Letters to the Editor

Particle with Spin 3/2 in an Electromagnetic Field

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(Submitted to JETP editor July 9, 1956)

J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 363-365 (February, 1957)

LET us consider the relativistically invariant equations for particles of spin 3/2 in an electromagnetic field:*

$$(\alpha^k \pi_k + i\varkappa)\Psi = 0, \qquad (1)$$

$$\pi_k = \partial/\partial x_k - ieA_k, \ (k = 0, 1, 2, 3).$$

We write the matrices α_k in the parametric form of Petras[1]:

$$\alpha^k = \gamma^k + \gamma_i (B^{ik} - B^{ki})/\sqrt{3}, \ \gamma^k\gamma^i + \gamma^i\gamma^k = 2g^{ik},$$
$$B^{ik}B^{ls} = g^{kl}B^{is}, \ \gamma^l B^{ik} = B^{ik}\gamma^l.$$

From Eqs. (1) we find the supplementary conditions in relativistically covariant form:

$$D\gamma_i B^{mi}\Psi/\sqrt{3} = -iaF_{nl}\gamma^n B^{ml}\Psi, \qquad (2)$$

$$D = I - {}^1\!/_2\, iaF_{ps}\gamma^p\gamma^s, \quad a = {}^2\!/_3 (2 + \sqrt{3})\, e/\varkappa^2;$$

$$(Q^{kls}\pi_k + i\varkappa R^{ls})\Psi = 0, \qquad (3)$$

$$Q^{kls} = -Q^{lks} = \gamma^k B^{sl} - \gamma^l B^{sk} + (\gamma^l\gamma^k - \gamma^k\gamma^l)\gamma_i B^{si}/2\sqrt{3},$$

$$R^{ls} = B^{sl} - \gamma^l \gamma_i B^{si}/(1 + \sqrt{3})$$

(F_{nl} is the tensor of the electromagnetic field).

Since $Q^{lls} = 0$, then the term with π_l is missing in condition (3). For $l = 0$, the term with a time derivative is absent from (3).

Multiplying Eq. (1) by the matrix

$$\omega = I - {}^1\!/_2 (2 - \sqrt{3})\, \gamma_s \gamma_i B^{si}$$

and applying condition (2), we get the relativistically invariant equation

$$(\mathscr{L}^k \pi_k + i\varkappa M)\Psi = 0, \qquad (4)$$

$$\mathscr{L}^k = \gamma^k - i\frac{1-\sqrt{3}}{2} aF_{nm}\gamma^k\gamma_i D^{-1}\gamma^n B^{im} + i(2 - \sqrt{3})\, aF_{nm} D^{-1}\gamma^n B^{km},$$

$$M = I + i\frac{2\sqrt{3}-3}{2} aF_{nm}\gamma_i D^{-1}\gamma^n B^{im} - i\frac{1-\sqrt{3}}{2}\frac{a}{\varkappa}\left(\frac{\partial}{\partial x_p} F_{nm}\gamma^p\gamma_i D^{-1}\right)\gamma^n B^{im} - (1-\sqrt{3})\left(\frac{\partial}{\partial x_p} F_{nm} D^{-1}\right)\gamma^n B^{pm}.$$

Condition (3) is now the condition for the compatibility of Eq. (4) and condition (2). The fundamental equation (1) is equivalent to Eq. (4) with conditions (2) and (3). We note that Eq. (4) is non-linear relative to the electromagnetic field.

Taking into account the smallness of the dimensionless parameter ($aF_{nm} = 10^{-19} F_{nm}$, where F_{nm} is measured in the Gaussian system of units), we solve the equation relative to $\partial/\partial x_0$, neglecting terms of second and higher order relative to aF_{nm}. We then get an equation in the Schrödinger form

$$i\partial\Psi/\partial x_0 = H\Psi,$$

where H is the Hamiltonian of the particle with spin 3/2 in the electromagnetic field in the linear approximation relative to the small parameter aF_{nm}. We remark that the extreme smallness of aF_{nm} guarantees as negligibly small the magnitude of the contribution in the Hamiltonian from the higher approximations in aF_{nm}, even for strong fields.

The Hamiltonian H has the form*

$$H = H_0 + F_{nm}R^{nm\nu}(p_\nu - eA_\nu) + F_{nm}P^{nm} + \frac{\partial F_{nm}}{\partial x_p} Q^{nmp},$$

$$H_0 = \gamma_0\gamma^\nu (p_\nu - eA_\nu) + e\varphi + \varkappa\gamma_0,$$
$$p_\nu = -i\partial/\partial x_\nu, \ \varphi = -A_0, \qquad (5)$$

$$R^{nm\nu} = i\frac{1+\sqrt{3}}{\sqrt{3}}\frac{e}{\varkappa^2}\{(\gamma_0\gamma^\nu\gamma_i\gamma^n - \gamma_i\gamma^n\gamma_0\gamma^\nu)B^{im} - (1-\sqrt{3})\gamma_0\gamma^n(B^{\nu m} - \gamma_0\gamma^\nu B^{0m})\},$$

