TABLE

E_0	Computed values					Experimental values
	$(EN_n)_p/(EN_n)_M$	N_{ep}	N_{eM}	N_{ep}/N_{eM}	$\frac{C_p}{C_M}=\left(\frac{N_{ep}}{N_{eM}}\right)^{\varkappa}$	
10^{14}eV	6.0	$3.8\cdot10^4$	$4.5\cdot10^3$	8.5	20 ($\varkappa=1.4$)	≈13
	5.33	$2.5\cdot10^4$	$4.2\cdot10^3$	6.0	12.4 ($\varkappa=1.4$)	
10^{16}eV	5.2	$6.7\cdot10^6$	$9.6\cdot10^5$	7.0	22 ($\varkappa=1.6$)	≈15
	5.30	$4.5\cdot10^6$	$9.0\cdot10^5$	5.0	13.0 ($\varkappa=1.6$)	
10^{18}eV	4.7	$9.2\cdot10^8$	$2.1\cdot10^8$	4.4	15 ($\varkappa=1.8$) 20 ($\varkappa=2.0$)	
	5.14	$6.2\cdot10^8$	$1.8\cdot10^8$	3.5	9.6 ($\varkappa=1.8$) 12.2 ($\varkappa=2.0$)	

particles with energies $E \approx 10^{18}$ ev, but the data on the magnitude of the barometric effect at sea level for showers from primary particles of the same energy[5] agree better with the calculations based on Landau's theory.

The results of the calculations of the altitude dependence depend slightly on the energy spectra of the created mesons, but are very sensitive to the amount of energy remaining with the nucleon; therefore the results present a weighty argument that in the region $E > 5 \times 10^{12}$ ev (at least, up to $E \approx 10^{14}$-10^{16} ev) the nucleon in collisions with nuclei of the atoms of the air loses, on the average, only about 1/3 of its energy. Conversely, for the region of very high energies ($E \approx 10^{18}$ ev), we conclude that a significant breaking down in the energy of the nucleon takes place in nuclear collisions.

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The Construction of a Phenomenological Scattering Matrix with Non-local Interaction

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IN recent years, detailed studies have been made on the possibility of introducing relativistically invariant cut–off form factors in the Schrödinger equation or in the equation of motion described in the Heisenberg representation.[1–3] It was shown that in all cases, multi–time equations become non–integrable, while the conditions of macroscopic causality are violated[2–4]. However, we can attempt to introduce the form factors directly into the relativistically invariant equation for the S-matrix which was obtained earlier in the theory with local interaction. Wataghin and Rayski [5] suggested the construction of the S–matrix, juxtaposing to each angle of the Feynman diagram (which corresponds to the theory with local interaction) a form factor while each internal line of this diagram is a causal function D^c or Δ^c [5].

Analytically, this suggestion can be formulated by writing the S matrix in the form*:

$$S = \sum_{n=0}^{\infty} \frac{(ie)^n}{n!} \tag{1}$$

$$\times \int_{-\infty}^{+\infty} P\{\varphi^*(x_1) A(x_1') \varphi(x_1'') \ldots \varphi^*(x_n) A(x_n') \varphi(x_n'')\}$$

$$\times F(x_1 x_1' x_1'') \ldots F(x_n x_n' x_n'') d^4(x_1 \ldots x_n'').$$

Actually, we can apply to the S matrix (1) the theorems of Wick on the reduction of the P derivative to the N derivative. The operator packets are expressed by D^c and Δ^c functions just as in the theory with local interaction. We cannot take the operator packets $\varphi^*(x_i)$ and (x_i) into account, since the corresponding matrix elements are equal to zero; this can easily be shown by a calculation in momentum space. However, it is not difficult to show that the S matrix (1) is not unitary, and, consequently, cannot be interpreted as the scattering matrix.**

