sists of the fact that under otherwise equal conditions the pencil is narrower in the magnetic field than without it. This follows from the fact that

$$\frac{4}{L^2}\sin^2\frac{L(z-\zeta)}{2} < (z-\zeta)^2,$$

and means $\gamma_2\,(L) < \gamma_2\,(0)$ if $L \neq 0$. This is also understandable because in the intervals between collisions in the magnetic field, the scattered particle moves in a helical path and departs from the axis of the pencil less than in the absence of the field, when it moves along a straight line.

The other effect is the following. In the absence of the magnetic field, the mean value of the angle $\varphi - \varphi_0$ is equal to zero. In this case we have a radial "polarization" of the pencil. In the magnetic field,

$$\overline{\varphi-\varphi_0} = -\operatorname{arctg}(\gamma_2 L/2\gamma_1).$$

This means that in the magnetic field, the axis of polarization is turned by this angle.

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Translated by R. T. Beyer
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s-d Exchange in Ferromagnetic Metals

F. M. GAL'PERIN

(Submitted to JETP editor October 22, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 381-382
(February, 1957)

LANDAU first pointed out the possibility of the fundamental phenomenon of "submagnetization" of the external *s*-electrons of a ferromagnetic crystal in the exchange "field" of the internal *d*-electrons of the atoms, as a result of exchange between the electrons under consideration (*s-d* exchange).

Vonsovskii developed in detail the theory of *s-d* exchange. His results contain exchange integrals between *s*- and *d*-electrons of a single atom (I_0) and of neighbor atoms (I), and also the transport integral of an *s*-electron. In the present state of the theory, these integrals cannot be calculated. Consequently, as the same author pointed out, a quantitative comparison with experiment is not possible; the theory gives only a qualitative explanation of many phenomena[1].

The aim of the present work is to show that the integrals mentioned can be found by an empirical method, and that substitution of their values in Vonsovskii's relations for pure ferromagnetic metals[1,2] gives satisfactory agreement with experiment.

We make the simple and natural assumption that the *s-d* exchange interaction depends on the interelectronic distances and on the number of electrons participating in the interaction. As an example, we consider the approximation of tight binding of the *s*-electron[1-3]. In this case, an *s*-electron spends most of the time in an *s*-state at distance R_s from the nucleus of some atom, and the *d*-electrons spend most of the time at a distance R_d from nuclei of atoms, where R_s and R_d are the radii of the *s*- and *d*-shells, respectively, of an isolated atom. Under these conditions the minimum distances between nearest *s*- and *d*-electrons in a single atom and in neighbor atoms (along a straight line connecting the nuclei of the atoms) are, respectively, R_s-R_d and r_1-R, where $R = R_s + R_d$, r_1 is the distance between an atom and the atoms nearest to it (first c.s., c.s. = coordination sphere), $(r_2 - R)$ is the distance between an *s*-electron and the next nearest *d*-electrons, r_2 is the distance between an atom and the next nearest atoms (second c.s.). In order to preserve the accuracy with which r_1 and r_2 are usually measured, the magnitude of R is computed with the same accuracy (to the fourth decimal place) by our relation[4]; the values of R thus obtained differ by no more than 1% from those calculated by Slater's method[5].

According to the sign of $r_i - R$, the metals divide into two groups: Co falls in group 1 [$(r_i/R) < 1$], Ni (Dy and Er) in group 2 [$(r_i/R) > 1$], Fe(Gd) with respect to its first c.s. in group 1, where $i = 1$ for Ni (second c.s. not taken into account because of the large distance of the next-nearest atom: $r_2/r_1 = 1.414$), $i = 1, 2$ for Co and Fe (second approximation); $i = 1, 2$ for Co ($r_2 \approx r_1$), $i = 1$ for the other metals (first approximation); i = number of the c.s.

We postulate that the exchange integral I is

$$I = 1 \pm \sum_i \Delta E_i; \quad \Delta E_i = 0.641\, n_i\,(r_i - R); \tag{1}$$

here and hereafter, the upper sign refers to group 1, the lower to group 2. We denote by I_1 the integral I in first approximation.