$$P^{nm} = \frac{ie}{\sqrt{3}\varkappa}\left\{\gamma_0\gamma_i\gamma^n B^{im} - \frac{1+\sqrt{3}}{\sqrt{3}}\gamma_i\gamma^n\gamma_0 B^{im} - \frac{2}{\sqrt{3}}\gamma_0\gamma^n\gamma_0 B^{0m}\right\},$$

$$Q^{nmp} = i\frac{1+\sqrt{3}}{\sqrt{3}}\frac{e}{\varkappa^2}\gamma_0\{\gamma^p\gamma_i\gamma^n B^{im} - (1-\sqrt{3})\gamma^n B^{pm}\}.$$

As can be seen, a particle with spin 3/2 possesses dipole and quadrupole kinematic moments and, moreover, the Hamiltonian H possesses an unusual term of a dipole type which depends on the momentum.**

Averaging is carried out over the charge density. Therefore the normalization condition has the form

$$\int \Psi^* \rho \Psi (dr) = 1,$$

$\rho = \eta\alpha^0$ is the Hermitian charge density matrix. The matrix η which determines the invariant bilinear Hermite form $(\Phi^* \eta \Psi)$ is found in the form

$$\eta = -\gamma_0 B^{ii} + (2 - \sqrt{3})\, \gamma_i \gamma_0 \gamma_k B^{ik}.$$

Correspondingly,

$$\rho = -B^{ii} + (1 - \sqrt{3})\, \gamma_i \gamma_k B^{ik} / \sqrt{3} + \gamma_i \gamma_0 B^{i0} / \sqrt{3} + \gamma_0 \gamma_i B^{0i} / \sqrt{3}.$$

The mean value of the energy of a particle with spin 3/2 in an electromagnetic field will be determined by the formula:

$$E = \int \Psi^* \rho i \frac{\partial}{\partial x_0} \Psi (dr) = \int \Psi^* \rho H \Psi (dr).$$

Hence for a guarantee of the reality of E we have the condition of the quasi-hermiticity of the operator H:

$$\int \{(H\Phi)^* \rho \Psi - \Phi^* \rho H \Psi\} (dr) = 0. \qquad (6)$$

For the Hermitian (5), the condition (6) is satisfied by taking into account the additional conditions (2) and (3).

The value of the energy in the linear approximation relative to the small parameter aF_{nm} is represented in the following form (at the same time we transform to the Gaussian units):

$$E = \int \Psi^* (-B^{ii}) \Big\{ H_0 + \frac{2}{3}(2 + \sqrt{3}) \frac{\hbar e}{M^2 c^2} \times [R_E^{\mu\nu} E_\mu + R_H^{\mu\nu} H_\mu] \Big(p_\nu - \frac{e}{c} A_\nu\Big) - \frac{2}{3}(2 + \sqrt{3}) \frac{\hbar e^2}{M^2 c^3} [R_E^{\mu 0} E_\mu + R_H^{\mu 0} H_\mu] \varphi + \frac{2}{3}(2 + \sqrt{3}) \frac{\hbar^2 e}{M^2 c^2} \Big[Q_E^{\mu\nu} \frac{\partial E_\mu}{\partial x_\nu} + Q_H^{\mu\nu} \frac{\partial H_\mu}{\partial x_\nu} \Big] \Big\} \Psi (dr). \qquad (7)$$

Thus a contribution to the energy, in addition to terms of the Hamiltonian of the Dirac type H_0, is made by supplementary terms of the dipole type, which depend on the momentum, and on the quadrupole electric and magnetic moments. Averaging in Eq. (7) is carried out over $\Psi^* (-B^{ii}) \Psi$ — the charge density of free particles.

For comparison we note that particles with spins 0 and ½ do not possess kinematic moments while a particle with spin 1 possesses kinematic dipole moments, which do not make a direct contribution to the energy but, in spite of this, are said to be electromagnetic interactions[3-5].

I consider it my pleasant duty to express my thanks to Dozent S. V. Izmailov for pointing out the theme and for his constant interest in the work.

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* The analysis is carried out in a system of units in which $\hbar = c = 1$. The metric tensor g^{ik} is chosen in the form

$$g^{00} = -g^{11} = -g^{22} = -g^{33} = 1, \quad g^{ik} = 0 \ (i \neq k).$$

* In the summation the Latin indices run, as above, over the values 0, 1, 2, 3, and the Greek over the values 1, 2, 3.

** Terms of such a type were first obtained by Darwin[2] in the non-relativistic approximation for particles with spin ½ in an electromagnetic field.

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Translated by R. T. Beyer
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Experimental Comparison of the Energy Spectra of γ–Quanta from the Decay of π°–Mesons Formed on Carbon and Lead Nuclei by 600 Mev Protons

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(Submitted to JETP editor November 2,1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 385–386 (February, 1957)

IN experiments conducted up to the present time there has not been discovered any noticeable