It was shown in Ref. 6 that it is possible to satisfy the unitarity condition and macroscopic causality simultaneously if we add to the original Hermite "Lagrange function" $L(x_1 x_2 x_3)$ an anti-Hermitian part which is expressed in the form of an infinite series in powers of the interaction constant. It will be shown below that we can construct a unitary and macroscopically causal expression for the S matrix without the introduction of additional series, the physical meaning and convergence of which are not explicit. For this purpose, we write down the expression for the S matrix in the form:

$$S=\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int_{-\infty}^{+\infty}P^*\{H_1H_2\ldots H_n\}\,d^4(x_1x_2\ldots x_n),$$

$$H_i\equiv H(x_i)=-ie\int\varphi^*(x_1)\,A(x_2)\,\varphi(x_3)\times\sum_{i=1}^{3}F_i(x_1x_2x_3)\,\delta(x_i-x)\,d^4(x_1x_2x_3) \qquad (2)$$

(for the definition of the operator p^*, see below). If we write the expression for the operator H_i in the form

$$H_i=-ie\int\varphi_p^*(x_i)\,A_k(x_i)\,\varphi_q(x_i)\,\{\Phi_1(k+q;q)+\Phi_2(p;q)+\Phi_3(p;p-k)\}\,d^4(pqk),$$

$$\varphi_p^*(x_i)=\varphi^*(p)\,e^{-ipx_i};\quad\varphi_q(x_i)=\varphi(q)\,e^{iqx_i};\quad A_k(x_i)=A(k)\,e^{ikx_i};$$

[$\varphi^*(p)$; $\varphi(q)$; $A(k)$; Φ_i are the Fourier components of the operators $\varphi^*(x)$; $\varphi(x)$; $A(x)$ and the form factors F_i], then we can apply the same rule for the calculation of the matrix elements (2) as in the theory with local interaction. We thus obtain for the matrix element of the energy eigenvalue of the field A:

$$\int\langle k\,|\,A_k(x_1)\,A_k(x_2)\,|\,k_1\rangle\langle 0\,|\,P^*\{\varphi_{p_2}^*(x_2)\,\varphi_{q_1}(x_1)\}\,|\,0\rangle\langle 0\,|\,p^*\{\varphi_{p_1}^*(x_1)\,\varphi_{q_2}(x_2)\}\,|\,0\rangle \qquad (3)$$
$$\times\{\Phi_1(k+q_1;q_1)+\Phi_2(p_1;q_1)+\Phi_3(p_1;P-k)\}\,\{\Phi_1(k+q_2;q_2)+$$
$$+\Phi_2(p_2;q_2)+\Phi_3(p_2;p_2-k_2)\}\,d^4(x_1x_2p_1p_2q_1q_2).$$

In space–time regions

$$e_{ij}^2\equiv(\bar{x}_i-\bar{x}_j)^2-c^2(t_i^2-i_j)^2<0;\quad e_{ij}^2\gg\lambda^2$$

(λ is a constant which defines the characteristic dimension in the theory), the operator P^* does not differ from the known chronological operator P, since in these regions the operator P in (2) can be defined in a relativistically invariant fashion.[1] In microscopic regions $e_{ij}^2\lesssim\lambda$ one can generalize the definition of the operator P in relativistically invariant fashion. This generalization is, to a marked degree, arbitrary. Thus, if we set

$$P^*\{H_iH_j\}=\tfrac{1}{2}[H_iH_j]_+ +\tfrac{1}{2}\varepsilon_{ij}^*[H_iH_j]_-; \qquad (4)$$
$$[H_iH_j]_\pm=H_iH_j\pm H_jH_i;$$
$$\varepsilon_{ij}^*\equiv\varepsilon^*(x_i-x_j)=\begin{cases}+1 & \text{for } e_{ij}^2\leqslant 0;\quad (t_i-t_j)>0;\\ 0 & \text{for } e_{ij}^2>0;\\ -1 & \text{for } e_{ij}^2\leqslant 0;\quad (t_i-t_j)<0;\end{cases}$$

then

$$\langle 0\,|\,P^*\{\varphi_p(x_1)\,\varphi_q(x_2)\}\,|\,0\rangle=$$
$$=\tfrac{1}{2}\langle 0\,|\,[\varphi_p^*(x_1)\,\varphi_q(x_2)]_+\,|\,0\rangle$$
$$+\tfrac{1}{2}\varepsilon^*(x_1-x_2)\langle 0\,|\,[\varphi_p^*(x_1)\,\varphi_q(x_2)]_-\,|\,0\rangle$$
$$=-\tfrac{1}{2}i\,(1+\varepsilon^*(x_2-x_1))\,\Delta_q^+(x_2-x_1)\,\delta(p-q)$$
$$-\tfrac{1}{2}i\,(1-\varepsilon^*(x_2-x_1))\,\Delta_q^-(x_2-x_1)\,\delta(p-q)$$

