Effect of a Longitudinal Magnetic Field on the Multiple Scattering of Particles

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LET there be a homogeneous magnetic field in a scattering medium. Let us consider how an infinitely narrow pencil, generally speaking, of relativistic particles, directed parallel to the magnetic induction vector, will be scattered. As is well known, the magnetic field does not change the energy of the particles. Thus, if we neglect the energy losses of the particles in the scattering, we can limit ourselves to the case in which all the particles have the same energy. We take the axis of the pencil to be the z axis. We assume that the properties of the medium do not depend on x and y, and that, in the mean, the angle between the velocities of the particles and the z axis is small. This problem has been solved by Fermi[1] in the case in which the medium is homogeneous and the magnetic field is absent.

Let a current of particles arise at the origin. For $z > 0$, the equation of the process under consideration has the form:

$$\frac{\partial n}{\partial z} + \alpha\frac{\partial n}{\partial x} + \beta\frac{\partial n}{\partial y} + L\left(\beta\frac{\partial n}{\partial \alpha} - \alpha\frac{\partial n}{\partial \beta}\right) + \frac{n}{l(z)} = \frac{a^2(z)}{4}\left[\frac{\partial^2 n}{\partial \alpha^2} + \frac{\partial^2 n}{\partial \beta^2}\right] \qquad (1)$$

with the boundary condition

$$n = A\delta(x)\,\delta(y)\,\delta(\alpha)\,\delta(\beta)$$

for $z = 0$, where α and β are the projections of the angular deviations of the particle velocity from the z axis on the planes zx and zy; n is the particle distribution function in x, y, z, α, β; A is the ratio of the particle flux through the plane $z = 0$ to its velocity; $1/l(z) = (mT/p) + \Sigma N_i(z)\,\sigma_i$ $N_i(z)$ is the number of atoms of type i per unit volume of the medium; σ_i is the absorption cross section of the particles by atoms of type i; m is the rest mass, p is the momentum, T is the characteristic lifetime of the particles relative to random decay. Let the cross section of single scattering of particles on the atoms of type i have the form $d\sigma_i = R_i(\theta)\, d(\theta$. Then

$$a^2(z) = \sum_i N_i(z)\int R_i(\theta)\,\theta^2 d\theta.$$

Finally, $L = eB/cp$, B = magnetic induction, e = electric charge of the particle, c = velocity of light.

Equation (1) is solved by a decomposition of the function

$$F = A^{-1} n \exp\left\{-\int_0^z l^{-1}(\zeta)\,d\zeta\right\}$$

into a Fourier integral in x, y, α, β; as a result,

$$F = \frac{\exp\left\{\dfrac{-\gamma_0 r^2 + 2\gamma_1 r\theta\cos(\varphi_0 - \varphi) + \gamma_2 L r\theta\sin(\varphi_0 - \varphi) - \gamma_2\theta^2}{\gamma_0\gamma_2 - \gamma_1^2 - \gamma_2^2 L^2/4}\right\}}{\pi^2\left[\gamma_0\gamma_1 - \gamma_1^2 - \gamma_2^2 L^2/4\right]}, \qquad (2)$$

$$r\cos\varphi_0 = x, \quad r\sin\varphi_0 = y, \quad \theta\cos\varphi = \alpha, \quad \theta\sin\varphi = \beta,$$

$$\gamma_0 = \int_0^z a^2(\zeta)\,d\zeta, \quad \gamma_1 = \frac{1}{L}\int_0^z a^2(\zeta)\sin L(z-\zeta)\,d\zeta,$$

$$\gamma_2 = \frac{4}{L^2}\int_0^z a^2(\zeta)\sin^2\frac{L(z-\zeta)}{2}\,d\zeta.$$

The angular distribution without reference to the transverse displacement, as is easy to understand, does not depend on L:

$$\int\!\!\int_{-\infty}^{+\infty} n\,dx\,dy = \frac{A}{\pi\gamma_0}\exp\left\{-\frac{\theta^2}{\gamma_0} - \int_0^z\frac{d\zeta}{l(\zeta)}\right\}. \qquad (3)$$

The spatial distribution without reference to the angles has the form:

$$\int\!\!\int_{-\infty}^{+\infty} n\,d\alpha\,d\beta = \frac{A}{\pi\gamma_2}\exp\left\{-\frac{r^2}{\gamma_2} - \int_0^z\frac{d\zeta}{l(\zeta)}\right\}. \qquad (4)$$