The exchange integral I_0 is similarly expressed:

$$I_0 = -[1 - 0.641\,(R_s - R_d) \pm 0.065\,(N_d - 2/e)]; \qquad (2)$$

$$R_s - R_d = 1.56\{1 - 0.13\,[eN_d/(N_d - 1) \mp (N_d/2 - 1)/e]\}, \qquad (2')$$

where N_d is the number of unpaired d-electrons in the atom, $e = 1$ for Co and Ni, $e = 2$ for Fe, $e = 7$ for Gd. In the tight binding approximation being considered, we neglect the transport integral of the s-electron. Upon evaluating (2) and (2′), we find $I_0 = \pm 0.13\,eN_{d(f)}/N_{d(f)} - 1$; the minus sign applies to Fe, Ni and Co, the plus to Gd.

By substituting (1) and (2) in Vonsovskii's relation (1.13)[1], we find the atomic magnetic moments

$$m/M_B = N_d + N_f + 0.15\,(I_0/I - 4)n_s \quad (n_s = 2). \qquad (3)^*$$

By reversing the sign before ΔE in (1) and substituting (1) in our relation

$$m_p/M_B = N_{d(f)} - n_s + \sum_i (1 \mp \Delta E_i); \qquad (3')$$

$$i = 1 \text{ for Ni and Fe}, i = 1, 2 \text{ for Co},$$

we find the paramagnetic atomic magnetic moments m_p. By differentiating m with respect to pressure P, and by assuming that under hydrostatic compression only the interatomic distances and the moments m change, we find

$$\frac{1}{m}\frac{dm}{dP} = \pm\, 0.0641\,\frac{xI_0}{mI^2}\,(n_1r_1 - n_2r_2), \qquad (4)$$

where x is the compressibility. It follows from (4) that $dm/dP < 0$ for Gd and Fe and > 0 for Ni. We furthermore get for exchange energy of the "3/2-power law" of Bloch and Möller

$$A = kn^2N_d^2(N_f)I_1/e^2. \qquad (5)$$

Here $e = m_0/M_B$ for group 1, $e = (m_0/M_B) + 1$ for group 2, where m_0 is the integral part of the atomic moment m (0, 1, 2 and 7 M_B for Ni, Co, Fe and Gd respectively).

Properties of pure ferromagnetic metals.

Metal		Gd	Ni	Co	Fe
$n_1(r_1,$ Å)		6(3.561)	12(2,4868)	6(2.499)	8(2.4777)
$n_2(r_2,$ Å)		6(3.629)		6(2.507)	6(2,8610
R, Å		3,566	2,4082	2.5382	2.7332
$N_{d(f)}; n_{d(f)}$		1(7);1(7)	2; 8	3; 7	4; 6
$\Delta E_1(\Delta E_2)$		—0,02(0,24)	0.60	—0.15(—0,12)	—1.31(0.49)
$\sum_i \Delta E_i$		0,22	0.60	—0.27	—0.82
$I(I_1); I'/K$		1,22(0,98)	0.40(0.40) ;1	0,73(0.73); 1	**0.18** (0.69); 1
$I_0; I'_0/K$		1,06; —1	—0.260; —1	—0.195; —1	—0.347; — 1
m/M_B	calc.	7,06	0.605	1.720	2,220
	expt.	7,10	0.605	1.715	2,217
m_p/M_B	calc.		1,6	3,27	3,31
	expt.		1,6	3,27	3.37
$\frac{1}{m}\frac{dm}{dP}\cdot 10^7\,\frac{\text{cm}^2}{\text{kg}}$	calc.		30	—1	—5
	expt.		60	—	—6
A/k	calc.	5	230	946	177
	expt.	4.5[6]	226[8], 220[7]	—	207[7], 183[8]
Θ°K	calc.	266	642	1385	1043
	expt.	289	631	1393	1043

The Curie point is calculated by a relation similar to (1.8) of Ref. 1:

$$\Theta = 2nA_{dd}\,[A_{sf}]\,/\,1 - \delta\,/\,nI'[nI_1])$$

(the quantities in square brackets are those for Gd, determined, according to our hypothesis, by *s-f* exchange), where $A_{sf} = -\frac{1}{2}(1 - N_f)(I_0 + nI_1)$, $A_{dd} = \frac{1}{2}(1 + N_d^2)(I'_0 + nI')$, $\delta = 0.15 \times (I_0 - 4I_1)^2 n_s \div I_1 n_{d(f)}$, cf. the Table and (22.1) of Ref. 2.