and Eq. (3) is rewritten in the form:

$$\int\langle k\,|\,A_k(x_1)\,A_k(x_2)\,|\,k\rangle\,\tilde{\Delta}^c(x_2-x_1;k)\,\tilde{\Delta}^c(x_1-x_2;k)\,d^4(x_1x_2);$$

$$\tilde{\Delta}^c(x;k)=\tfrac{1}{2}(1+\varepsilon^*(x))\,\tilde{\Delta}^+(x;k),$$
$$+\tfrac{1}{2}(1-\varepsilon^*(x))\,\tilde{\Delta}^-(x;k);$$
$$\tilde{\Delta}^\pm(x;k)=\int\Delta^\pm(q)\,\{\Phi_1(k+q;q)+\Phi_2(q;q)+\Phi_3(q;q-k)\}\,e^{iqx}d^4q.$$

It follows from the properties of the form-factors

F_i that the commutator $[H_i H_j] \approx 0$ in the macroscopic regions $e^2_{ij} >> \lambda^2$, i.e., we can replace ϵ^*_{ij} by ϵ_{ij} in all regions $|e^2_{ij}| >> \lambda^2$ and, consequently, the function $\tilde{\Delta}^c_{ij}$ will contain only positive frequencies for $t_i > t_j$. In other words, the S matrix (2), in macroscopic regions of space–time, satisfies the conditions of causality. The unitarity of (2) can be shown in the same way as for the S matrix without the form factor, where the operator P^* can always be replaced by P[3].

We can also determine P^* by the normal product in correspondence with wick's theorem:

$$P^* \{\varphi^*_p(x_1)\varphi_q(x_2)\} = N\{\varphi^*_p(x_1)\varphi_q(x_2)\} + {}^1/_2\tilde{\Delta}^c(x_1 - x_2; p)\delta(p-q).$$

From the viewpoint of computation, such a definition of the operator P^* is much more satisfactory than (4).**

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*As an example, we consider the simple case of the interaction of the scalar charged and neutral fields φ and A.

**The unitary condition $SS^+ = 1$ in the second approximation of perturbation theory reduces to the requirement that all the matrix elements of the expression $S_2^+ + S_2 + S_1 S_1^+$ vanish. In a theory with a form function, this is impossible. We can establish this fact by considering, for example, the matrix element of the eigenvalue of the energy of the field A or φ[3]

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An Experiment on the Measurement of the Viscosity of an Expanded Liquid

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STUDY of the viscosity of expanded liquids would be of considerable interest for the development of the theory of viscosity. In the literature there is not indication of work in this field. We therefore set ourselves the task of investigating whether such measurements could be carried out. Reported below are some results of experiments we performed.

The substance chosen for investigation was benzene. The sample used had physical constants that agreed fully with handbook values. The benzene was put through an additional distillation just before the experiments for removal of absorbed water and for partial degasification, and the first and last fractions were rejected. Since we did not undertake to attain maximum expansion of the liquid complete degasification of the benzene was not carried out.

The method of Stokes was chosen for making the measurements as being easiest to apply. For manufacture of glass beads of the necessary diameter, we adopted the method of fusion of glass powder granules in the flame of a gas burner. This method was used by Bloomquist and Clark[1]; we modified and simplified their method to some extent. For our task we selected a few beads of 3C-5K molybdenum glass of diameter 0.05 to 0.06 mm, of accurately spherical shape and containing no gas inclusions. Since glass beads in benzene are hard to see, the ones chosen were coated with aluminum by evaporation in a vacuum. The density of the spheres was determined by the method of free flotation and was 2.268 g/cm^3 at 25° C.

The experiments on measurement of the viscosity of the expanded benzene were carried out in cylindrical ampoules, made of 3C-5K molybdenum glass, of inside diameter 6 mm. At the ends of the ampoules were intakes, 150 mm apart. The inside diameter of the intakes was about 1 mm. The ampoule was filled with benzene in such a way that after sealing, a bubble of gas remained in it. The glass bead was inserted into the ampoule