We note two effects, produced by the magnetic field in the narrow beam. The principal effect consists of the fact that under otherwise equal conditions the pencil is narrower in the magnetic field than without it. This follows from the fact that

$$\frac{4}{L^2}\sin^2\frac{L(z-\zeta)}{2} < (z-\zeta)^2,$$

and means $\gamma_2(L) < \gamma_2(0)$ if $L \neq 0$. This is also understandable because in the intervals between collisions in the magnetic field, the scattered particle moves in a helical path and departs from the axis of the pencil less than in the absence of the field, when it moves along a straight line.

The other effect is the following. In the absence of the magnetic field, the mean value of the angle $\varphi - \varphi_0$ is equal to zero. In this case we have a radial "polarization" of the pencil. In the magnetic field,

$$\overline{\varphi - \varphi_0} = -\operatorname{arctg}(\gamma_2 L/2\gamma_1).$$

This means that in the magnetic field, the axis of polarization is turned by this angle.

1 B. Rossi and K. Gresisen, *The interaction of cosmic rays with matter*, p. 45 (Russian translation).

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s-d Exchange in Ferromagnetic Metals

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LANDAU first pointed out the possibility of the fundamental phenomenon of "submagnetization" of the external *s*-electrons of a ferromagnetic crystal in the exchange "field" of the internal *d*-electrons of the atoms, as a result of exchange between the electrons under consideration (*s-d* exchange).

Vonsovskii developed in detail the theory of *s-d* exchange. His results contain exchange integrals between *s*- and *d*-electrons of a single atom (I_0) and of neighbor atoms (I), and also the transport integral of an *s*-electron. In the present state of the theory, these integrals cannot be calculated. Consequently, as the same author pointed out, a quantitative comparison with experiment is not possible; the theory gives only a qualitative explanation of many phenomena[1].

The aim of the present work is to show that the integrals mentioned can be found by an empirical method, and that substitution of their values in Vonsovskii's relations for pure ferromagnetic metals[1,2] gives satisfactory agreement with experiment.

We make the simple and natural assumption that the *s-d* exchange interaction depends on the interelectronic distances and on the number of electrons participating in the interaction. As an example, we consider the approximation of tight binding of the *s*-electron[1-3]. In this case, an *s*-electron spends most of the time in an *s*-state at distance R_s from the nucleus of some atom, and the *d*-electrons spend most of the time at a distance R_d from nuclei of atoms, where R_s and R_d are the radii of the *s*- and *d*-shells, respectively, of an isolated atom. Under these conditions the minimum distances between nearest *s*- and *d*-electrons in a single atom and in neighbor atoms (along a straight line connecting the nuclei of the atoms) are, respectively, R_s-R_d and r_1-R, where $R = R_s + R_d$, r_1 is the distance between an atom and the atoms nearest to it (first c.s., c.s. = coordination sphere), $(r_2 - R)$ is the distance between an *s*-electron and the next nearest *d*-electrons, r_2 is the distance between an atom and the next nearest atoms (second c.s.). In order to preserve the accuracy with which r_1 and r_2 are usually measured, the magnitude of R is computed with the same accuracy (to the fourth decimal place) by our relation[4]; the values of R thus obtained differ by no more than 1% from those calculated by Slater's method[5].

According to the sign of $r_i - R$, the metals divide into two groups: Co falls in group 1 $[(r_i/R) < 1]$, Ni (Dy and Er) in group 2 $[(r_i/R) > 1]$, Fe(Gd) with respect to its first c.s. in group 1, where $i = 1$ for Ni (second c.s. not taken into account because of the large distance of the next-nearest atom: $r_2/r_1 = 1.414$), $i = 1, 2$ for Co and Fe (second approximation); $i = 1, 2$ for Co ($r_2 \approx r_1$), $i = 1$ for the other metals (first approximation); i = number of the c.s.

We postulate that the exchange integral I is

$$I = 1 \pm \sum_i \Delta E_i; \quad \Delta E_i = 0.641\, n_i (r_i - R); \tag{1}$$

here and hereafter, the upper sign refers to group 1, the lower to group 2. We denote by I_1 the integral I in first approximation.