary for $n > 2$. For $n = 3$, this dependence will be still weak—logarithmic. The principal energy dependence is contained in the factor $(ak)^{n-3} \sim E^{(n-3)/2}$ (E is the nucleon energy, which multiplies the integral. It follows that for $n = 1$ (Coulomb law), the cross section will depend on the energy as E^{-2}; for $n = 2$ (quadratic law), as E^{-1} and, finally, for $n = 3$, it is constant (not counting a weak dependence on the cutoff at the lower limit which will lead to a logarithmic rise of the cross section).

For laws with a stronger r—dependence than $1/r^3$, the cross section will increase appreciably with energy.

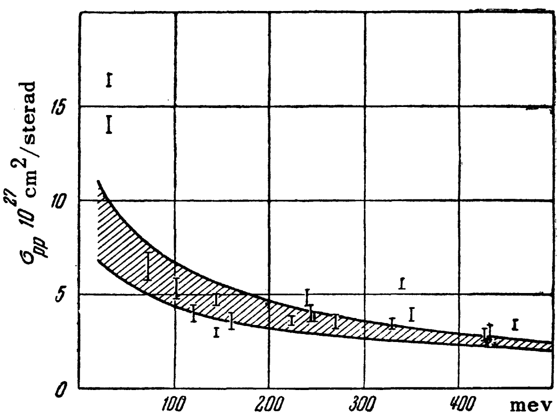
The qualitative behavior of the cross section for $V \approx r^{-3}$ shows that this law can be brought into agreement with the experimental data on $p-p$ scattering. Such an interaction law would, however, be in disagreement with meson theory, which requires the presence of an exponential factor $e^{-\varkappa r}$, where $\varkappa = \mu c/\hbar^2$ and where μ is the meson mass. This factor introduces deviations from the pure Coulomb law which are the smaller, the larger the proton energy. Indeed, if we write the interaction $e^{-\varkappa r}/r^3$ in a dimensionless form, we get $V(\rho) \sim e^{-\alpha\rho}/\rho^3$ where $\alpha = \varkappa/k$: hence the larger the value of k, the smaller the α and the smaller the role of the exponential.

The qualitative analysis reported above is the basis of an exact quantative calculation (not using the Born approximation) of $^1S_0$ and $^3P_0$ proton elastic scattering for the interactions

$$V = V_0 e^{-\alpha\rho}/\rho^3 \quad \text{for} \quad \rho > ak, \tag{4}$$
$$V = V_0 e^{-\alpha ak}/(ak)^3 \quad \text{for} \quad \rho < ak,$$

i.e., for a cut–off cubic law with an exponential. It was assumed that $a = \hbar/mc = 2 \times 10^{-14}$ cm, $\varkappa = \mu c/\hbar = 7 \times 10^{12}$ cm^{-1} (for π-mesons); two values were assumed for V_0 : 15 and 30.

The graph shows the calculated proton elastic scattering versus energy (the cross–hatched area is between the curves for V_0 = 15 and V_0 = 30). The vertical lines represent a compilation[1] of the expermental data. The calculations for the energies above 400 mev are doubtful because the relativistic corrections become excessively large. For a reasonable evaluation of the obtained agreement with the experimental data, it should be noted that at small energies ($E \ll 100$ mev), the interaction will contain terms with singularities of lower degree ($V \sim 1/r^2$ and $V \sim 1/r$). In the energy range between 100 and 300 mev, however, the cross section would have changed 9 times for a $V \sim 1/r$ interaction law, and 3 times for $V \sim 1/r^2$. For $V \sim 1/r^3$ the cross section is almost constant and close to the experimental values. This result can be considered as an indication of the substantial role of terms of the type $e^{-\varkappa r}/r^3$ in the proton interaction.



Region between the curves denotes the calculated cross section as a function of the energy. Experimental data are shown as vertical lines.

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2 Meshcheriakov, Bogachev, Neganov and Piskarev, Dokl. Akad. Nauk SSSR 99, 955 (1954).

3 Meshcheriakov, Neganov, Soroko and Vzorov, Dokl. Akad. Nauk SSSR 99, 959 (1954).

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