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Translated by J. L. Herson
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Force Fluctuations in an Electron Gas

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(Submitted to JETP editor November 15, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 386–387 (February, 1957)

FOR the calculation of the microscopic parameters of an electron gas, it is important to known the fluctuation of the random force distribution $w(\mathbf{F})$ which acts on the individual electron. A calculation of w (F) under the assumption that the a priori probability $\tau(\mathbf{r})$ of the particle distribution about an individual particle is a constant, i.e., that there is no correlation in the positions of the particles, was carried out by Holtsmark.[1] In this case a distribution function was obtained such that all its moments diverge, beginning with the second. On the other hand, it is clear that there will be an appreciable correlation only at distances $r_0 \sim e^2 / kT$, at which the mean kinetic energy of the particles is of the order of the value of the potential barrier. For $T \sim 10^5$, the value of r_0 is $\sim 10^{-8}$.

At large distances, there is virtually no correlation, since the gas is assumed to be in a state of statistical equilibrium with constant density. Since the relaxation length and the mean distance between particles are appreciably greater than r_0 under ordinary circumstances, we can neglect the interaction of the surrounding particles located in a sphere of radius r_0. In other words, the motion of the particles in the vicinity of the isolated one is the ordinary Rutherford scattering. The distribution of the particles near the particle under examination can be found by solving the kinetic equation

$$v_i \frac{\partial f}{\partial x_i} - \frac{1}{m} \frac{\partial u}{\partial x_i} \frac{\partial f}{\partial v_i} = 0 \qquad (1)$$

with boundary condition $f(\mathbf{r}, \mathbf{v}) = \exp(-mv^2/kT)$ for $u = 0$, where $u(\mathbf{r})$ is the potential established by the isolated electron. This equation has the integral $f = \Omega(mv^2/2 + u)$ where Ω is an arbitrary function. Consequently, solving this equation, we get

$$\tau(\mathbf{r}) = \exp(-e^2/rkT). \qquad (2)$$

It is of interest to note that in the limiting case of very strong interaction, for which the particles out to the distance $<< r_0$ are in equilibrium with the field of the isolated electron, $\tau(\mathbf{r})$ is obtained in the same fashion.

Making use of the Markov procedure:

$$w(\mathbf{F}) = \frac{1}{8\pi^3} \int \exp(-i\rho\mathbf{F})\, A(\rho)\, d\rho,$$

$$A(\rho) = \lim_{N \to \infty} \left[\int \exp(-e^2 \rho \mathbf{r}/r^3)\, \tau(\mathbf{r})\, d\mathbf{r} \right]^N,$$

we can show that for large F the function w (F) falls off more rapidly than any power of F. Consequently, the moments of all orders of w (F) must exist. These can be computed by expanding the eigenfunction $A(\rho)$ in a series in powers of ρ. For example,

$$\langle \mathbf{F}^2 \rangle = \int \mathbf{F}^2 w(\mathbf{F})\, d\mathbf{F} = -(\Delta_\rho A(\rho))_{\rho=0} = 4\pi e^2 nkT.$$

In a similar fashion, all the moments of even order can be calculated. [Moments of odd order are equal to zero because of spherical symmetry of w (F).] The value obtained can serve for a rough estimate of the coefficient of viscosity:

$$\mu \sim V \overline{\langle \mathbf{F}^2 \rangle}/m\bar{v}^2 = V \overline{4\pi e^2 n/kT}.$$

An exact calculation ought to take into account the correlation of force fluctuations at different points of space, for which it is necessary to compute $< F_i(x) F_k(x') >$. However, the consideration just given shows that the principal contribution to the viscosity, the diameter, the relaxation length for particles with long range interaction will not be given by an account of well regulated, averaged forces (pair collisions, interaction with vibrations) since this contribution is proportional to the first moment of w (F) and the interaction with the fluctuations of the electric field. For these quantities, a consideration of the fluctuation scattering is more important than consideration of the well regulated deceleration.

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Translated by R. T. Beyer
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