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* The same value of m is given by our relation m/M_B

$$m\,/\,M_B = N_{d(f)} - n_s + {}^1\!/_2\,(N_d \mp 1)I.$$

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Translated by W. F. Brown, Jr.
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Quasi-Magnetic Interaction of the Spin of a Nucleon with the Rotation of the Nucleus

D. F. ZARETSKII AND A. V. SHUT'KO

(Submitted to JETP editor August 31, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 370–371 (February, 1957)

BOHR and Mottelson[1,2] have interpreted the rotational levels of nuclei with spin 1/2 by starting out from the assumption that the Ω-projection of the total angular momentum of nucleons on the axis of symmetry of the nucleus is an integral of the motion. We can conceive of a different interpretation of the rotational levels of the nuclei, however, if we start from the following two assumptions: 1) there are absent from these nuclei the ${}^2\Sigma$ states which are considered according to a coupling scheme which corresponds to the case of *b* coupling according to Gund.[3] Then in the first approximation, the levels with total momenta $I = K \pm 1/2$ are degenerate (K is the rotational quantum number); 2) this degeneracy is taken into account by the introduction into the Hamiltonian of Bohr and Mottelson of an interaction of the type

$$H_{Rs} = -(\lambda\,/\,mc^2)\,\mathrm{s}[(\nabla U \mathrm{v}_{\mathrm{coll}}],$$

where λ is the dimensionless phenomenological constant which has the same meaning and value as in the usual nuclear spin–orbit coupling (see, for example, Ref. 4); s is the spin vector of the nucleon, U (r) = self-consistent potential of the nucleus, mc^2 = rest energy of the nucleon, $\mathrm{v}_{\mathrm{coll}}$ = velocity with which the nucleon takes part in the collective motion. Introduction of the interaction (1) into the Hamiltonian of the system can be justified if we start out from the model of independent particles which move in a rotating self-consistent field.[5,6]

First, we shall make clear the meaning of $\mathrm{v}_{\mathrm{coll}}$. Consideration of the rotation of the field, as is well known, leads to the appearance in the Hamiltonian of the system (which is written relative to the rotating system of coordinates) of a perturbation of the form $H' = -\hbar\,\omega_x L_x$, where ω_x is the frequency of rotation of the field U(r) relative to the x–axis, perpendicular to the axis of symmetry of the nucleus, L_x is the projection of the orbital momentum of the nucleons on the corresponding axis. The unperturbed Hamiltonian H_0 describes the motion of particles in the non–rotating field. We can write the perturbed wave functions, with accuracy up to first order in ω_x, in the form

$$\psi = \psi_0 + i\omega_x\psi_1, \qquad (2)$$

where ψ_0 is the unperturbed wave function, which is real for the Σ state; ψ_1 is also a real function. Then, with the same accuracy as (2), we get an expression for the current density:

$$\mathrm{j} = \psi_0^2\,\mathrm{v}_{\mathrm{coll}}, \quad \mathrm{v}_{\mathrm{coll}} = (h\omega_x\,/\,m)\,\mathrm{grad}\,(\psi_1\,/\,\psi_0). \qquad (3)$$

We can interpret this expression that for a rotation of the nucleus, each nucleon acquires an additional irrotational velocity $\mathrm{v}_{\mathrm{coll}}$.

We now make clear the meaning and origin of the interaction (1). We represent the wave function (2), with corresponding accuracy, in the form

$$\psi = \psi_0 \exp\{i\omega_x\psi_1\,/\,\psi_0\}. \qquad (4)$$

Then the Schrödinger equation in the new representation has the form